# W1/L1 mängder

den 25 augusti 2020 13:21

**L1 bakgrund/mängder**

- mängder (objekt som delar egenskaper. OO class)
- mängder av tal
  - naturliga $\mathbb{N}$ $A=\{n \mid n \text{ jämt tal}\}$
  - hela $\mathbb{Z}$
  - rationella (decimal form med upprepning) $\mathbb{Q}$
  - irrationella (decimal form utan upprepning) $\mathbb{I}$
- reella talen = rationella + irrationella $\mathbb{R} = \mathbb{Q} \cup \mathbb{I}$
- egenskaper av tal
  - algebra regler: varifrån kommer dessa?
  - varför är (-1)*(-1)=1?
  - varför är (-1)*2=-2?
  - kvadratregel
  - konjugatregel
  - kvadrat komplettering

$A=\{a \mid \ldots\}$ $\cup \ \cap \ \setminus$

$A=\{a,b,c\}$

class bild{ }

mängd = samling av objekt som delar egenskaper

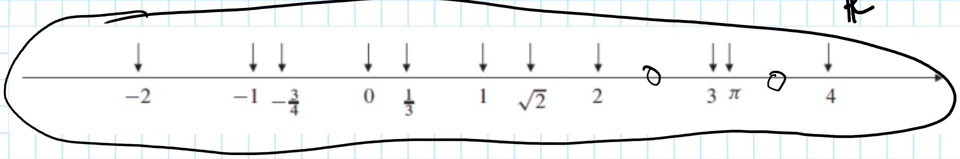
{Adam, Rasmus, Julia, Hedy}

## P Preliminaries



1d, 2d

## P.1 Real Numbers and the Real Line



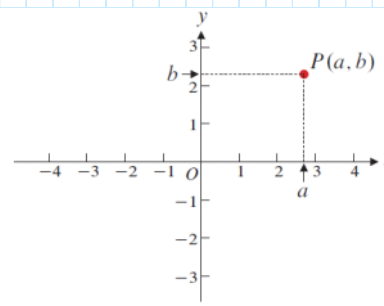




**Figure P.8** The coordinate axes and the point $P$ with coordinates $(a, b)$

Calculus depends on properties of the real number system. **Real numbers** are numbers that can be expressed as decimals, for example,

$$5 = 5.00000\ldots$$
$$-\frac{3}{4} = -0.750000\ldots$$
$$\frac{1}{3} = 0.3333\ldots$$
$$\sqrt{2} = 1.4142\ldots$$
$$\pi = 3.14159\ldots$$

In each case the three dots (...) indicate that the sequence of decimal digits goes on forever. For the first three numbers above, the patterns of the digits are obvious; we know what all the subsequent digits are. For $\sqrt{2}$ and $\pi$ there are no obvious patterns.

$5.0$, $0.750$, $0.3$

$x = 0.123123123$

$x = p/q$

$x = 0.123123$

$1000x = 123.123 = 123 + 0.123$

$1000x = 123 + x$

$(1000-1)x = 123$

$999x = 123$

$x = \frac{123}{999}$

varför är (-1)*2 = -2? -> (1+(-1))*2=0

varför är (-1)*(-2)=2? -> (1+(-1))*(-2)=0

därför att: $(-1)(-1) = 1$

$1 + (-1) = 0 \quad (-2)$

$1\cdot(-2) + (-1)(-2) = 0$

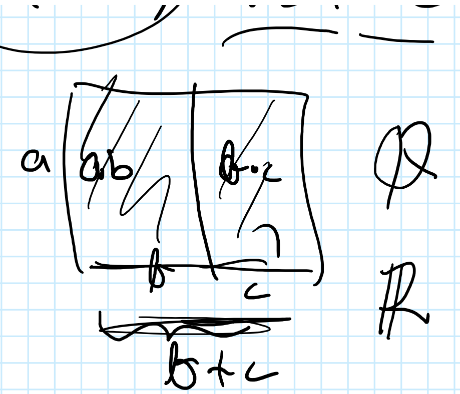
$(-\sqrt{2})(-\sqrt{2}) = \sqrt{2}\cdot\sqrt{2} = 2$

$a\cdot(b+c) = a\cdot b + b\cdot c$

- Associativity of addition and multiplication: $a + (b + c) = (a + b) + c$, and $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.
- Commutativity of addition and multiplication: $a + b = b + a$, and $a \cdot b = b \cdot a$.
- Additive and multiplicative identity: there exist two different elements 0

$a \cdot b = b \cdot a$.

- Additive and multiplicative identity: there exist two different elements $0$ and $1$ in $F$ such that $a + 0 = a$ and $a \cdot 1 = a$.
- Additive inverses: for every $a$ in $F$, there exists an element in $F$, denoted $-a$, called the *additive inverse* of a, such that $a + (-a) = 0$.
- Multiplicative inverses: for every $a \neq 0$ in $F$, there exists an element in $F$, denoted by $a^{-1}$ or $1/a$, called the *multiplicative inverse* of a, such that $a \cdot a^{-1} = 1$.
- Distributivity of multiplication over addition: $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$.



