MVE045 W2-RÖ1

kontinuitet

ADAMS Problem 1.4:1,3

EXERCISES 1.4

Exercises 1–3 refer to the function g defined on $[-2, 2]$, whose graph is shown in Figure 1.33.

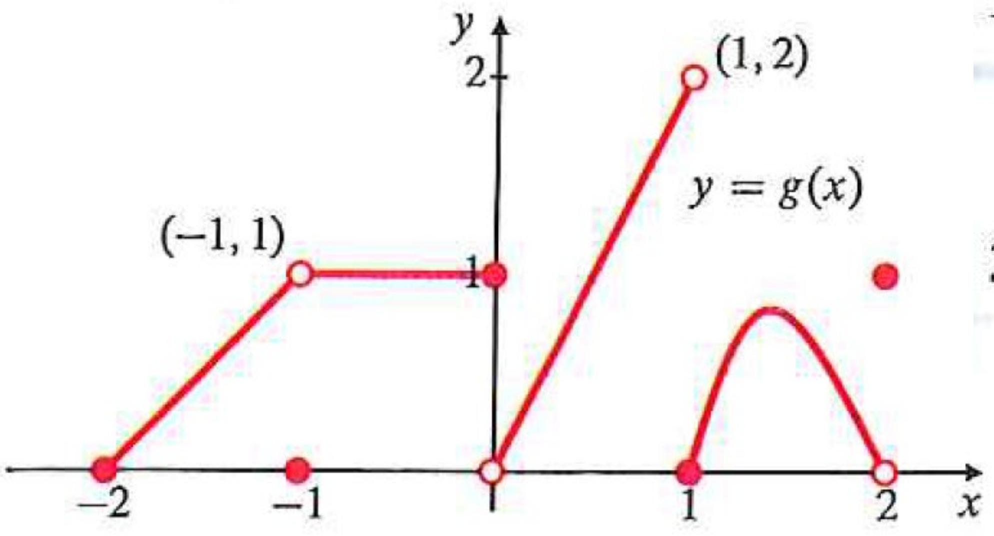
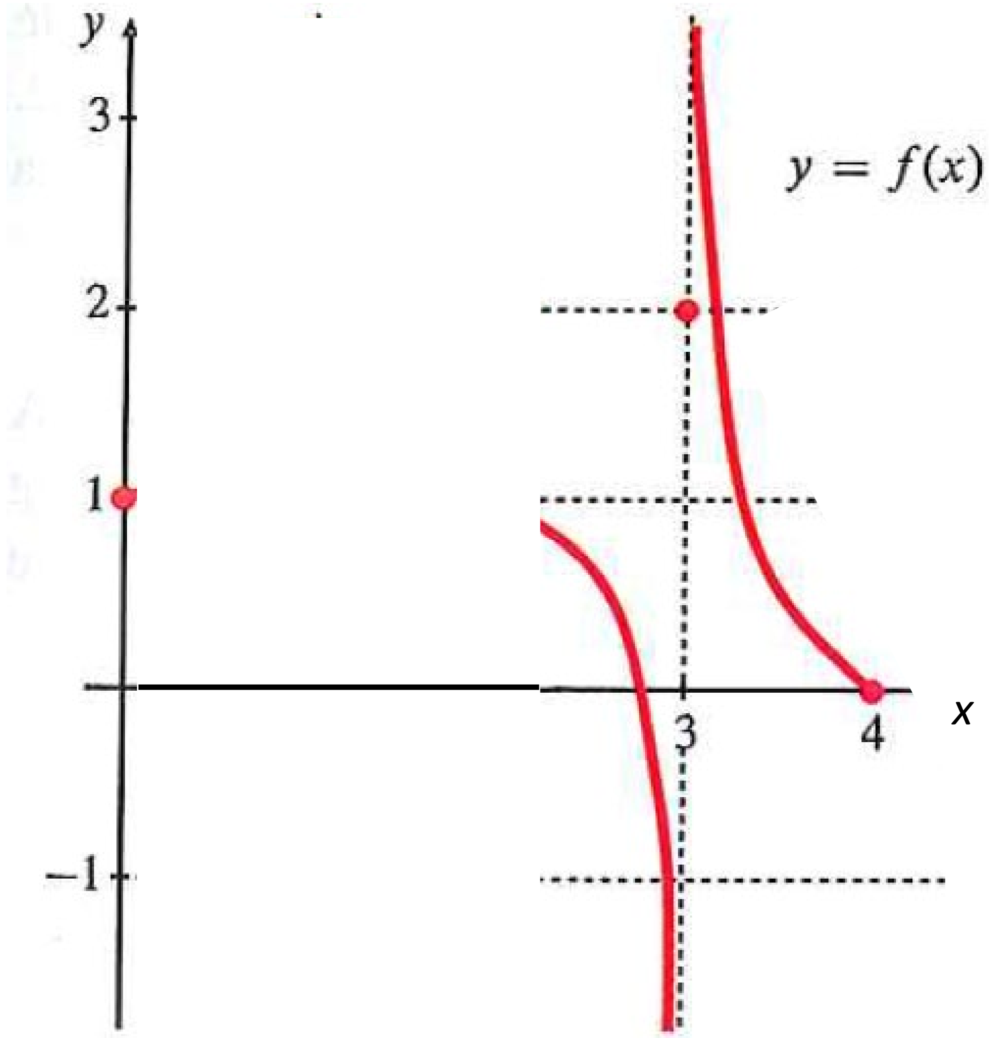


