den exponentiella funktionen

den 21 september 2020 11:35

3.2 Exponential and Logarithmic Functions

Exponential functions

If $a > 0$, then

$$a^0 = 1$$

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}} \quad \text{if } n = 1, 2, 3, \ldots$$

$$a^{-n} = \frac{1}{a^n} \quad \text{if } n = 1, 2, 3, \ldots$$

$$a^{m/n} = \sqrt[n]{a^m} \quad \text{if } n = 1, 2, 3, \ldots \quad \text{and } m = \pm 1, \pm 2, \pm 3, \ldots.$$

In this definition, $\sqrt[n]{a}$ is the number $b > 0$ that satisfies $b^n = a$.

$(ab)^{n/m} = a^{n/m} b^{n/m}$

$a^x \quad a^{3/2} \quad a^{7/12} \quad a^n$

$a^{n-m} = \frac{a^n}{a^m}$

$a^{0-m} = \frac{a^0}{a^m}$

$a^{-m} = \frac{1}{a^m}$

$a^{n-n} = \frac{a^n}{a^n} = 1$

$a^0 = 1$

$n = m$

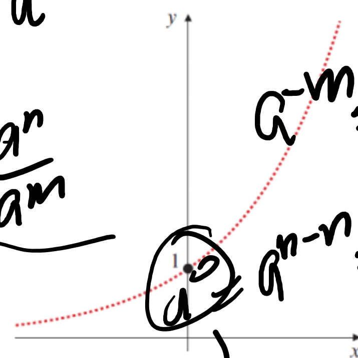

Figure 3.7 $y = 2^x$ for rational x

$x \in \mathbb{R}$

$$a^x = \lim_{\substack{r \to x \\ r \text{ rational}}} a^r$$

$x = \lim_{\substack{r \to x \\ r \in \mathbb{Q}}} r \quad \Big| \quad r = \lim_{\substack{x \to r \\ x \in \mathbb{Q}}} x$

EXAMPLE 1 Since the irrational number $\pi = 3.141\,592\,653\,59\ldots$ is the limit of the sequence of rational numbers

2^π

$$r_1 = 3, \quad r_2 = 3.1, \quad r_3 = 3.14, \quad r_4 = 3.141, \quad r_5 = 3.1415, \quad \ldots,$$

we can calculate 2^π as the limit of the corresponding sequence

$$2^3 = 8, \quad 2^{3.1} = 8.574\,187\,7\ldots, \quad 2^{3.14} = 8.815\,240\,9\ldots, \quad \ldots \to$$

This gives $2^\pi = \lim_{n\to\infty} 2^{r_n} = 8.824\,977\,827\ldots.$

algoritmen

$$\boxed{a^x = \lim_{\substack{r \to x \\ r \text{ rational}}} a^r}$$

Laws of exponents

If $a > 0$ and $b > 0$, and x and y are any real numbers, then

(i) $a^0 = 1$ (ii) $a^{x+y} = a^x a^y$

(iii) $a^{-x} = \frac{1}{a^x}$ (iv) $a^{x-y} = \frac{a^x}{a^y}$

(v) $(a^x)^y = a^{xy}$ (vi) $(ab)^x = a^x b^x$

Exponential functions

If $a > 0$, then

$$a^0 = 1$$

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}} \quad \text{if } n = 1, 2, 3, \ldots$$

$$a^{-n} = \frac{1}{a^n} \quad \text{if } n = 1, 2, 3, \ldots$$

$$a^{m/n} = \sqrt[n]{a^m} \quad \text{if } n = 1, 2, 3, \ldots \quad \text{and } m = \pm 1, \pm 2, \pm 3, \ldots.$$

In this definition, $\sqrt[n]{a}$ is the number $b > 0$ that satisfies $b^n = a$.

EXERCISES 3.2

In this definition, $\sqrt[n]{a}$ is the number $b > 0$ that satisfies $b^n = a$.

