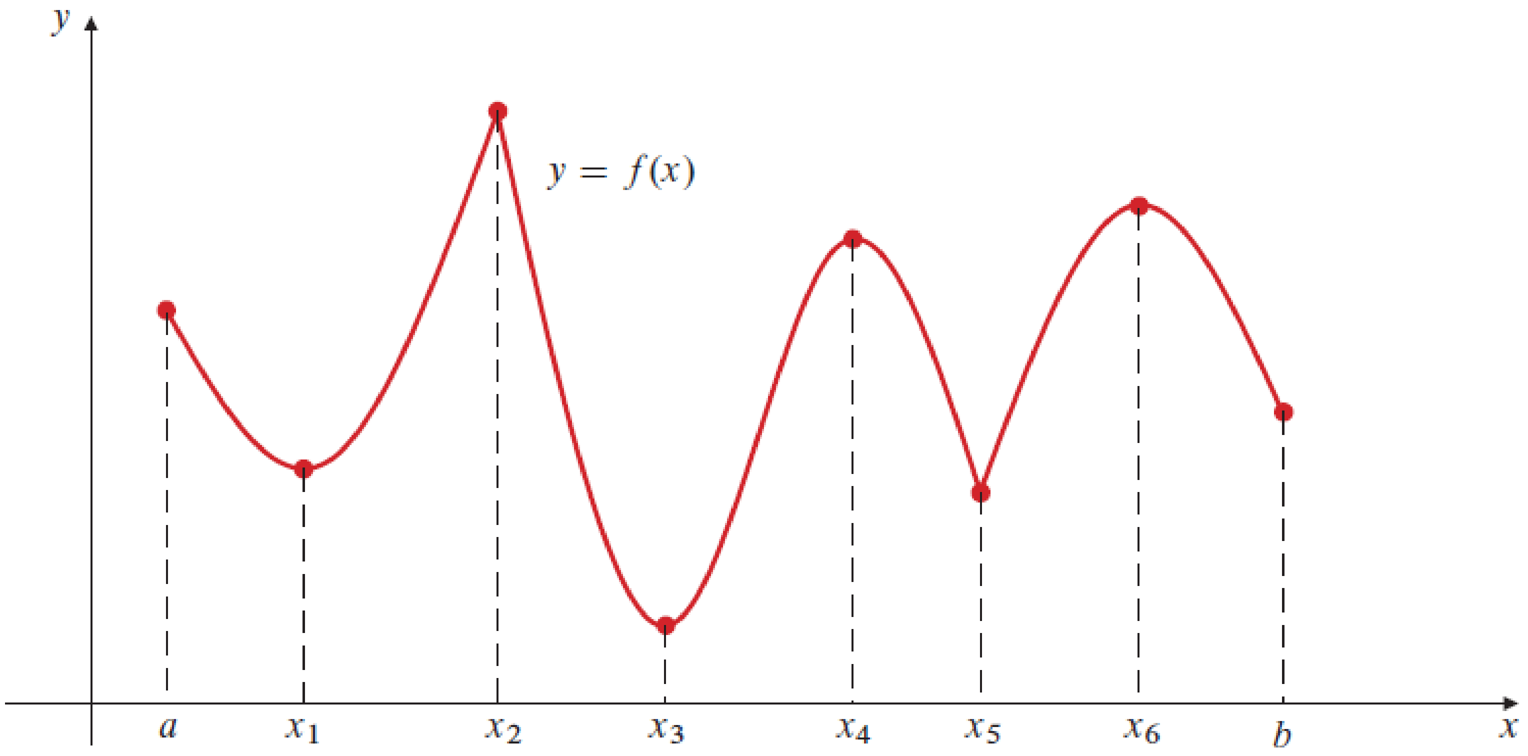
W5 – RÖ2 derivata tillämpningar

rita grafer; kommentarer i grått eller grafer, skall inte finnas tillgängliga på tentan. de finns nu för att underlätta övningen.