Figure 1.33

1. State whether g is (a) continuous, (b) left continuous, (c) right continuous, and (d) discontinuous at each of the points -2, -1, 0, 1, and 2.

3. Does g have an absolute maximum value on $[-2, 2]$? an absolute minimum value?

ADAMS Problem 1.4: 5



5. Can the function *f* graphed in the figure be redefined at the single point *x=3* so that it becomes continuous there?

ADAMS Problem 1.4: 7

State where in its domain the given function is continuous, where it is left or right continuous, and where it just discontinuous

7. $f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

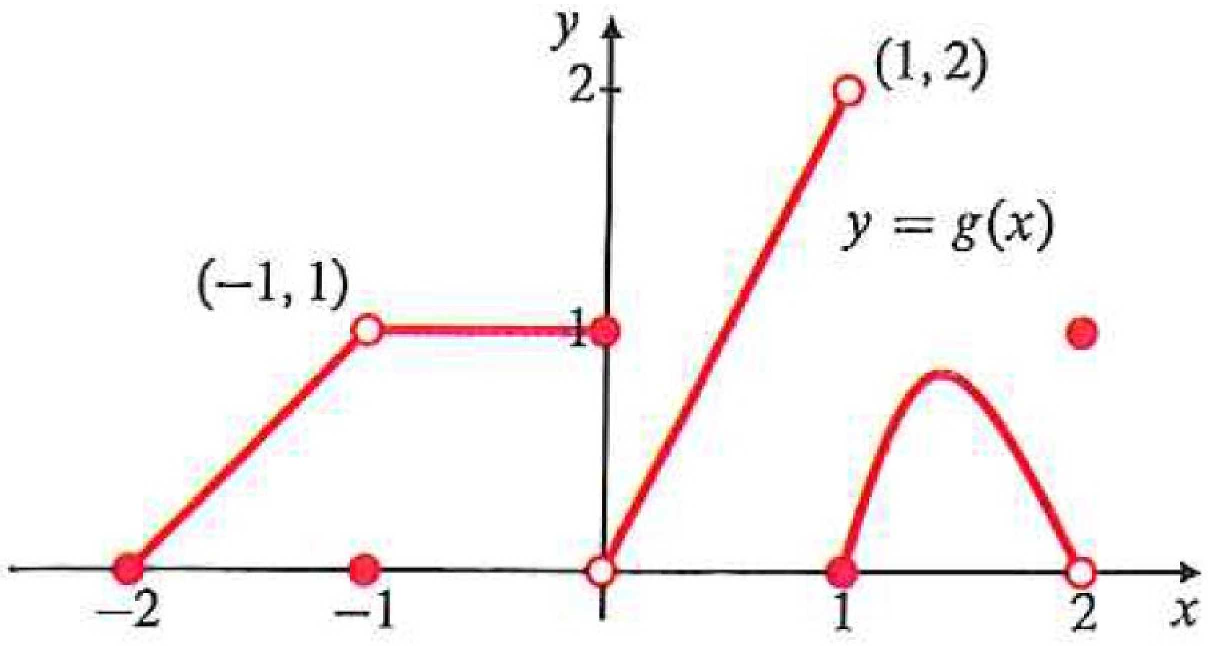
10. $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 0.987 & \text{if } x > 1 \end{cases}$

