MVE045 W1-RÖ12

mängder, tal, funktioner

ADAMS Problems P.1

Solve the equations in Exercises 27–32.

27. $|x| = 3$

28. $|x - 3| = 7$

29. $|2t + 5| = 4$

30. $|1 - t| = 1$

31. $|8 - 3s| = 9$

32. $\left|\frac{s}{2} - 1\right| = 1$

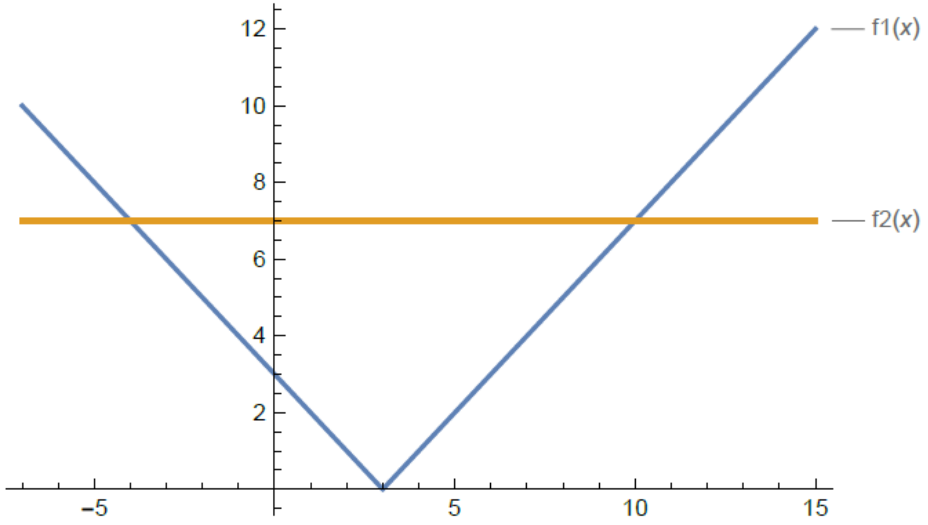
In Exercises 41–42, solve the given inequality by interpreting it as a statement about distances on the real line.

41. $|x + 1| > |x - 3|$

42. $|x - 3| < 2|x|$

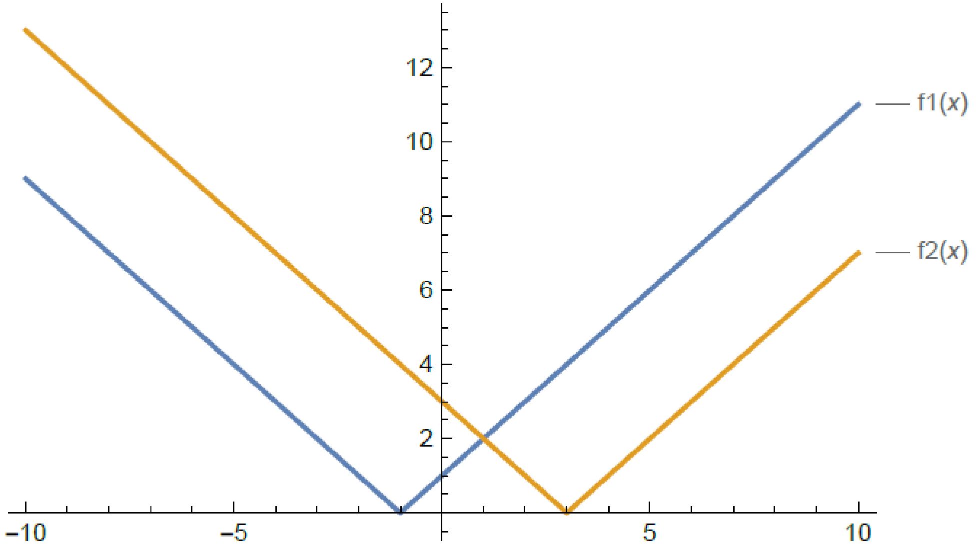
P.1:28 graphical solution

```
f1[x_] := Abs[x - 3];
f2[x_] := 7;
Plot[{f1[x], f2[x]}, {x, -7, 15}, PlotLabels → "Expressions"]
```



