derivata i en punkt = tangent, linjär approximation

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MVE045 todo

W3 Dugga kommer snart

Kursrepresentanter

TKITE	stefan.krstic2@hotmail.com	Stefan Krstic
~~TKITE~~	~~lenia@student.chalmers.se~~	~~Lenia Malki~~
TKITE	lisasamuelssons@gmail.com	Lisa Samuelsson
TKITE	erikwessman10@gmail.com	Erik Wessman
~~TKITE~~	~~malte@akvist.com~~	~~Malte Åkvist~~
josefine nord		

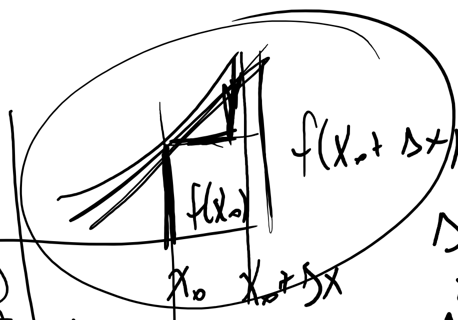
Hugo Ganelius

LG

kallelse för de som är nya.

2.2 The Derivative

2.1 Tangent Lines and Their Slopes



$$\cancel{\lim_{dx\to 0}} \frac{f(x+dx)-f(x)}{dx} = df = dy$$

$$\Delta f = f(x_0+\Delta x) - f(x_0) \equiv \Delta y$$

$\underline{\underline{A}} \cdot \underline{x}$

$\hat{A} \equiv \frac{d}{dx}$

$\hat{A} \cdot f(x) = \frac{d}{dx} f(x)$

notation ε, δ

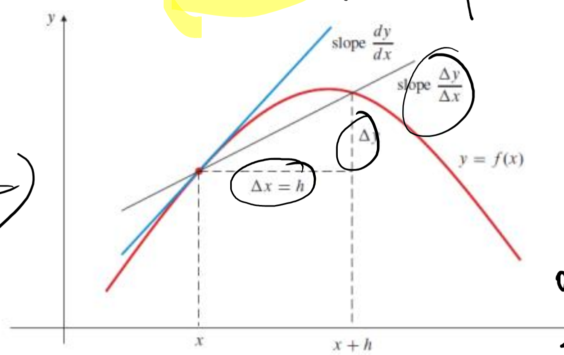
$$\frac{dy}{dx} = \lim_{\Delta x\to 0} \frac{\Delta y}{\Delta x}.$$

Leibniz Notation

Because functions can be written in different ways, it is useful to have more than one notation for derivatives. If $y = f(x)$, we can use the dependent variable y to represent the function, and we can denote the derivative of the function with respect to x in any of the following ways:

$$D_x y = y' = \frac{dy}{dx} = \frac{d}{dx} f(x) = f'(x) = D_x f(x) = Df(x).$$

slope $\frac{dy}{dx}$

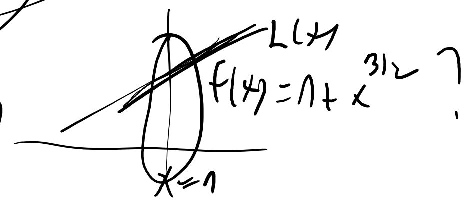


$$(y)'_x \equiv \frac{dy}{dx}$$

operator form

$\Delta x = h \to 0$

$f(x) \approx L(x)$



Differentials

sv → differentialer

The Newton quotient $\Delta y/\Delta x$ is actually the quotient of two quantities, Δy and Δx. It is not at all clear, however, that the derivative dy/dx, the limit of $\Delta y/\Delta x$ as Δx approaches zero, can be regarded as a quotient. If y is a continuous function of x, then Δy approaches zero when Δx approaches zero, so dy/dx appears to be the meaningless quantity $0/0$. Nevertheless, it is sometimes useful to be able to refer to quantities dy and dx in such a way that their quotient is the derivative dy/dx. We can justify this by regarding dx as a new *independent* variable (called **the differential of** x) and defining a new *dependent* variable dy (**the differential of** y) as a function of x and dx by

$$dy = \frac{dy}{dx}\,dx = f'(x)\,dx.$$

For example, if $y = x^2$, we can write $dy = 2x\,dx$ to mean the same thing as $dy/dx = 2x$. Similarly, if $f(x) = 1/x$, we can write $df(x) = -(1/x^2)\,dx$ as the equivalent differential form of the assertion that $(d/dx)f(x) = f'(x) = -1/x^2$. This *differential notation* is useful in applications (see Sections 2.7 and 12.6), and especially for the interpretation and manipulation of integrals beginning in Chapter 5.

