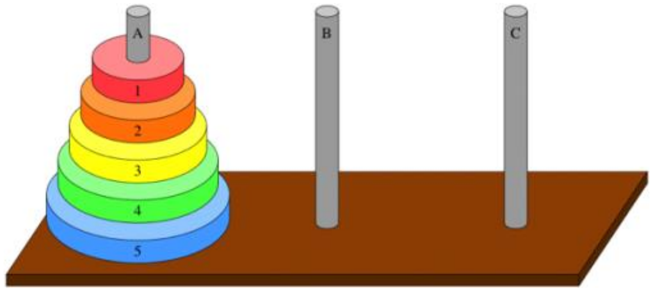
komplicerade volymer: towers of hanoi

den 6 oktober 2020 17:47



https://www.khanacademy.org/computing/computer-science/algorithms/towers-of-hanoi/a/towers-of-hanoi

7.1 Volumes by Slicing—Solids of Revolution

$$\int dV = \int_0^H dh \cdot A = A\int_0^H dh = A \cdot h\Big|_0^H = A \cdot H$$

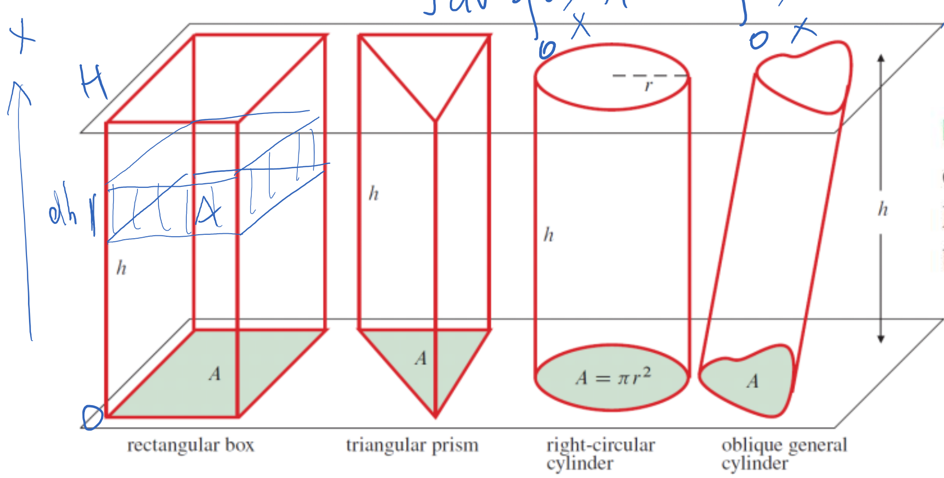


Figure 7.1 The volume of any prism or cylinder is the area A of its base times its height h (measured perpendicularly to the base): $V = Ah$

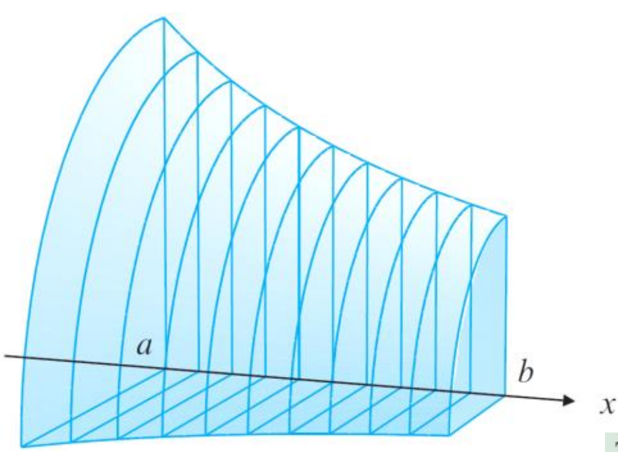


Figure 7.2 Slicing a solid perpendicularly to an axis

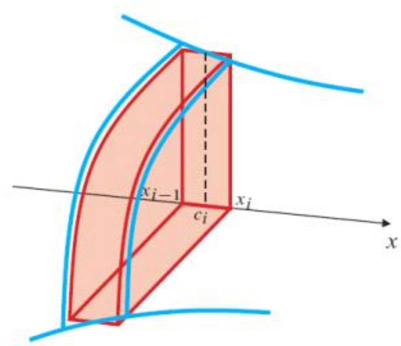


Figure 7.3 The volume of a slice

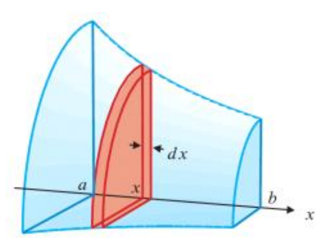


Figure 7.4 The volume element

The volume V of a solid between $x = a$ and $x = b$ having cross-sectional area $A(x)$ at position x is