Typer av viktiga punkter: identifiera alla typer i grafen



KP – kritisk punkt

SP – singulär punkt

IP – inflektions punkt

locMin – lokal minimum

locMax – lokal maximum

gMin – global minimum

gMax – global maximum

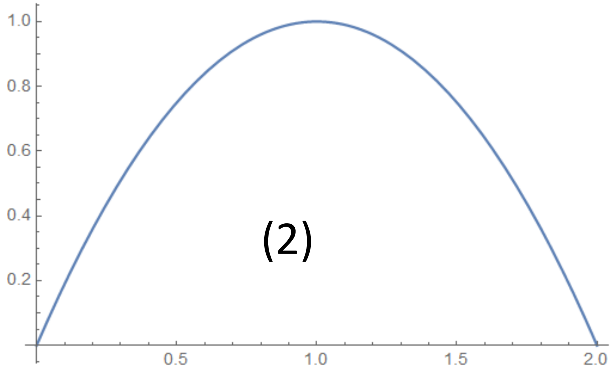
ADAMS/4.5 concavity and critical points

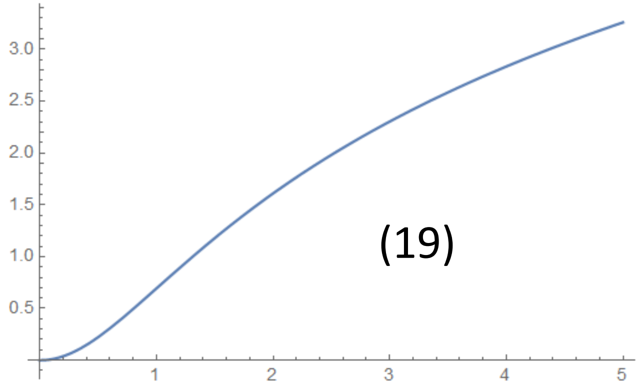
Determine the intervals of ”constant” concavity of the given function, and locate any inflection points

(G) AD/4.5-Prob 1: $f(x) = x$

(G) AD/4.5-Prob 2: $f(x) = 2x - x^2$

(VG) AD/4.5-Prob 19: $f(x) = \ln(1 + x^2)$





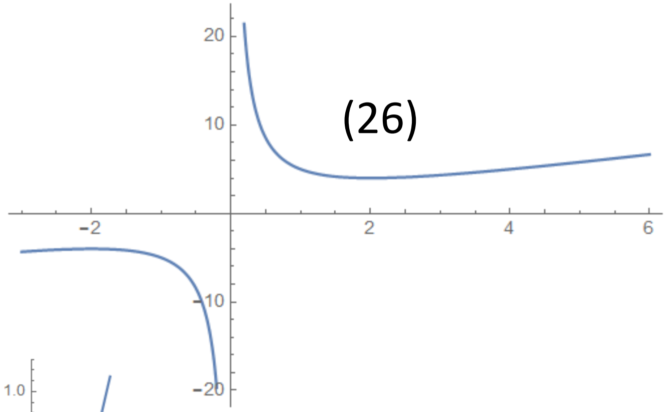
“constant concavity” = där f’’(x) har samma tecken

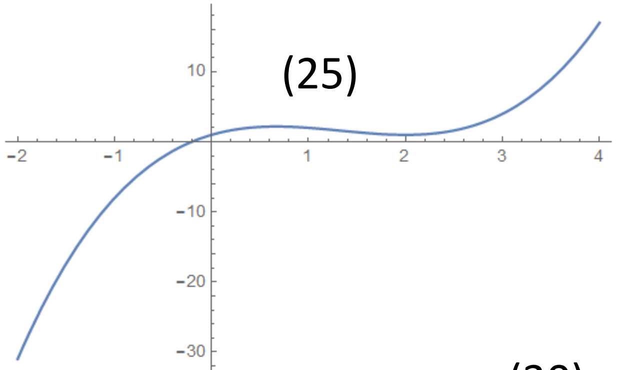
Classify the critical points of the functions in the Exercises below using the Second Derivative test (whenever possible).