$$\frac{dy}{dx} = f'(x) \quad \Big| \cdot dx$$

$$\frac{dy}{\cancel{dx}} \cdot \cancel{dx} = f'(x)\,dx$$

$$dy = f'(x)\,dx$$

$$\frac{dy}{dx} = f'(x)$$

$$y = x^2, \quad dy = d(x^2) = (x^2)'_x \cdot dx$$

$$dy = 2x\,dx$$

man kan använda dy/dx i integralsubstitution tror jag?

ja!

$d(x^3) = ?$ ADAMS Recept:

$$d(x^3) = (x^3)'_x\,dx = 3x^2\,dx$$

$$df = df(x) = \big(f(x+\Delta x) - f(x)\big)_{\Delta x \to 0}$$

$f(x) = x^3$

0,01 0,001

$(\ldots \Delta x \ldots)_{\Delta x \to 0}$

|||

oändlig liten

$\ldots dx \ldots$ tal

$\Delta x \to 0$ $\quad$ $\ldots dx \ldots$ $1d$

$f(x) = x^3$

$$\Delta f = f(x+\Delta x) - f(x) = (x+\Delta x)^3 - x^3 = \cancel{x^3} + 3x^2\Delta x + 3x\,\Delta x^2 + \Delta x^3 - \cancel{x^3}$$

$x \sim 1$

$\cancel{3\cdot 1^2}\cdot 0{,}01$ $\quad \sim 1 \cdot 0{,}0001 \gg$ $\quad \sim 1\cdot 0{,}000001$

~ 1

$0.000001 \gg 0.000000000\,00000000\,0.000\,000\,000\,001$

$$\doteq 3x\,\Delta x + O(\Delta x^2, \cancel{\Delta x^3})$$

$$\Delta f \doteq 3x^2\,\Delta x + O(\Delta x^2)$$

$\Delta x \to 0$ $\quad df = 3x^2 dx + \cancel{O(dx^2)}$

$\Delta x \to dx$

$dx^3 = 3x^2 dx$ $\quad$ notation

$dx^3 = \begin{cases} (dx)^3 \leftrightarrow \Delta x^3 \\ d(x^3) \leftrightarrow ((x+\Delta x)^3 - x^3) \end{cases}$

$dx^3 \equiv (dx)^3 \qquad d(x^3) = 3x^2 dx$

derivata: grafisk tolkning

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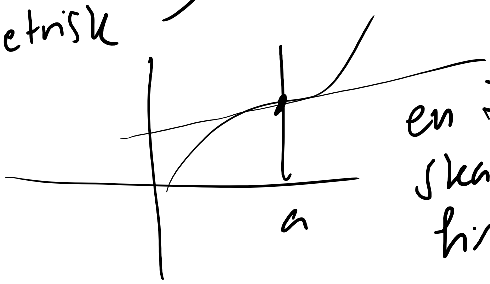
f(x) är deriverbar i $x = x_0$ ($x = a$):

$$\cancel{\lim \frac{\Delta y}{dx}} = \lim \frac{\Delta f}{\Delta x} \text{ finns.}$$

geometrisk



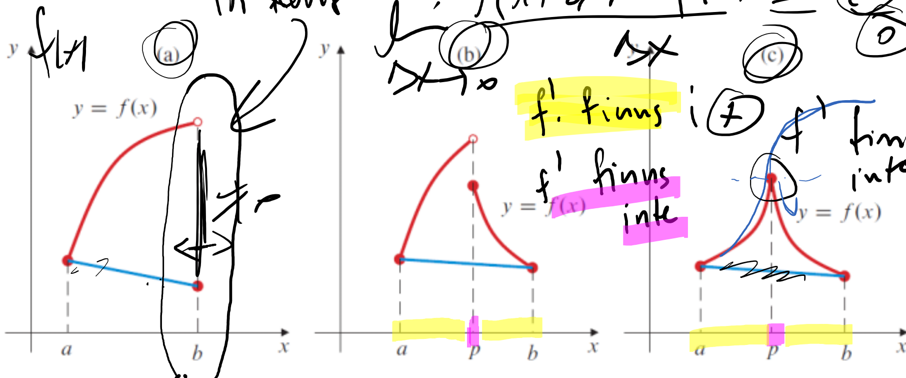
en tangent skall finnas

$$\lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x)}{\Delta x}$$

in kont'