ADAMS Problem 1.4: 19, 3

related to min/max theorem

19. Does the function x^2 have a maximum value on the open interval $-1 < x < 1$? a minimum value? Explain.

19’. A similar question but for the closed interval $-1 \leq x \leq 1$?



3. Does g (in the figure) have an absolute maximum value on [-2,2]? an absolute minimum value?

ADAMS Problem 1.4: 15

How should the function be defined at the given point to be continuous there? Give a formula for the continuous extension to that point.

15. $\dfrac{t^2 - 5t + 6}{t^2 - t - 6}$ at 3

Answer in the form $f(t) = \begin{cases} \text{expression in } t & \text{condition on } t \\ \text{another expression in } t & \text{another condition on } t \end{cases}$

ADAMS Problem 1.4: 23
min/max

23. A software company estimates that if it assigns x programmers to work on the project, it can develop a new product in T days, where

$$T = 100 - 30x + 3x^2.$$

How many programmers should the company assign in order to complete the development as quickly as possible?

ADAMS Problem 1.4: 29, 30, 31

intermediate value theorem application

29. Show that $f(x) = x^3 + x - 1$ has a zero between $x = 0$ and $x = 1$.

!**30.** Show that the equation $x^3 - 15x + 1 = 0$ has three solutions in the interval $[-4, 4]$.

An excellent exam question! How would you solve such a problem? Hint: you can always try to evaluate the expression in some values for x.

!**31.** Show that the function $F(x) = (x - a)^2(x - b)^2 + x$ has the value $(a + b)/2$ at some point x.

Another excellent exam question! How would you solve such a problem? Hint: intermediate value theorem, of course. Try evaluating the function as some points.

ADAMS Problem 1.4: 23

min/max

23. A software company estimates that if it assigns x programmers to work on the project, it can develop a new product in T days, where

$$T = 100 - 30x + 3x^2.$$

How many programmers should the company assign in order to complete the development as quickly as possible?

ADAMS Problem 1.5: 3,4
absolute value practice (and a bit more that you will appreciate in the math courses that will follow)

In Exercises 3–6, in what interval must x be confined if $f(x)$ must be within the given distance ϵ of the number L?

3. $f(x) = 2x - 1$, $L = 3$, $\epsilon = 0.02$

4. $f(x) = x^2$, $L = 4$, $\epsilon = 0.1$

5. $f(x) = \sqrt{x}$, $L = 1$, $\epsilon = 0.1$

6. $f(x) = 1/x$, $L = -2$, $\epsilon = 0.01$

ADAMS Problem 1.5: 7,8
absolute value practice (and a bit more that you will appreciate in the math courses that will follow)

In Exercises 7–10, find a number $\delta > 0$ such that if $|x - a| < \delta$, then $|f(x) - L|$ will be less than the given number ϵ.

7. $f(x) = 3x + 1, \quad a = 2, \quad L = 7, \quad \epsilon = 0.03$
8. $f(x) = \sqrt{2x + 3}, \quad a = 3, \quad L = 3, \quad \epsilon = 0.01$
9. $f(x) = x^3, \quad a = 2, \quad L = 8, \quad \epsilon = 0.2$
10. $f(x) = 1/(x + 1), \quad a = 0, \quad L = 1, \quad \epsilon = 0.05$