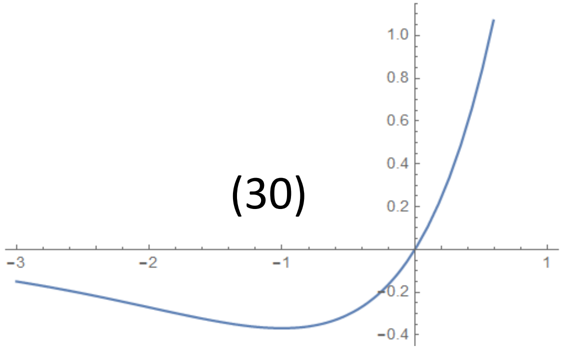
(G) AD/4.5-Prob 25: $f(x) = x(x-2)^2 + 1$

(G) AD/4.5-Prob 26: $f(x) = x + \frac{4}{x}$

(VG) AD/4.5-Prob 30: $f(x) = xe^x$







“critical points” = där f’(x) =0

ADAMS/4.6 sketching graphs

Sketch the graph of a given function, making use of any suitable information you can obtain from the function and its first and second derivatives

(G) AD/4.6-Prob 10: $f(x) = \frac{x^2-2}{x^2-1}$

(G) AD/4.6-Prob 30: $f(x) = e^{-x^2}$

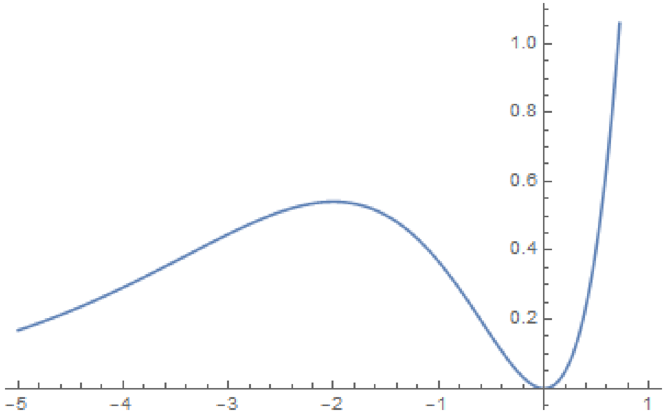
(MVG) AD/4.6-Prob 32: $f(x) = e^{-x} \sin x$

Detta är en funktion som är svårt att rita utan analys. Man skulle kunna tro att den inte har flera lokala max/min värdena med den har ett oändlig antal sådana

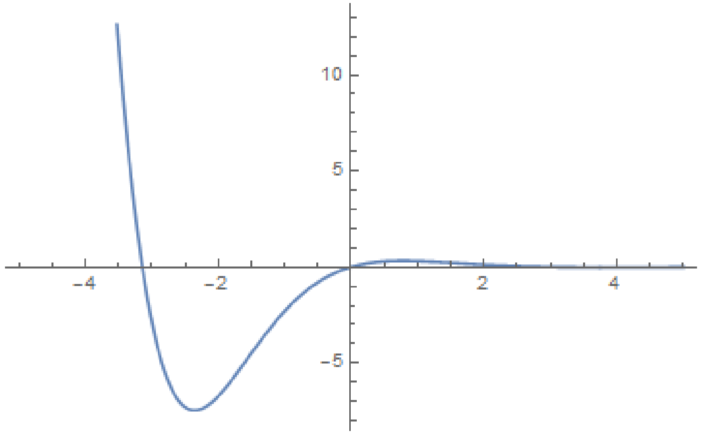
(VG) AD/4.6-Prob 34: $f(x) = x^2 e^x$

(MVG) AD/4.6-Prob 29: $f(x) = x + 2 \sin x$

$\text{Plot}[x\wedge 2\text{Exp}[x], \{x, -5,1\}]$



$\text{Plot}[\text{Exp}[-x]\text{Sin}[x], \{x, -5,5\}]$



tenta övning (rita grafer, MVG)

Betrakta funktionen:

$$f(x) = \sin(e^x)$$

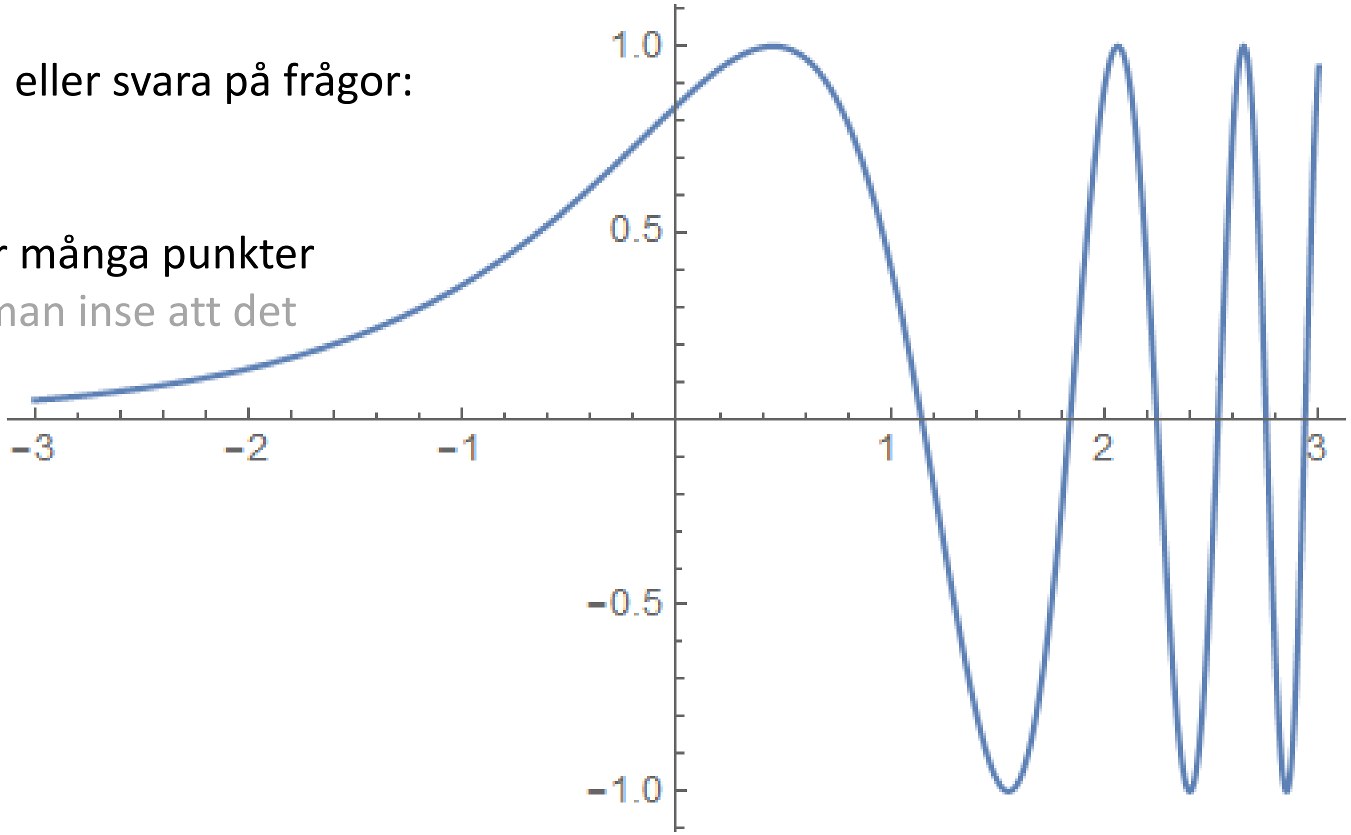
Använd alla dina kunskaper i analys för att göra följande, eller svara på frågor:

(a) Vad är definitions mängden?

(b) Rita grafen av funktionen (det bästa du kan).

(c) Har funktionen den globala max värden? Om ja, i hur många punkter förekommer den? I vilken punkt/punkter? Här skall man inse att det förekommer i oändlig många punkter.
Det går att hitta en explicit formeln för alla punkter.

(d) Samma som i (c) men för den globala min värdena.