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{\pm 0}{0}$$

kan vi teckna deriv. i punkt som är angiven?



(a) f' finns

(b) f' finns inte

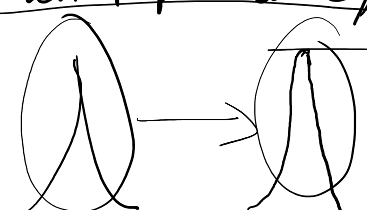
ADAMS 2.8 mean value theorem (medelvärdesattsen)

Två sidig der. , x_0 = finns + kont punkt $\overset{\text{def}}{\Rightarrow}$ derivatan finns

Eftersom den inte har någon "acceleration" i den punkten? Den bara är typ?

Finns den inte men är 0?

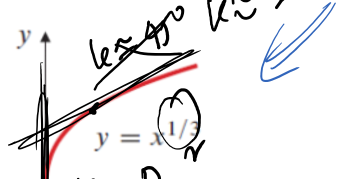
hastighet



det är ju lite svårt att veta om funktionen verkligen är "spetsig" eller om den faktiskt är rundad fast man måste zooma in mycket för att se det

oändlig spetsig

$$\frac{dy}{dx} = \frac{1}{3} \cdot x^{\frac{1}{3}-1}$$



$$\frac{dy}{dx} = \frac{1}{3}\cdot x^{\frac{1}{3}-1} = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3}\,\frac{1}{x^{2/3}}$$

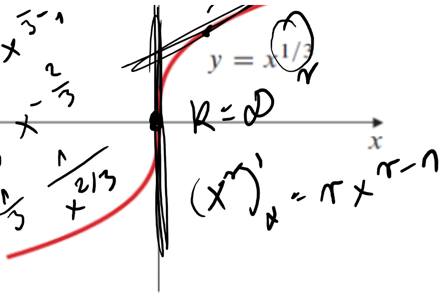
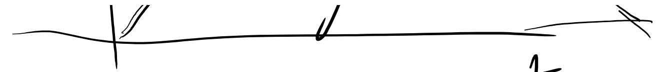


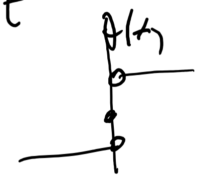
Figure 2.6 The y-axis is tangent to $y = x^{1/3}$ at the origin

$$\left(\frac{dy}{dx}\right)\Bigg|_{x=0} = \left(\frac{1}{3}\,\frac{1}{x^{2/3}}\right)\Bigg|_{x=0} = \left(\frac{1}{3}\cdot\frac{1}{0^{2/3}}\right) = +\infty$$



derivatan hoppar! t

$$\theta(x) = \begin{cases} 1 & x > 0 \\ & x = 0 \\ -1 & x < 0 \end{cases}$$



$\theta(0) = ?$

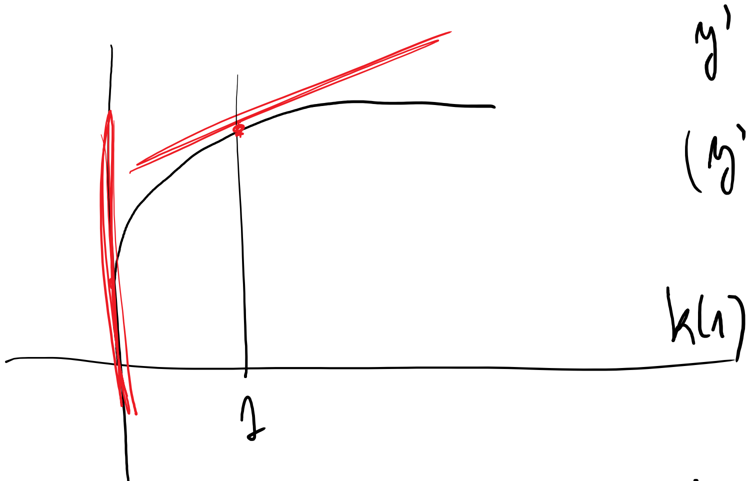
$$\left(x^2 + 2x + 5\right)\Big|_{x=3} = 3^2 + 2\cdot 3 + 5$$