EXERCISES 3.2

1. $\frac{3^3}{\sqrt{3^5}} = \frac{3^3}{\left(3^{\frac{1}{2}}\right)^5} = \frac{3^3}{3^{\frac{5}{2}}} = 3^{3-5/2} = 3^{\frac{6}{2}-\frac{5}{2}} = 3^{\frac{1}{2}} = \sqrt{3}$

2. $2^{1/2}8^{1/2} = 2^{1/2}\left(2^3\right)^{\frac{1}{2}} = 2^{\frac{1}{2}}\, 2^{\frac{3}{2}} = 2^{\frac{1}{2}+\frac{3}{2}} = 2^{\frac{4}{2}} = 2^2 = 4$

26. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

27. $\log_a(x^y) = y \log_a x$

2^{π}

$\lim_{\substack{r \in \mathbb{Q} \\ r \to \pi}} 2^r$

double pi

pi = 2 * arcsin(1.0)

$\boxed{\text{power}(2, \text{pi})}$

pi = 2 * arcsin(1)

Notation:

$a \equiv e$

$f(x) = e^x \equiv \exp(x)$

den naturliga konstanten e

den 21 september 2020 12:00

https://en.wikipedia.org/wiki/E_(mathematical_constant)

The number e is the limit

det

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

Similarly:

$$e = \lim_{t\to 0}(1+t)^{\frac{1}{t}}$$

The number e is the sum of the infinite series

$$e = \sum_{n=0}^{\infty}\frac{1}{n!} = \frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\cdots, \; ?$$

where $n!$ is the factorial of n. (By convention $0! = 1$.)

$n = \cancel{0}, 1, 2, 3, \ldots$

variabel byte

$$\lim_{t\to 0}(1+a\cdot t)^{\frac{1}{t}} = \lim_{t\to 0}(1+at)^{\frac{1}{a\cdot t}\cdot a}$$

$at = \tilde{t}$

$$= \lim_{\tilde{t}\to 0}\left((1+\tilde{t})^{\frac{1}{\tilde{t}}}\right)^a$$

$$= \left(\lim_{\tilde{t}\to 0}(1+\tilde{t})^{\frac{1}{\tilde{t}}}\right)^a$$

$$= \underline{e^a}$$

logaritmen: den inversa funktionen till exponentiering

den 21 september 2020 11:43

$a^x = f(x) = y$

$a^y = f(y) = x$

$y = \ldots$

$y = f^{-1}(x)$

$\log_a x$

Logarithms

The function $f(x) = a^x$ is a one-to-one function provided that $a > 0$ and $a \neq 1$. Therefore, f has an inverse which we call a *logarithmic function*.

If $a > 0$ and $a \neq 1$, the function $\log_a x$, called **the logarithm of** x **to the base** a, is the inverse of the one-to-one function a^x:

$$y = \log_a x \iff x = a^y, \qquad (a > 0, \quad a \neq 1).$$

Laws of logarithms

$\log_a a = 1$

If $x > 0$, $y > 0$, $a > 0$, $b > 0$, $a \neq 1$, and $b \neq 1$, then

(i) $\log_a 1 = 0$

(ii) $\log_a(xy) = \log_a x + \log_a y$

(iii) $\log_a\left(\frac{1}{x}\right) = -\log_a x$

(iv) $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

(v) $\log_a(x^y) = y \log_a x$

(vi) $\log_a x = \frac{\log_b x}{\log_b a}$

$5 \cdot 5 = 25$

$5 \cdot 5 \cdot 5 = 25 \cdot 5$

$5^3 = 25 \cdot 4 + 25$
$= 100 + 25$
$= 125$

EXERCISES 3.2

5. $\log_5 125 = \log_5 5^3 = 3 \log_5 5 = 3 \cdot 1 = 3$

6. $\log_4\left(\frac{1}{8}\right) = \log_4 4^{?} = ? \log_4 4 = ? \cdot 1 = ?$

7. $\log_{1/3} 3^{2x}$

$8^{-1} = \frac{1}{8} = 4^{?}$

$2^{-3} = 4^{?}$

$(2^3)^{-1} = 2^{-3} = 8^{-1} = 4^{?}$

$\log_a 1 = 0$

$x = a^y \iff y = \log_a x$

$1 = a^y \iff y = \log_a 1$

$\Downarrow$

$y = 0$

$\log_a a = y$

$x = a^y \iff y = \log_a x$

$\log_a a = 1$

$a = a^y$

uzmi $x = a$

$y = \log_a a$

$y = ?$

$y = 1$

derivata av a^x

den 21 september 2020 11:58

$h_- \; e^a$

,

EXERCISES 3.2

35. Suppose that $f(x) = a^x$ is differentiable at $x = 0$ and that $f'(0) = k$, where $k \neq 0$. Prove that f is differentiable at any real number x and that

$$f'(x) = k\,a^x = k\,f(x).$$

$k = \ln a$

36. Continuing Exercise 35, prove that $f^{-1}(x) = \log_a x$ is differentiable at any $x > 0$ and that

$$(f^{-1})'(x) = \frac{1}{kx}.$$

https://en.wikipedia.org/wiki/E_(mathematical_constant)

$a^{x+h} = a^x \cdot a^h$

$$\frac{d}{dx}a^x \overset{DD}{=} \lim_{h\to 0}\frac{a^{x+h}-a^x}{h} = \lim_{h\to 0}\frac{a^x a^h - a^x}{h}$$

$$= a^x \cdot \underbrace{\left(\lim_{h\to 0}\frac{a^h-1}{h}\right)}_{f'(0) = k = \ln a} = a^x \cdot k$$

$$DD:\; f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$k = \lim_{h\to 0}\frac{a^h-1}{h} = \left\{\begin{array}{l} \text{variabel byte:} \\ a^h - 1 = u \\ a^h = u+1 \quad \mid \log_a \\ \log_a a^h = \log(u+1) \\ \quad\shortparallel \\ h\log_a a = h\cdot 1 = h \end{array}\right\} = \lim_{u\to 0}\frac{u}{\log_a(1+u)} =$$

$L = \log_a(1+u)$

$$\frac{u}{L} = \frac{u/u}{L/u} = \frac{1}{L/u} = \frac{1}{\frac{L}{u}} = \frac{1}{\frac{1}{u}\cdot L}$$

$x\log y = \log y^x$

$$= \lim_{u\to 0}\frac{1}{\frac{1}{u}\log_a(1+u)} = \lim_{u\to 0}\frac{1}{\log_a(1+u)^{\frac{1}{u}}}$$

$$\lim\frac{f}{g} = \frac{\lim f}{\lim g}$$

$\lim \quad 1$

1

$$= \frac{\lim_{u\to 0} 1}{\lim_{u\to 0} \log_a (1+u)^{\frac{1}{u}}} = \frac{1}{\lim_{u\to 0} \log_a (1+u)^{\frac{1}{u}}}$$

$$= \log_a \left[\lim_{u\to 0} (1+u)^{\frac{1}{u}} \right]$$

OK

APPENDIX III Continuous Functions

THEOREM 1

Combining continuous functions

(a) ~~If f and g are continuous at the point a, then so are $f+g$, $f-g$, fg, and, if $g(a) \neq 0$, f/g.~~