optimering EXERCISES 4.8

2. Two positive numbers have product 8. What is the smallest possible value for their sum?

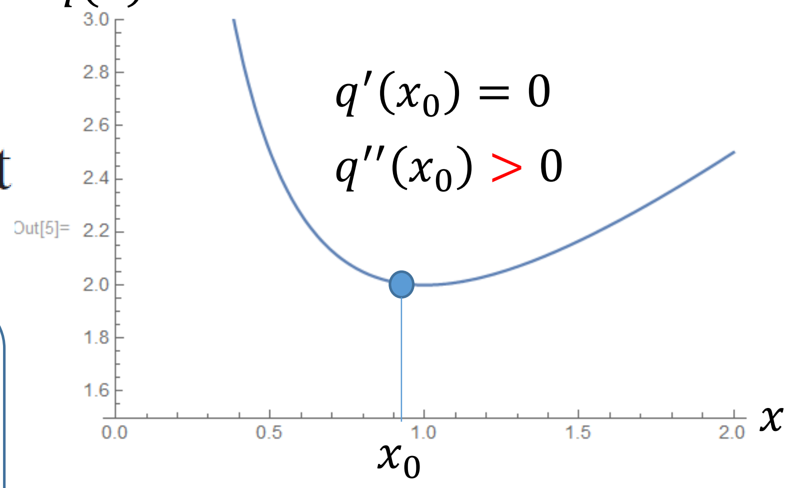
G

x, y och $xy = 8$, $q = x + y$

villkoren definierar $q(x) = x + y(x)$ med $y(x) = 8/x$ alltså $q(x) = x + 8/x$

$\min\limits_{x>0} \boldsymbol{q}(\boldsymbol{x}) = ?$

```
Plot[x + 1 / x, {x, 0, 2},
 PlotRange → {1.5, 3}]
```



4. Two numbers have sum 16. What are the numbers if the product of the cube of one and the fifth power of the other is as large as possible?

G

x, y och $x + y = 16$, $q(x) = x^3 y(x)^5 = x^3(16 - x)^5$

$\boldsymbol{x}_* = \underset{x>0}{\mathrm{argmax}}\, \boldsymbol{q}(\boldsymbol{x}) = ?$

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Plot[x^3 (16 - x) ^5 , {x, -2, 20}]
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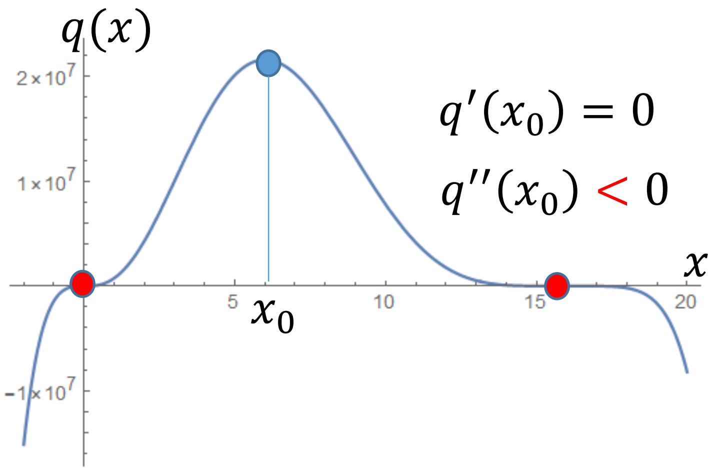
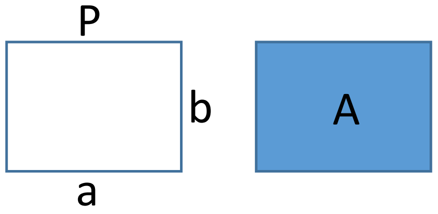


optimering EXERCISES 4.8

VG **8.** Among all rectangles of given perimeter, show that the square has the greatest area.

perimeter = The total distance around the outside of a rectangle



$P = 2(a+b)$ $A = ab$

$P = konst \ \rightarrow b = B_P(a)$

$B_P(a) = P/2 - a$

$A = f_P(a) = a(P/2 - a)$

$a_* = \underset{a}{\operatorname{argmax}} f_P(a)$

28. **(Getting around a corner)** Find the length of the longest beam that can be carried horizontally around the corner from a hallway of width a m to a hallway of width b m. (See Figure 4.56; assume the beam has no width.)