P.1:41 graphical solution

```
f1[x_] := Abs[x + 1];
f2[x_] := Abs[x - 3];
Plot[{f1[x], f2[x]}, {x, -10, 10}, PlotLabels → "Expressions"]
```



ADAMS Problems P.2

In Exercises 31–32, find equations for the lines through P that are (a) parallel to and (b) perpendicular to the given line.

31. $P(2, 1)$, $y = x + 2$

32. $P(-2, 2)$, $2x + y = 4$

41. By calculating the lengths of its three sides, show that the triangle with vertices at the points $A(2, 1)$, $B(6, 4)$, and $C(5, -3)$ is isosceles.

44. Find the coordinates of the midpoint on the line segment P_1P_2 joining the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$.

In Exercises 47–48, interpret the equation as a statement about distances, and hence determine the graph of the equation.

47. $\sqrt{(x-2)^2 + y^2} = 4$

48. $\sqrt{(x-2)^2 + y^2} = \sqrt{x^2 + (y-2)^2}$

ADAMS Problems P.3

Describe the regions defined by the inequalities and pairs of inequalities in Exercises 9–16.

9. $x^2 + y^2 > 1$

10. $x^2 + y^2 < 4$

11. $(x+1)^2 + y^2 \leq 4$

12. $x^2 + (y-2)^2 \leq 4$

13. $x^2 + y^2 > 1, \quad x^2 + y^2 < 4$

14. $x^2 + y^2 \leq 4, \quad (x+2)^2 + y^2 \leq 4$

15. $x^2 + y^2 < 2x, \quad x^2 + y^2 < 2y$

16. $x^2 + y^2 - 4x + 2y > 4, \quad x + y > 1$

In Exercises 35–38, write an equation for the graph obtained by shifting the graph of the given equation as indicated.

