Tentamen MVE255

Mathematical sciences, Chalmers University of Technology.

Date:	–
Examiner:	Axel Flinth
Allowed aids:	Any.
Grade limits:	20 points for the grade 3. 30 points for the grade 4 40 points for the grade 5.

There are in total 50 points to collect.

Calculations and arguments should be presented in full. **Only providing an answer will normally not be rewarded with points.** Solutions may be written in Swedish or English (or German).

If you use any external tool or resource, you should reference it. See the Canvas Page for more information. Not referencing properly may result in point deduction.

The exam consists of **eight (8)** problems. They are distributed over **four (4)** sheets.

Good luck!

PRACTICE EXAM

Solutions will be posted on October 19th.

Problem 1 (6p)

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined through

$$f(x, y) = xe^{xy}, \quad \langle x, y\rangle \in \mathbb{R}^2.$$

(a) Calculate the gradient ∇f of f. (2p)

(b) Calculate the Hessian f'' of f. (2p)

(c) Evaluate f, ∇f and f'' in $< 0, 0 >$. (2p)

Problem 2 (7p)

Calculate $\iint_T xy \, \mathrm{d}x\mathrm{d}y$, where T is the triangle with corners $< 0, 0 >$, $< 1, 1 >$ and $< 2, 0 >$.

Problem 3 **(6p)**

Calculate the center of mass of the part of the unit disk which lies in the region $\{\langle x, y\rangle \mid x, y \geq 0\}$.

Problem 4 **(7p)**

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined through

$$f(x, y) = xe^y.$$

The optimization problem

$$\max f(x, y) \text{ subject to } x + y = 1$$

has a unique solution (this does not need to be shown).

(a) Find the solution. (4p)

(b) Does the corresponding minimization problem

$$\min f(x, y) \text{ subject to } x + y = 1$$

have a solution? (3p)

Problem 5 **(6p)**

A circle has a known center in $\langle 0, 1\rangle$. In order to determine it fully, we measure the position of one point $\mathbf{p}$ on its perimiter. The measurement reads

$$\mathbf{p} =< 1, 1 > .$$

Each coordinate of the measurement has an error, which is smaller than 0.1 length units. Use the Schrankensatz to give an estimate of the radius of the circle, with error bounds.

Problem 6 **(6p)**

Let $\alpha \in \mathbb{R}$ be a parameter. Consider the curves given by the parametrisations

$$\gamma_\alpha(t) = \begin{bmatrix} t\cos(\pi\alpha t) \\ t\sin(\pi\alpha t) \\ t^2(1+2\sin^2(t)) \end{bmatrix}, \quad t \in [0,1]$$

and the vector field

$$\mathbf{v}(x,y,z) = \begin{bmatrix} 0 \\ 0 \\ z\cos(z)+\sin(z) \end{bmatrix}.$$

(a) Prove that $\mathbf{v}$ is conservative, and calculate a potential of $\mathbf{v}$. (3p)

(b) Prove that the curve integral of $\mathbf{v}$ is the same along all the curves. I.e. prove that

$$\int_{\gamma_\alpha} \mathbf{v} \cdot d\mathbf{s}$$

does not depend on α. (3p)

Problem 7 **(6p)**

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function

$$f(x,y) = \begin{cases} y & \text{if } y = x^2 \\ 0 & \text{else.} \end{cases}$$

(a) Is f partially differentiable in $\langle 0,0\rangle$? (3p)

(b) Is f continuous in $\langle 0,0\rangle$? (3p)

Problem 8 **(6p)**

Let $\mathbf{v} : \mathbb{R}^2 \setminus \{\langle 0,0\rangle\} \to \mathbb{R}^2$ be the vector field

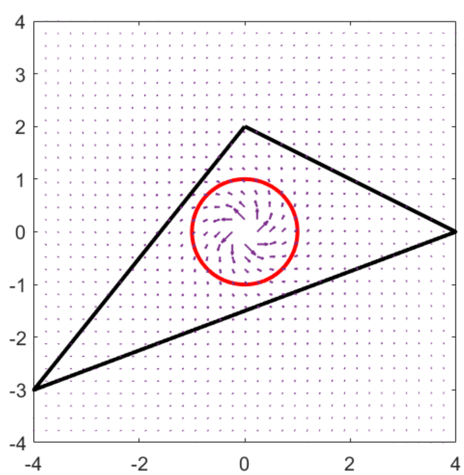
$$\mathbf{v}(x,y) = \begin{bmatrix} \frac{x-y}{x^2+y^2} \\ \frac{y+x}{x^2+y^2} \end{bmatrix}.$$

(a) Calculate the value of (1p)

$$\frac{\partial v_1}{\partial y} - \frac{\partial v_2}{\partial x}$$

(b) Calculate the line integral of $\mathbf{v}$ along the unit circle, traversed counter-clockwise. (2p)

(c) Now consider a triangle T with two corners corners $\langle 0,2\rangle$, $\langle 4,0\rangle$ and a third corner $\langle a,b\rangle$ which is chosen so that the triangle contains the unit circle, as in the figure. Calculate the line integral of $\mathbf{v}$ along the border of T, again traversed counter-clockwise. (3p)



Tip Neither of the line integrals are equal to zero.