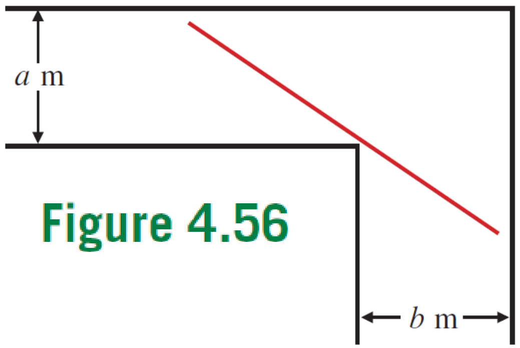
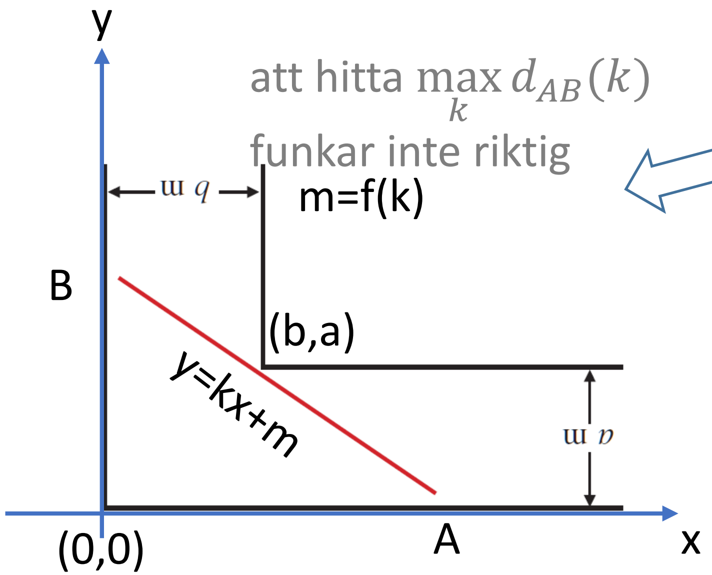


Figure 4.56

MVG[2]

att hitta $\max_{k} d_{AB}(k)$

funkar inte riktig

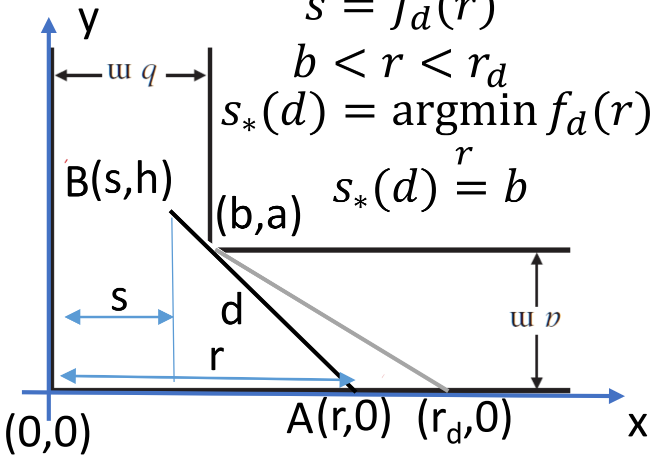


$s = f_d(r)$

$b < r < r_d$

$s_*(d) = \underset{r}{\operatorname{argmin}} f_d(r)$

$s_*(d) \overset{?}{=} b$



EXERCISES 4.1

4.1 Related rates (enklare problem)

1. Find the rate of change of the area of a square whose side is 8 cm long, if the side length is increasing at 2 cm/min.

2. The area of a square is decreasing at 2 ft^2/s. How fast is the side length changing when it is 8 ft?

9. How fast is the surface area of a cube changing when the volume of the cube is 64 cm^3 and is increasing at 2 cm^3/s?

How to solve related-rates problems

1. Read the problem very carefully. Try to understand the relationships between the variable quantities. What is given? What is to be found?
5. Differentiate the equation(s) implicitly with respect to time, regarding all variable quantities as functions of time. You can manipulate the equa-

4.1 Related rates (svårare problem)

EXERCISES 4.1

15. The point P moves so that at time t it is at the intersection of the curves $xy = t$ and $y = tx^2$. How fast is the distance of P from the origin changing at time $t = 2$?

18. The top of a ladder 5 m long rests against a vertical wall. If the base of the ladder is being pulled away from the base of the wall at a rate of 1/3 m/s, how fast is the top of the ladder slipping down the wall when it is 3 m above the base of the wall?