35. $y = 1 - x^2$ down 1, left 1

36. $x^2 + y^2 = 5$ up 2, left 4

37. $y = (x-1)^2 - 1$ down 1, right 1

38. $y = \sqrt{x}$ down 2, left 4

29. Figure P.34 shows the graph $y = x^2$ and four shifted versions of it. Write equations for the shifted versions.

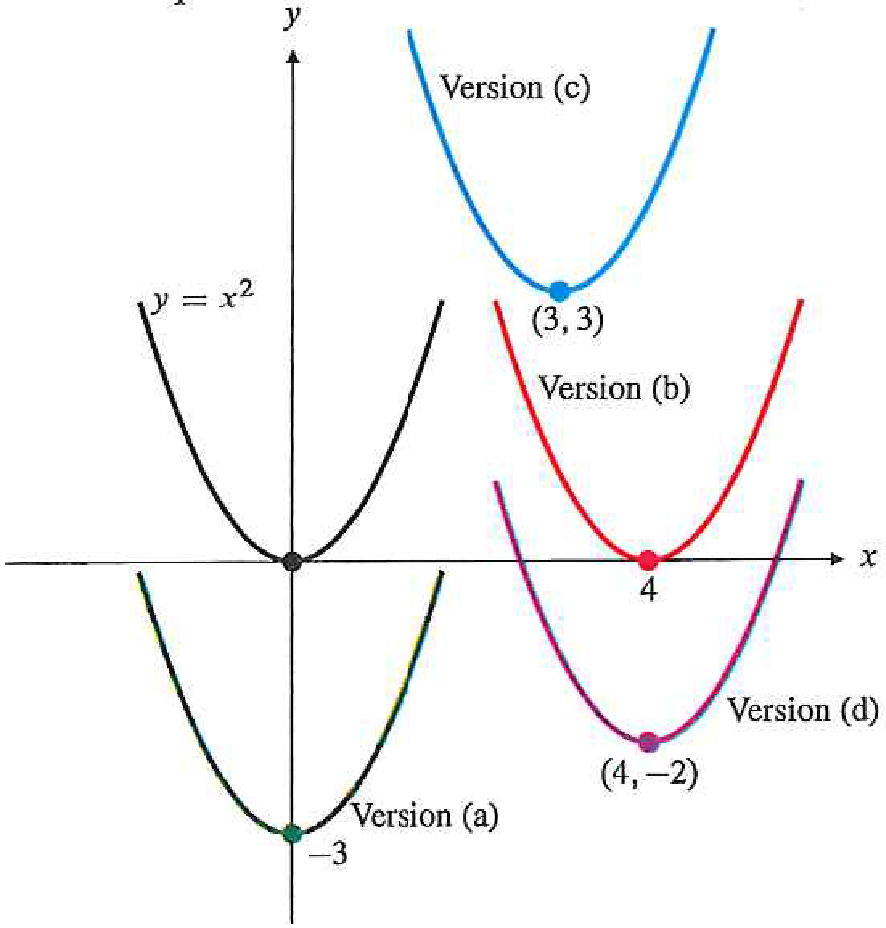


Figure P.34

ADAMS Problems P.4

7. Which of the graphs in Figure P.56 are graphs of functions $y = f(x)$? Why?