ett exempel till:

$$y = \sqrt{x} = x^{1/2}$$

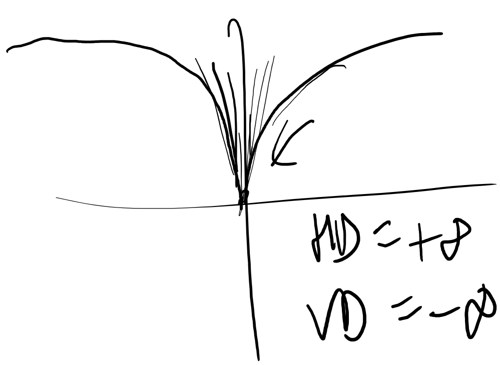
$$y' = \frac{1}{2\sqrt{x}}$$

$$(y')_{x=0} = \infty$$

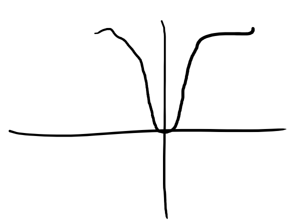


$$k(1) = \left(\frac{1}{2\sqrt{x}}\right)\Bigg|_{x=1} = \frac{1}{2\sqrt{1}} = \frac{1}{2}$$

$$k(0) = \left(\frac{1}{2\sqrt{x}}\right)_{x=0} = \frac{1}{0} = \infty$$



$$f(x) = \sqrt{|x|}$$



2.3 Differentiation Rules

några typiska mistag studenter gör på tentan

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$df = x$ s.o. 1

varför är det fel att skriva så?

s.o.0

$df = x \cdot dx$

s.o.0 s.o.1 s.o.0

"0 = 1 × 0"

$d\delta \sim$ av vilken "storleksordning" är detta tal

fysiker

10 s.o.=1

100 s.o.=2

1 000 = s.o 3

0,001 ?

Dx

102

35

0,6

1

s.o. = 1

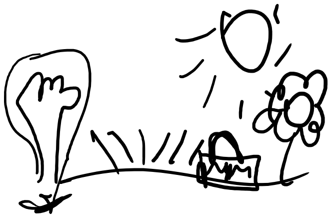
Vad hette det engleksa ordet för "Storleksordning", hitta inget på nätet

"order of magnitude"

$d\delta =$ s.o. 0

$$\boxed{\text{varför är det ok att skriva så här:} \quad dx + dx^2 = dx}$$

Du sa att dx^2 var försumligt. dx >> dx^2



Du sa att dx^2 var försumligt. dx >> dx^2

$$\overset{VS}{dx} \gg \overset{HS}{dx^2} \qquad dx = 0.1$$

$$0.1 \gg 0.01 \qquad = 0.01$$

$$0{,}01 \gg 0.0001$$

Stämmer detta endast då: x->0?
ja

$$HS \to 0 \qquad VS \qquad dx \to 0 \qquad \frac{HS}{VS} = \frac{(dx)^2}{dx} = dx \to 0$$

$$\varepsilon \gg \varepsilon^2$$

ex:

$$\underset{\sim\varepsilon}{da} + \underset{\sim\varepsilon}{db} + \cancel{\underset{\sim\varepsilon\cdot\varepsilon}{da \cdot db}} = \begin{cases} da + da \cdot db \leftrightarrow (a) \\ db + da \leftrightarrow (b) \\ db + da \cdot db \leftrightarrow (c) \\ \cancel{db\ da} \ (d) \end{cases}$$

