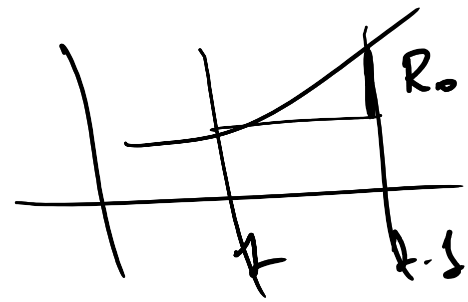
W4 Sammanfattning

den 24 september 2020 15:20

$f(1) \longrightarrow f(x)$ $\qquad x = 1 + \underbrace{0.1}_{\Delta x}$

$f(1.1)$



$f(1.1) \approx f(1)$

$f(1.1) = f(1) + R_0$

Med. v. satsen. $\qquad$ * f kont $[a,b]$

* f der (a,b)

$$\frac{f(b)-f(a)}{b-a} = f'(c) \qquad c \in (a,b)$$

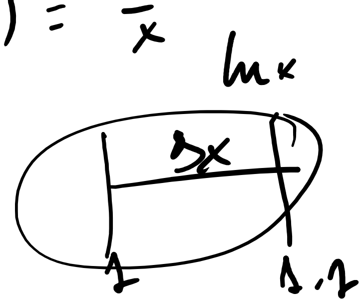
$f(x) = \ln x \qquad \ln 1 = 0 \qquad \ln 1.1 \approx \ln 1$

$f(1),\ f(1.1)$ $\qquad f(x)$ kont i $[1, 1.1]$

$f(x)$ der. i $(1, 1.1)$ $\qquad c \in (1, 1.1)$

$$\frac{f(1.1)-f(1)}{1.1-1} = f'(c) = \frac{1}{c} \qquad (\ln x)' = \frac{1}{x}$$

alg.



$f(1.1) - f(1) = \frac{1}{c} \cdot \underbrace{(1.1 - 1)}_{\Delta x}$

$f(1.1) = f(1) + \frac{1}{c}\, 0.1$

$$f(1.1) = f(1) + \frac{1}{\xi}\,0.1$$
$$f(1.1) = f(1) + R$$
$$R = \frac{1}{\xi}\,0.1$$

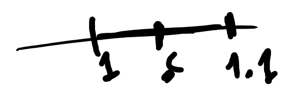
$$1 < \xi < 1.1$$

$$\frac{1}{1} > \frac{1}{\xi} > \frac{1}{1.1}$$

$$0.909 = \frac{1}{1.1} < \frac{1}{\xi} < 1 \quad \Big| \cdot 0.1$$

$$0.909 \cdot 0.1 < \overset{R}{\frac{1}{\xi}\,0.1} < 1 \cdot 0.1$$

Δx

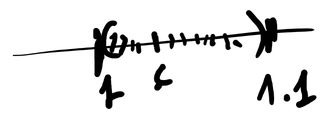


$$f(1.1) = f(1) + R_0 \qquad R_0 = f'(\xi)\,(1.1 - 1)$$

$$R_0 = f'(\xi) \cdot \Delta x$$

$$f(1.1) = \underbrace{f(1) + f'(1)\,\Delta x}_{T_1} + \overset{\text{error}}{R_1} \qquad \boxed{R_1 = \frac{1}{2} f''(\xi)\,\Delta x^2}$$

x ≈ Linjär approximation



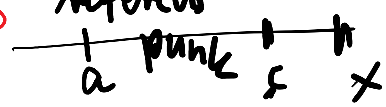
$$f(x) = T_1(x) + R_1 \; ; \qquad T_1(x) = f(a) + f'(a)(x-a)$$

det jag sa i videon är fel.

$$\cancel{R_1 = f'(\xi)(x-a) \quad \xi \in (a,x)}$$

$$R_1 = \frac{1}{2!} f''(\xi)(x-a)^2 \quad \xi \in (a,x)$$

referens punkt



$$x = a + \Delta x$$

$$f(x) = T_2(x) + R_2 \qquad T_2(x) = f(a) + f'(a)(x-a)$$

$$f(x) = T_2(x) + R_2 \qquad T_2(x) = f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2$$

$$R_2 = \frac{1}{3!} f^{(3)}(\xi)(x-a)^3$$

$\xi \in (a, x)$

$$\boxed{f(x) = T_n(x) + R_n \qquad \xi \in (a,x) \qquad T_n(x) = f(a) + \frac{1}{1!}f'(a)(x-a) + \dots + \frac{1}{n!} f^{(n)}(a)(x-a)^n \qquad R_n = \frac{1}{(n+1)!} f^{(n+1)}(\xi)(x-a)^{n+1}}$$

Varför ser T_n ut som den ser ut?

$P(x)$,

$P(a) = f(a)$ $\quad n=0$

$P'(a) = f'(a)$ $\quad n=1$

$P''(a) = f''(a)$ $\quad n=2$

$\vdots$

$$(x^n)^{(1)} = n x^{n-1}$$

$$(x^n)^{(2)} = n(n-1) x^{n-2}$$

$\vdots$

$$(x^n)^{(n)} = n(n-1)(n-2)\cdots 1\, x^{n-(n)} = n!$$