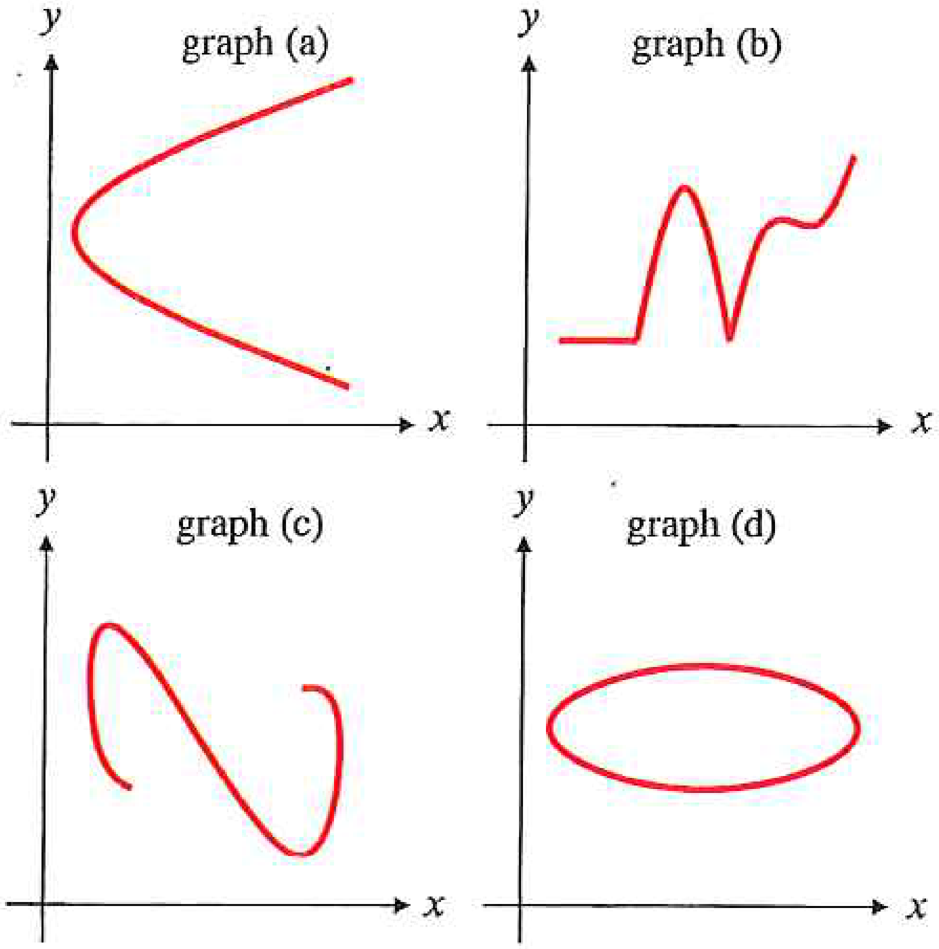


Figure P.56

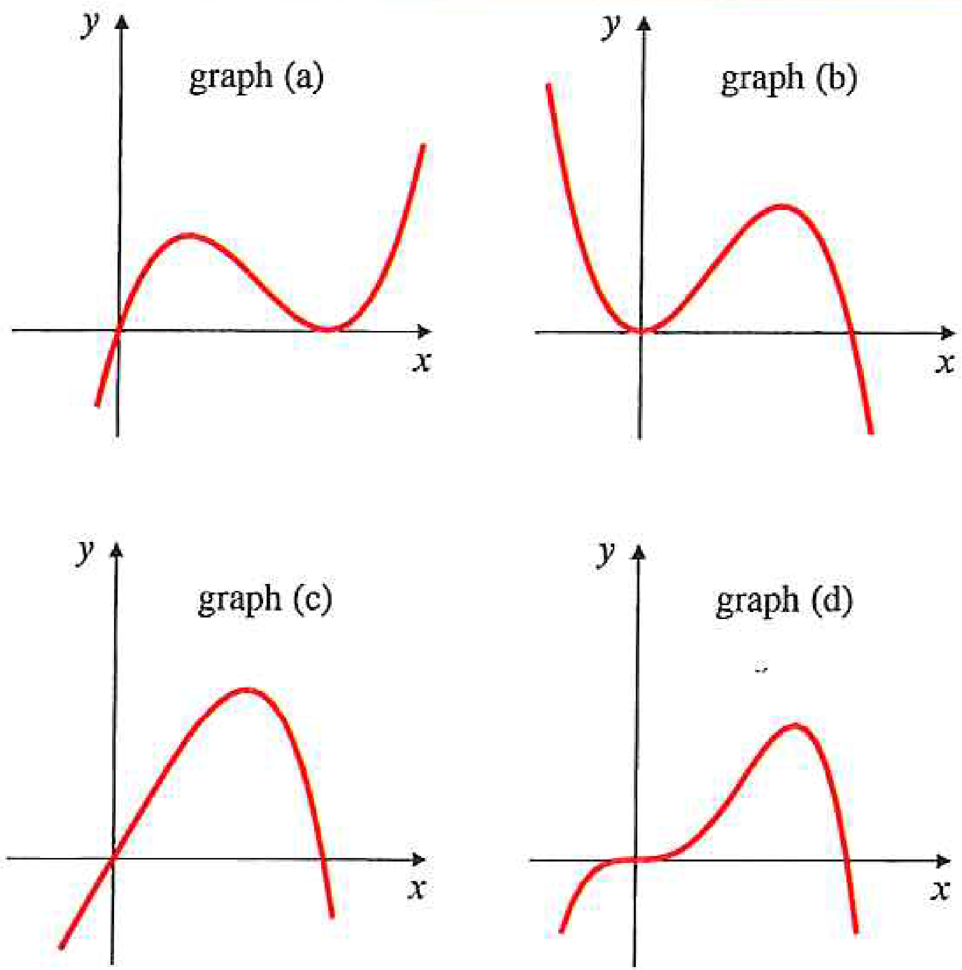


Figure P.57

8. Figure P.57 shows the graphs of the functions: (i) $x - x^4$, (ii) $x^3 - x^4$, (iii) $x(1-x)^2$, (iv) $x^2 - x^3$. Which graph corresponds to which function?

ADAMS Problems P.5

Sketch the graphs of the functions in Exercises 3–6 by combining the graphs of simpler functions from which they are built up.

