How to write a mathematical text – tips and tricks

How do you write a good mathematical text? In general terms, the same rules apply as when writing any text – the aim is to express oneself clearly an exact, while keeping readability as high as possible. If one succeeds, the text will be less misunderstood, and will also convince the reader to greater extent. Also, if you force yourself to express yourself in a very exact manner,it will be easier to spot flaws in your argument. There are however a few things to think about specifically when writing mathematics.

Use complete sentences

One way to make the readability of your text to sky-rocket is to **use complete sentences**. For instance, it is easier to understand

> Since P is on the line ℓ, there exists an $t \in \mathbb{R}$ so that $P = Q + t\mathbf{n}$.

than

> Line $\rightarrow P = Q + t\mathbf{n}$

Note that **formulas**, even those that are written on their own lines, **are a part of the text**. If they end a sentence, they should hence be ended by a period. The same applies for commas.

Example: If x is greater than zero, we have

$$x^3 > 0,$$

but if x is negative, we must instead have

$$x^3 < 0.$$

Define your variable and your notation

A good rule of thumb is to **everytime you introduce a new object in your text**, **explain which type** of object it is. A cliché like

> Let x be an arbitrary real number.

or even

> Let $x \in \mathbb{R}$.

suffices. When a symbol emerges from nowhere, it is up to the reader to guess if it is a vector, matrix, set or something completely different. If the writer is lucky, the reader will understand it, possibly only with a lot of effort. If the writer is unlucky, the reader might instead misunderstand, and interpret a completely correct reasoning as jibberish.

On the same note, it is always good to **explain notation which is not standard**. If one is to reason about triangles and their circumferences, for instance, it can be a good idea to start with a phrase like this

For a triangle T, we let $o(T)$ denote its circumference.

This should of course not be exaggerated, then the language will get unnecessarily cluttered, and lose its flow. It is for instance totally OK to write

Let $\sum_{i=1}^{n} a_i \mathbf{v}_i$ be a linear combination of $\mathbf{v}_1, \ldots \mathbf{v}_n$

without explicitly pointing out that $\mathbf{v}_1, \ldots \mathbf{v}_n$ are vectors and that $a_1, \ldots, a_n$ are members of $\mathbb{R}$ (or for that matter use the symbol $\mathbb{R}$ without explaining that it denotes the set of real numbers). There are no absolute rights and wrongs here – a lot is up to style, experience and common sense.

A last hint is that it is totally allowed to **define and use own notions** in order to make the text easier to read. E.g., one can state that all vectors $\mathbf{v}$ for which all elements v_i are greater than zero will be called 'positive', although the claim "$\mathbf{v} > 0$" formally does not have a meaning. Not having to explain that each time such a vector is used in the text can help a lot to make the text easy to read.

Divide into, or bring back to, special cases

This is maybe rather a strategy for problem solving, but it is a good idea also when writing down an argument to **divide ones reasoning** in a few special cases. It is often easier to find a simple and concise argument for each special case that one that applies to each case. In order to show that $x^2 > 0$ for each $x \neq 0$, it is probably the most convenient to first treat the case $x > 0$ and to then move on to the case $x < 0$.

Another suggestion in the same genre is that it is sometimes obvious that a **simplifying argument** can be made. Maybe it is obvious that a function $f(\mathbf{v})$ only gets larger as $|\mathbf{v}|$ grows – if we are trying to prove the inequality

$$f(\mathbf{v}) \leq 1 \text{ for all } \mathbf{v} \text{ with } \mathbf{v} \leq 1,$$

it is then sufficient to only concentrate on vectors with norm exactly one. In situations like this, when 'estetic', but not really restricting, assumptions are made, one can use the acronym "**WLOG**"[1] – Withoug Loss of Generality. This should however be done with caution – you really need to be sure that the assumption that you make is really not missing something.

Even is this is more relevant when writing a longer text than for an assignment in a course on an undergraduate level, it can be nice to now that the essence, or a technical or formula-heavy part, of an argument can be "outsourced" to a ***lemma***. It may for instance be easier to first show that a formula holds for all symmetric matrices A, and then apply it to one (or more) special matrix $B_{i,j}^k$ in the actual reasoning.

Use figures, but in the right way

Often, especially in such a geometrical subject as calculus, it can be good to include a figure to help ones reasoning. If you choose to include a figure, think it through. Does it show all the geometrical aspects I want to use in my argument? Are all objects in them correctly named?

[1] In german, one instead uses oBdA "ohne Beschränkung der Allgemeinheit".

It is however important to **not only** use a figure. Arguments of the type " as can be seen in the figure" can suffice, but it is much safer to use a formal mathematical argument. The reason for this is that one can make wrong conclusions from a figure. It may not always be perfectly drawn, and things are not always as they seem. If we for instance plot the function

$$f(t) = 1 + \frac{10^{-64}}{t}$$

in a reasonably large region around 0, we would have to use a ridicolously dense sampling to see that $\lim_{t\to 0} f(t) \neq 1$ (try it!). Hence, use figures as a complement to showcase your arguments, not as arguments themselves.

Software

In this course, it is of course totally adequate to turn in your solutions as photos of handwritten text. It is namely not trivial to typeset a mathematical texts. There is a built in equation editor in word, but it is not very simple to use when writing something slightly more advanced.

The best way to typeset mathematics is instead to learn to write **LaTeX**. For a programmer, this does not need to be super hard – in principle, you first 'code' your text and then let LaTeXautomatically typeset it (by 'compiling' the code). There are many programs that facilitate the coding available for free, for instance by coloring formulas. Two examples are Kyle and texmaker, but there are a lot of other examples. An interesting alternative, especially when making a collaborative work, is to use the web-based tool `www.overleaf.com`. With this tool, several people can work on the same document at the same time. Also, there are drop-down menues (in the "Rich Text" mode) to typeset math (although the power of LaTeXonly shines through when you learn to typeset it without such a menu).

To learn writing in LaTeXis however not something that you do in five minutes, and is absolutely no requisite for passing this course. For those that are interested, the intro "The not so short introduction to LaTeX" by Tobi Oetiker, which is available for download at `https://tobi.oetiker.ch/lshort/lshort.pdf`, is highly recommended. To facilitate finding which one of the hundreds (if not thousands) of shorthands to typeset a certain symbol, the service detexify, found at `http://detexify.kirelabs.org/classify.html` (there is also an app for android) is an excellent help.