The *completeness* property of the real number system is more subtle and difficult to understand. One way to state it is as follows: if $A$ is any set of real numbers having at least one number in it, and if there exists a real number $y$ with the property that $x \leq y$ for every $x$ in $A$ (such a number $y$ is called an **upper bound** for $A$), then there exists a *smallest* such number, called the **least upper bound** or **supremum** of $A$, and denoted $\sup(A)$. Roughly speaking, this says that there can be no holes or gaps on the real line—every point corresponds to a real number. We will not need to deal much with completeness in our study of calculus. It is typically used to prove certain important results—in particular, Theorems 8 and 9 in Chapter 1. (These proofs are given in Appendix III but are not usually included in elementary calculus courses; they are studied in more advanced courses in mathematical analysis.) However, when we study infinite sequences and series in Chapter 9, we will make direct use of completeness.

APPENDIX III

# Continuous Functions

> “Geometry may sometimes appear to take the lead over analysis, but in fact precedes it only as a servant goes before his master to clear the path and light him on the way. The interval between the two is as wide as between empiricism and science, as between the understanding and the reason, or as between the finite and the infinite.”

J. J. Sylvester 1814–1897
from *Philosophic Magazine*, 1866

**The completeness axiom for the real numbers**

A nonempty set of real numbers that has an upper bound must have a least upper bound.

Equivalently, a nonempty set of real numbers having a lower bound must have a greatest lower bound.

We stress that this is an *axiom* to be assumed without proof. It cannot be deduced from the more elementary algebraic and order properties of the real numbers. These other properties are shared by the rational numbers, a set that is not complete. The completeness axiom is essential for the proof of the most important results about continuous functions, in particular, for the Max-Min Theorem and the Intermediate-Value

In Section 9.1 we stated a version of the completeness axiom that pertains to *sequences* of real numbers; specifically, that an increasing sequence that is bounded above converges to a limit. We begin by verifying that this follows from the version stated above. (Both statements are, in fact, equivalent.) As noted in Section 9.1, the sequence

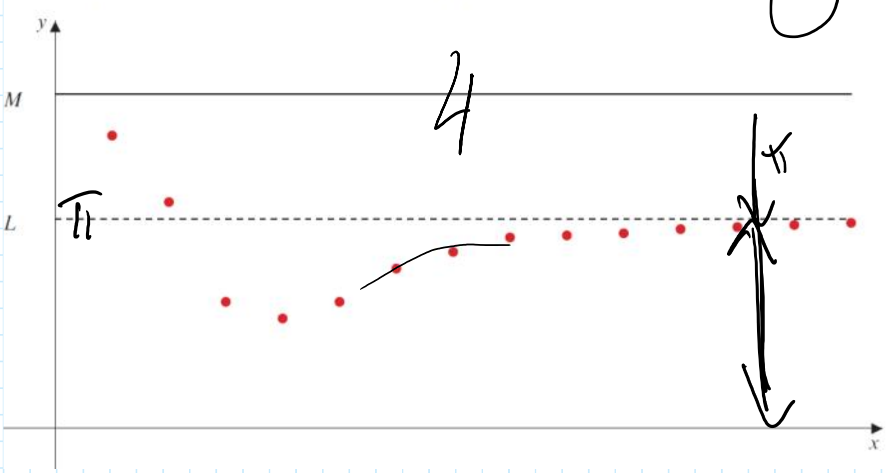
## 9.1 Sequences and Convergence

The *completeness property* of the real number system (see Section P.1) can be reformulated in terms of sequences to read as follows:

**Bounded monotonic sequences converge**

If the sequence $\{a_n\}$ is bounded above and is (ultimately) increasing, then it converges. The same conclusion holds if $\{a_n\}$ is bounded below and is (ultimately) decreasing.

Thus, a bounded, ultimately monotonic sequence is convergent. (See Figure 9.2.)




1. $\pi = 3.1415926...$

$\pi_1 = 3.1$

$\pi_2 = 3.14$

⋮

$\pi_i < 4$

Naturliga tal = $\{0, 1, 2, 3, \ldots\}$ $N$ $\mathbb{N}$

Hela tal = $\{0, \pm 1, \pm 2, \ldots\}$ $\mathbb{Z}$

Rationella tal = $\{ p/q \mid p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0 \}$

Irrationella tal = $\{ ? \}$

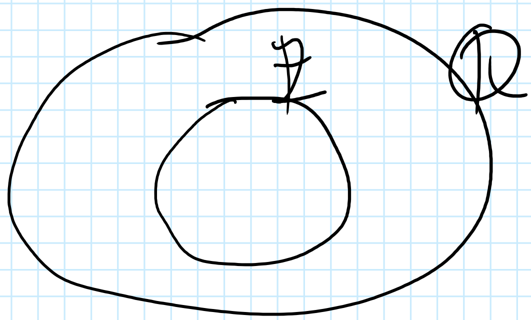
$1/3 =$

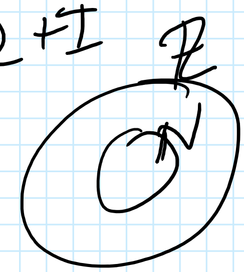
$\mathbb{Z}$ $\mathbb{Q}$

$\mathbb{N} \subset \mathbb{Z}$

$\mathbb{Z} \subset \mathbb{Q}$

$\mathbb{R} = \mathbb{Q} + \mathbb{I}$




~~Kvadrat~~ Kvadrerings regel

$a(b+c) = a \cdot b + a \cdot c$

$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2$

$a^2 - b^2 = (a-b)(a+b)$

$x + 2x = (x+1)^2 + 1$

$(x+1)^2 = x^2 + 2x + 1$

$x^2 + 2x = (x+1)^2 - 1$