3. $x - x^2$

4. $x^3 - x$

5. $x + |x|$

6. $|x| + |x - 2|$

7. If $f(x) = x + 5$ and $g(x) = x^2 - 3$, find the following:

(a) $f \circ g(0)$

(b) $g(f(0))$

(c) $f(g(x))$

(d) $g \circ f(x)$

(e) $f \circ f(-5)$

(f) $g(g(2))$

(g) $f(f(x))$

(h) $g \circ g(x)$

ADAMS Problems Complex numbers

In Exercises 1–4, find the real and imaginary parts ($\mathrm{Re}(z)$ and $\mathrm{Im}(z)$) of the given complex numbers z, and sketch the position of each number in the complex plane (i.e., in an Argand diagram).

1. $z = -5 + 2i$

2. $z = 4 - i$

3. $z = -\pi i$

4. $z = -6$

16. If $\mathrm{Arg}(z) = 3\pi/4$ and $\mathrm{Arg}(w) = \pi/2$, find $\mathrm{Arg}(zw)$.

17. If $\mathrm{Arg}(z) = -5\pi/6$ and $\mathrm{Arg}(w) = \pi/4$, find $\mathrm{Arg}(z/w)$.

56. Find all solutions of $z^5 + a^5 = 0$, where a is a positive real number.

57. Show that the sum of the n nth roots of unity is zero. *Hint:* Show that these roots are all powers of the principal root.

Simplify the expressions

40. $\dfrac{2-i}{2+i}$

41. $\dfrac{1+3i}{2-i}$

42. $\dfrac{1+i}{i(2+3i)}$

43. $\dfrac{(1+2i)(2-3i)}{(2-i)(3+2i)}$

44. Prove that $\overline{z+w} = \overline{z} + \overline{w}$.

45. Prove that $\overline{\left(\dfrac{z}{w}\right)} = \dfrac{\overline{z}}{\overline{w}}$.

51. Find the three cube roots of -1.

ADAMS Problems P.6

Find the roots of the polynomials in Exercises 1–12. If a root is repeated, give its multiplicity. Also, write each polynomial as a product of linear factors.

1. $x^2 + 7x + 10$

2. $x^2 - 3x - 10$

3. $x^2 + 2x + 2$

4. $x^2 - 6x + 13$

5. $16x^4 - 8x^2 + 1$

6. $x^4 + 6x^3 + 9x^2$

7. $x^3 + 1$

8. $x^4 - 1$

23. Show that $x - 1$ is a factor of a polynomial P of positive degree if and only if the sum of the coefficients of P is zero.

25. The complex conjugate of a complex number $z = u + iv$ (where u and v are real numbers) is the complex number $\bar{z} = u - iv$. It is shown in Appendix I that the complex conjugate of a sum (or product) of complex numbers is the sum (or product) of the complex conjugates of those numbers. Use this fact to verify that if $z = u + iv$ is a complex root of a polynomial P having real coefficients, then its conjugate $\bar{z}$ is also a root of P.

ADAMS Problems P.7

Find the values of the quantities in Exercises 1–6 using various formulas presented in this section. Do not use tables or a calculator.

1. $\cos \frac{3\pi}{4}$ **2.** $\tan -\frac{3\pi}{4}$ **3.** $\sin \frac{2\pi}{3}$

4. $\sin \frac{7\pi}{12}$ **5.** $\cos \frac{5\pi}{12}$ **6.** $\sin \frac{11\pi}{12}$

In Exercises 13–16, prove the given identities.

13. $\cos^4 x - \sin^4 x = \cos(2x)$

14. $\frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x} = \tan \frac{x}{2}$

15. $\frac{1-\cos x}{1+\cos x} = \tan^2 \frac{x}{2}$

16. $\frac{\cos x - \sin x}{\cos x + \sin x} = \sec 2x - \tan 2x$

23. Sketch the graph of $y = 2\cos\left(x - \frac{\pi}{3}\right)$.