välj

Vet du kanske vart det står om storleksordning i boken?
skall försöka peka på

ett exempel från adams

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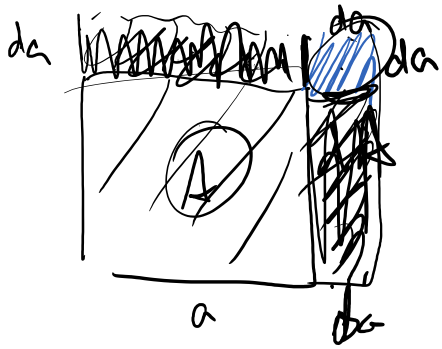

$$A + dA \longleftrightarrow A$$

den stora den små

$$dA = 2a \cdot da + da^2$$

$a \cdot da$
$a \cdot da$

$da^2 \ll 2a \cdot da$

$$\underbrace{\boxed{a+da}_{\,a+da} - \boxed{}_{\,a}^{\,a}}_{dA} = (a+da)^2 - a \cdot a$$

$$= a^2 + da^2 + 2a \cdot da - a^2$$

mekanisk recept:

$$A = a^2 \qquad dA = (A)'_a \cdot da = 2a \cdot da$$

derivata egenskaper

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$$\lim (f+g) = \lim f + \lim g \quad (*)$$

$$\frac{df}{dx} = \lim \frac{\Delta f}{\Delta x}$$

hur många
kan detta?
(a) ja
(b) nej

$$(f+g)' = f' + g'$$

$$h = f+g \qquad \Delta h = h(x+\Delta x) - h(x) = \ldots = \Delta f + \Delta g$$

$$\Delta h = \Delta f + \Delta g \quad \Big| : \Delta x$$

algebra med bråk

$$\frac{\Delta h}{\Delta x} = \frac{\Delta f + \Delta g}{\Delta x} = \frac{\Delta f}{\Delta x} + \frac{\Delta g}{\Delta x}$$

$$\frac{\Delta h}{\Delta x} = \frac{\Delta f}{\Delta x} + \frac{\Delta g}{\Delta x} \quad \Big| \lim_{\Delta x \to 0}$$

$$\lim_{\Delta x \to 0} \frac{\Delta h}{\Delta x} = \lim_{\Delta x \to 0} \left(\frac{\Delta f}{\Delta x} + \frac{\Delta g}{\Delta x} \right) \; (*)$$

$$= \lim \frac{\Delta f}{\Delta x} + \lim \frac{\Delta g}{\Delta x}$$

$(f+g)' = h' = f' + g'$

$(f \cdot g)' = f' \cdot g + f \cdot g'$ Stefan

$(f/g)' = \frac{f'g - fg'}{g^2}$ Daemon

$(k \cdot f)' = k \cdot f'$

$(x^3)' = 3x^2$

$$(x^3)' = (\underset{f}{x^2} \cdot \underset{g}{x})' = (x^2)' x + x^2 \cdot x'$$

$$= 2x\,x + x^2\,1$$

$$= 2 \cdot x^2 + x^2 = 3x^2 \checkmark$$

$\star$ $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} = f' \frac{g}{g^2} - f \frac{g'}{g^2}$

$$= f' \frac{1}{g} - f \frac{g'}{g^2} \qquad \left(\frac{1}{g}\right)' = -\frac{g'}{g^2}$$

$$\left(f\cdot\left(\frac{1}{g}\right)\right)' = f'\cdot\frac{1}{g} + f\left(\frac{1}{g}\right)'$$

$$\frac{d\left(\frac{f}{g}\right)}{dx} = \frac{\frac{f+df}{g+dg} - \frac{f}{g}}{dx} = \frac{\frac{(f+df)g - f(g+dg)}{(g+dg)\cdot g}}{dx}$$

$$= \frac{\cancel{f\cdot g} + df\cdot g - (\cancel{f\cdot g} + f\cdot dg)}{dx(g+dg)}$$

$$= \frac{df\cdot g - f\cdot dg}{dx(\underbrace{g}_{\sim 1} + \underbrace{\cancel{dg}}_{\sim 0})} = \frac{df\cdot g - f\cdot dg}{dx\cdot g}$$

$$= \frac{\frac{df}{dx}\cdot g - f\cdot\frac{dg}{dx}}{g} = \frac{f'\cdot g - f\cdot g'}{g}$$