(b) If f is continuous at the point L and if $\lim_{x\to a} g(x) = L$, then we have

$$\lim_{x\to a} f(g(x)) = f(L) = f\left(\lim_{x\to a} g(x)\right).$$

$\log_a e$

$f(\delta) = \log_a \delta$

$g(u) = (1+u)^{\frac{1}{u}}$

$f(e) = \lim_{\delta\to e} f(\delta)$

$$\lim_{u\to 0} g(u) = \lim_{u\to 0} (1+u)^{\frac{1}{u}} = e = L$$

$$= \frac{1}{\log_a \left[\lim_{u\to 0} (1+u)^{\frac{1}{u}} \right]} =$$

$$= \frac{1}{\log_a e}$$

$$\boxed{\frac{d}{dx} a^x = \frac{a^x}{\log_a e} = \dots = a^x \ln a}$$

def $a = e$

$\log_e \overset{\text{def}}{=} \ln$

$\log_e a = \ln a$

(vi) $\log_a x = \dfrac{\log_b x}{\log_b a}$

$b \to a$

$$\ln a = \log_e a = \frac{\log_b a}{\log_b e} = \frac{\log_a a}{\log_a e} = \frac{1}{\log_a e}$$

$\log_a a = 1$

$a \to e$
$x \to a$
b lämna kvar

borde du inte skriva att det ska vara (e^x)*x' $= 1$

$y = e^x \quad y' = e^x \ln e = e^x 1 = e^x$

$$\boxed{(e^x)' = e^x}$$

$f(x)$, $f'(x) = f(x)$, hur många f finns?

derivatan av 0 är 0

$f(x) = 0$, $f'(x) = 0$

$f(x) = 1$, $f'(x) = 1' = 0 \neq f(x)$

$f(x) = e^x$, $f(x) = 10e^x$ $f'(x) = 10(e^x)' = 10e^x$

$\stackrel{*}{=} f(x)$

$f(x) = A \cdot e^x$, $f'(x) = f(x)$

$f(x) = Ae^{2x}$, $f'(x) \stackrel{?}{=} f(x)$ Svar: nej derivata av log_a x

den 21 september 2020 11:58

EXERCISES 3.2

36. Continuing Exercise 35, prove that $f^{-1}(x) = \log_a x$ is differentiable at any $x > 0$ and that

$$(f^{-1})'(x) = \frac{1}{kx}.$$

35. Suppose that $f(x) = a^x$ is differentiable at $x = 0$ and that $f'(0) = k$, where $k \neq 0$. Prove that f is differentiable at any real number x and that

$$f'(x) = k\,a^x = k\,f(x).$$

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If $a > 0$ and $a \neq 1$, the function $\log_a x$, called **the logarithm of** x **to the base** a, is the inverse of the one-to-one function a^x:

$$y = \log_a x \iff x = a^y, \qquad (a > 0,\quad a \neq 1).$$

Strategi: att använda implicit derivata.

$$x = a^{y(x)} \quad \Big|\ \frac{d}{dx}$$

$$\frac{d}{dx}x = \frac{d}{dx}a^y$$

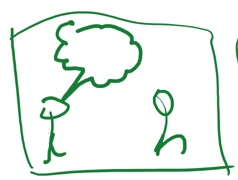
$$1 = \frac{da^y}{dx} = \underbrace{\frac{da^y}{\boxed{y}} \cdot \frac{\boxed{y}}{dx}}_{\text{leder ingenstans}}$$

$$\frac{da^y}{y} \leftrightarrow \frac{0}{1} = 0 \quad \text{ger ingenting}$$

$$\frac{y}{dx} \leftrightarrow \frac{1}{0} = \infty$$

$$\frac{da^y}{y} \cdot \frac{y}{dx} = 0 \cdot \infty$$

det här blir bra: $\underbrace{\frac{0}{0}}_{1} \quad \underbrace{\frac{0}{0}}_{1}$



$$1 = \frac{da^y}{dy} \cdot \frac{dy}{dx}$$

$$= (a^y)'_y\, y'_x$$

$$1 = a^y \cdot \ln a\, y'$$

$$y' = \frac{1}{a^y \ln a} = \frac{1}{x \ln a}$$

$$y = \log_a x \implies \frac{dy}{dx} = \frac{1}{x \ln a}$$

$\log_e = \ln$

$$y = \ln x \implies y' = \frac{1}{x \cdot \ln e} = \frac{1}{x}$$

den naturliga logaritmen