W4 – RÖ2 derivata tillämpningar

super komplicerade limes som är svåra att räkna utan derivata (L’Hopital) eller lim f(g)=f(lim g) trick, eller som kräver en varierande teknik för att hitta (tex man måste ta ln först); vi tar ”saftiga” problem från ADAMS och gamla tentor

Johann Bernoulli

Guillaume de l'Hôpital

Sur L'Hopital = Regeln går INTE att använda

Glad L'Hopital = Regeln går att använda

Table 4. Types of indeterminate forms

Type	Example
$[0/0]$	$\lim_{x\to 0} \dfrac{\sin x}{x}$
$[\infty/\infty]$	$\lim_{x\to 0} \dfrac{\ln(1/x^2)}{\cot(x^2)}$
$[0\cdot\infty]$	$\lim_{x\to 0+} x \ln \dfrac{1}{x}$
$[\infty-\infty]$	$\lim_{x\to(\pi/2)-} \left(\tan x - \dfrac{1}{\pi - 2x}\right)$
$[0^0]$	$\lim_{x\to 0+} x^x$
$[\infty^0]$	$\lim_{x\to(\pi/2)-} (\tan x)^{\cos x}$
$[1^\infty]$	$\lim_{x\to\infty} \left(1+\dfrac{1}{x}\right)^x$

The first l'Hôpital Rule

Suppose the functions f and g are differentiable on the interval (a, b), and $g'(x) \neq 0$ there. Suppose also that

(i) $\boxed{\lim_{x\to a+} f(x) = \lim_{x\to a+} g(x) = 0}$ and

(ii) $\lim_{x\to a+} \dfrac{f'(x)}{g'(x)} = L$ (where L is finite or ∞ or $-\infty$).

Then

$$\lim_{x\to a+} \frac{f(x)}{g(x)} = L.$$

Similar results hold if every occurrence of $\lim_{x\to a+}$ is replaced by $\lim_{x\to b-}$ or even $\lim_{x\to c}$ where $a < c < b$. The cases $a = -\infty$ and $b = \infty$ are also allowed.

The second l'Hôpital Rule

Suppose that f and g are differentiable on the interval (a, b) and that $g'(x) \neq 0$ there. Suppose also that

(i) $\boxed{\lim_{x\to a+} g(x) = \pm\infty \text{ and}}$

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Then

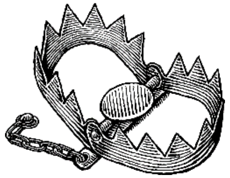
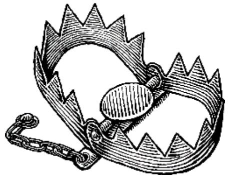
$$\lim_{x\to a+} \frac{f(x)}{g(x)} = L.$$

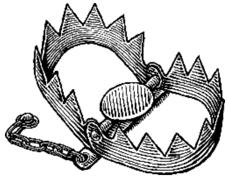
Again, similar results hold for $\lim_{x\to b-}$ and for $\lim_{x\to c}$, and the cases $a = -\infty$ and $b = \infty$ are allowed.

gamla limes som kan fixas algebraisk, men nu räknar vi de en gång till med L'Hopital; AD1.3

1. $\displaystyle\lim_{x\to\infty} \frac{x}{2x-3}$ **4.** $\displaystyle\lim_{x\to-\infty} \frac{x^2-2}{x-x^2}$ **5.** $\displaystyle\lim_{x\to-\infty} \frac{x^2+3}{x^3+2}$ 

7. $\displaystyle\lim_{x\to\infty} \frac{3x+2\sqrt{x}}{1-x}$  **10.** $\displaystyle\lim_{x\to-\infty} \frac{2x-5}{|3x+2|}$ 

 **15.** $\displaystyle\lim_{x\to-5/2} \frac{2x+5}{5x+2}$  **16.** $\displaystyle\lim_{x\to-2/5} \frac{2x+5}{5x+2}$ 

 **19.** $\displaystyle\lim_{x\to2+} \frac{x}{(2-x)^3}$ 

ADAMS/4.3 limes som kräver L'Hopital (att värma upp)

Evaluate the limits

G **1.** $\lim_{x\to 0} \frac{3x}{\tan 4x}$

G **2.** $\lim_{x\to 2} \frac{\ln(2x-3)}{x^2-4}$

G **3.** $\lim_{x\to 0} \frac{\sin ax}{\sin bx}$

G **4.** $\lim_{x\to 0} \frac{1-\cos ax}{1-\cos bx}$

G **5.** $\lim_{x\to 0} \frac{\sin^{-1} x}{\tan^{-1} x}$

G **12.** $\lim_{x\to 1} \frac{\ln(ex)-1}{\sin \pi x}$

G **9.** $\lim_{t\to \pi} \frac{\sin^2 t}{t-\pi}$

G

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ADAMS/4.3 lite svårare limes som kräver L'Hopital + logaritmering

Evaluate the limits

VG **24.** $\lim_{x \to 0+} x^{\sqrt{x}}$

VG **10.** $\lim_{x \to 0} \frac{10^x - e^x}{x}$

VG $\lim_{x \to \infty} \frac{\ln x^2}{2 + \ln x}$

VG **6.** $\lim_{x \to 1} \frac{x^{1/3} - 1}{x^{2/3} - 1}$

VG **16.** $\lim_{x \to 0} \frac{2 - x^2 - 2\cos x}{x^4}$

VG **33.** **(A Newton quotient for the second derivative)** Evaluate $\lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$ if f is a twice differentiable function.

VG **34.** If f has a continuous third derivative, evaluate

$$\lim_{h \to 0} \frac{f(x+3h) - 3f(x+h) + 3f(x-h) - f(x-3h)}{h^3}$$

ADAMS/4.3 väldigt svåra limes som kräver L'Hopital och/eller logaritmering

Evaluate the limits

MVG **25.** $\lim_{x \to 0+} (\csc x)^{\sin^2 x}$

MVG **26.** $\lim_{x \to 1+} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right)$

MVG **27.** $\lim_{t \to 0} \frac{3 \sin t - \sin 3t}{3 \tan t - \tan 3t}$

MVG **28.** $\lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{1/x^2}$

MVG **31.** $\lim_{x \to 1-} \frac{\ln \sin \pi x}{\csc \pi x}$

MVG **32.** $\lim_{x \to 0} (1 + \tan x)^{1/x}$

svåra limes från gamla tentor, kräver en viss känsla, mvg problem

MVG $$\lim_{x\to\infty} \frac{1+\sin x + x}{x \arctan x}$$

MVG $$\lim_{x\to 0+} \frac{e^{-\frac{1}{x}}}{x^2}$$