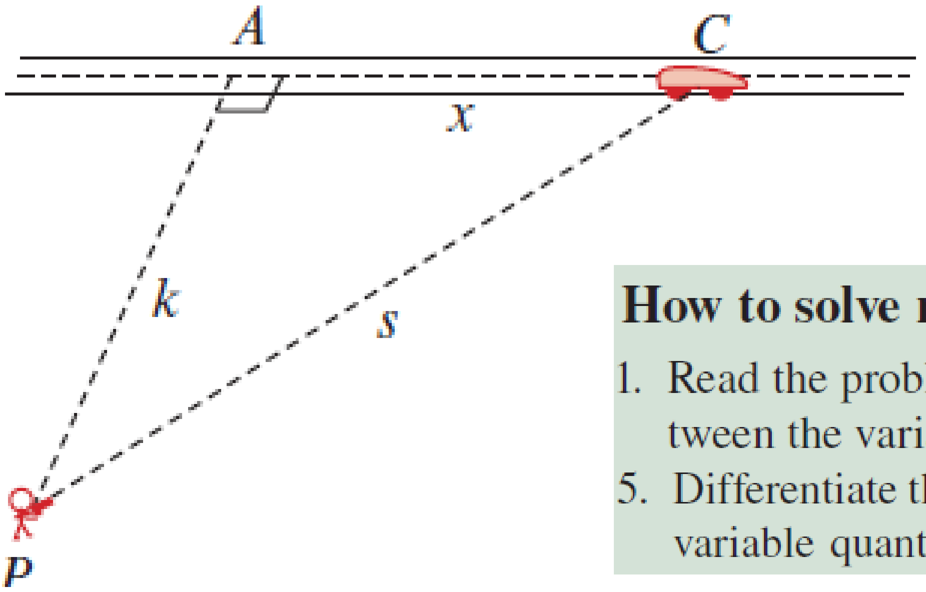
How to solve related-rates problems

1. Read the problem very carefully. Try to understand the relationships between the variable quantities. What is given? What is to be found?
5. Differentiate the equation(s) implicitly with respect to time, regarding all variable quantities as functions of time. You can manipulate the equa-

4.1 Related rates (svårare problem)

EXERCISES 4.1

16. **(Radar guns)** A police officer is standing near a highway using a radar gun to catch speeders. (See Figure 4.6.) He aims the gun at a car that has just passed his position and, when the gun is pointing at an angle of 45° to the direction of the highway, notes that the distance between the car and the gun is increasing at a rate of 100 km/h. How fast is the car travelling?



How to solve related-rates problems

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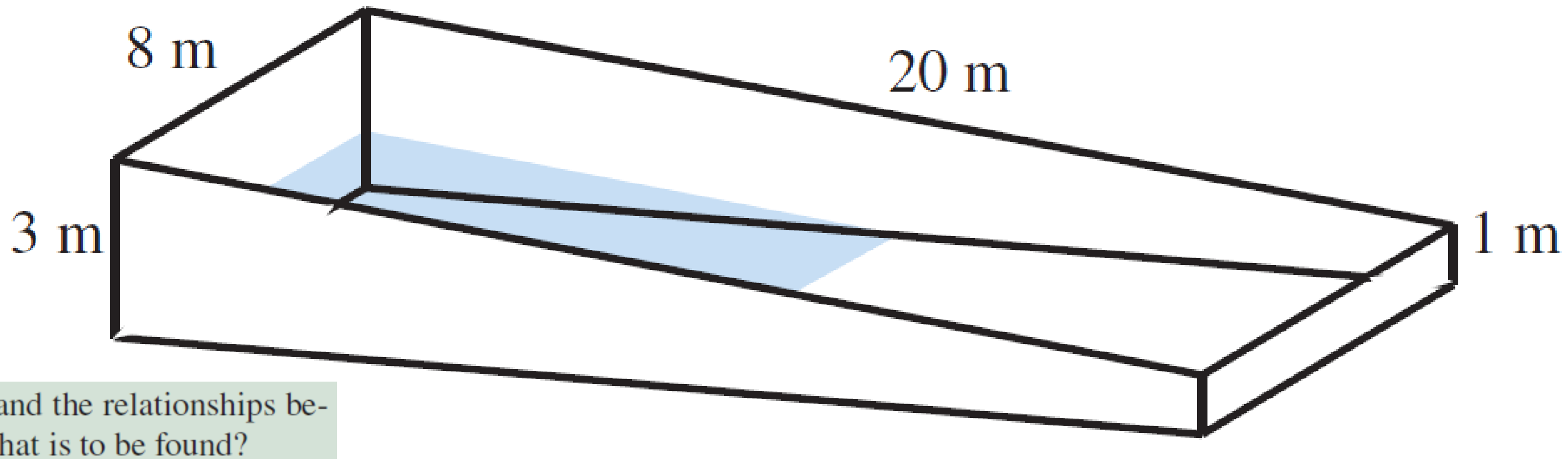
4.1 Related rates (svårare problem)

36. (Draining a pool) A rectangular swimming pool is 8 m wide and 20 m long. (See Figure 4.7.) Its bottom is a sloping plane, the depth increasing from 1 m at the shallow end to 3 m at the deep end. Water is draining out of the pool at a rate of 1 m^3/min. How fast is the surface of the water falling when the depth of water at the deep end is (a) 2.5 m? (b) 1 m?



How to solve related-rates problems

1. Read the problem very carefully. Try to understand the relationships between the variable quantities. What is given? What is to be found?
5. Differentiate the equation(s) implicitly with respect to time, regarding all variable quantities as functions of time. You can manipulate the equa-

4.1 Related rates (mycket svår problem)

38. Two crates, A and B, are on the floor of a warehouse. The crates are joined by a rope 15 m long, each crate being hooked at floor level to an end of the rope. The rope is stretched tight and pulled over a pulley P that is attached to a rafter ' above a point Q on the floor directly between the two crates. (See Figure 4.9.) If crate A is 3 m from Q and is being pulled directly away from Q at a rate of 1/2 m/s, how fast is crate B moving toward Q?

How to solve related-rates problems

1. Read the problem very carefully. Try to understand the relationships between the variable quantities. What is given? What is to be found?
5. Differentiate the equation(s) implicitly with respect to time, regarding all variable quantities as functions of time. You can manipulate the equa-

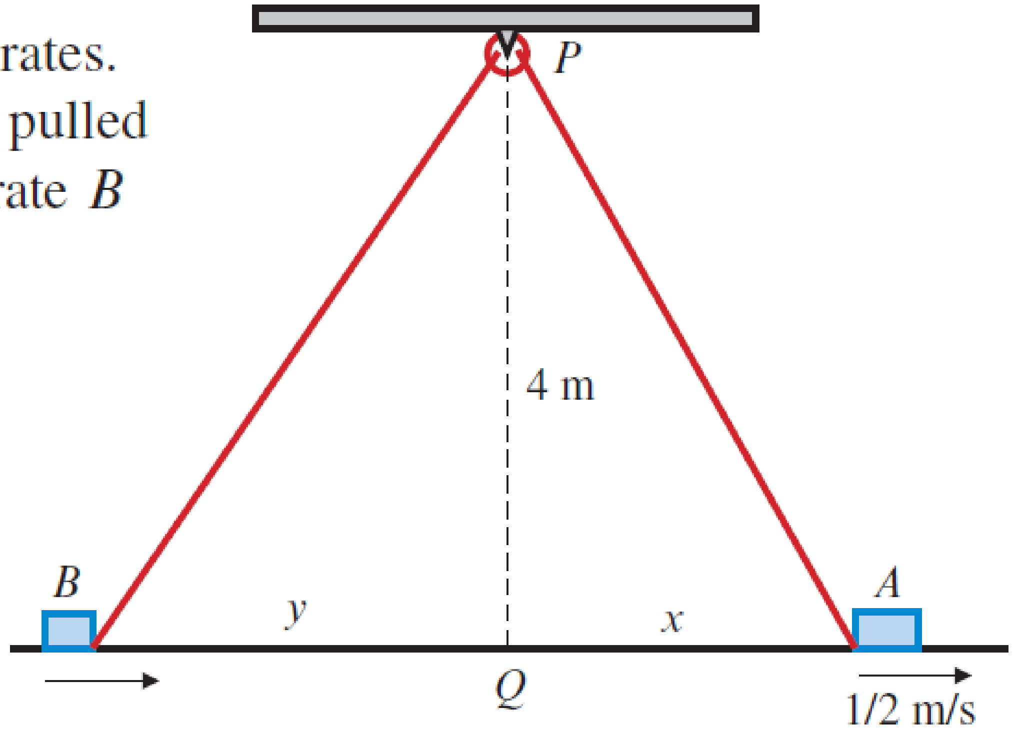


Figure 4.9