$$V = \int_a^b A(x)\,dx.$$

$$V = \int_{x=a}^{x=b} dV, \qquad \text{where} \qquad dV = A(x)\,dx$$

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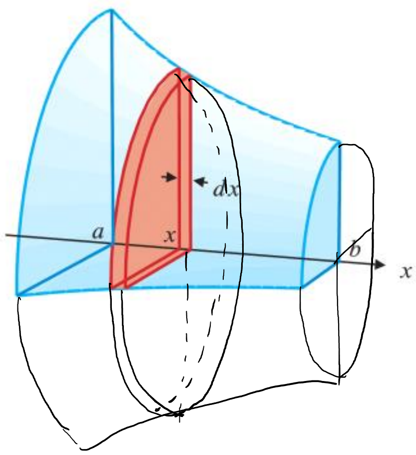
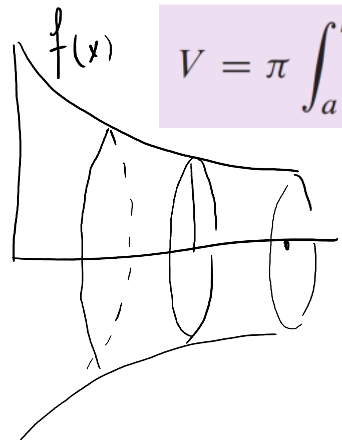
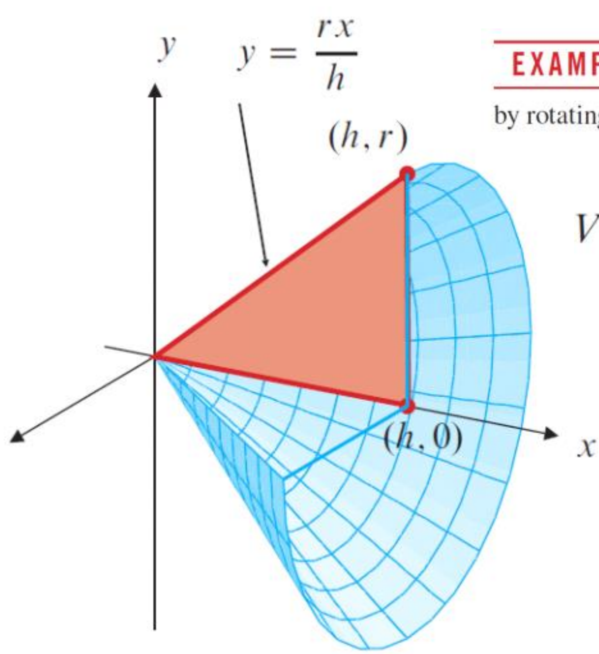
Solids of Revolution





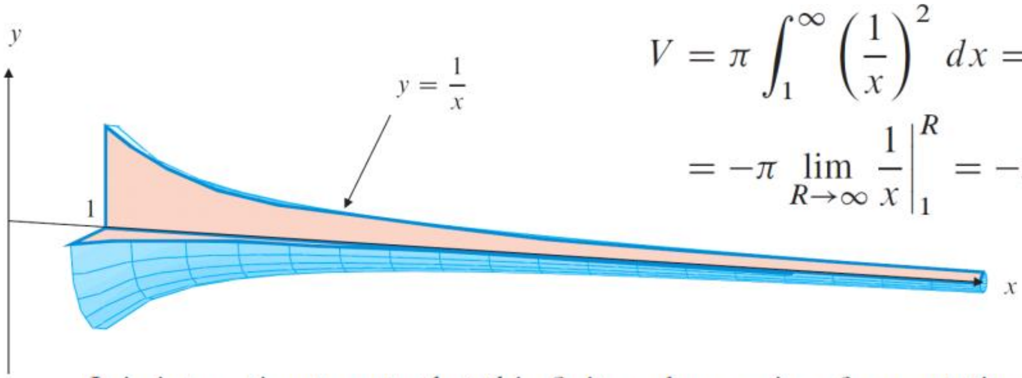
$$V = \pi \int_a^b (f(x))^2 \, dx$$



EXAMPLE 2 **(The volume of a right-circular cone)** Find the volume of the right-circular cone of base radius r and height h that is generated by rotating the triangle with vertices $(0, 0)$, $(h, 0)$, and (h, r) about the x-axis.

$$V = \pi \int_0^h \left(\frac{rx}{h}\right)^2 dx = \pi \left(\frac{r}{h}\right)^2 \frac{x^3}{3}\bigg|_0^h = \frac{1}{3}\pi r^2 h$$

EXAMPLE 3 Find the volume of the infinitely long horn that is generated by rotating the region bounded by $y = 1/x$ and $y = 0$ and lying to the right of $x = 1$ about the x-axis. The horn is illustrated in Figure 7.6.

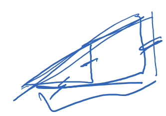


$$V = \pi \int_1^{\infty} \left(\frac{1}{x}\right)^2 dx = \pi \lim_{R\to\infty} \int_1^R \frac{1}{x^2}\, dx$$

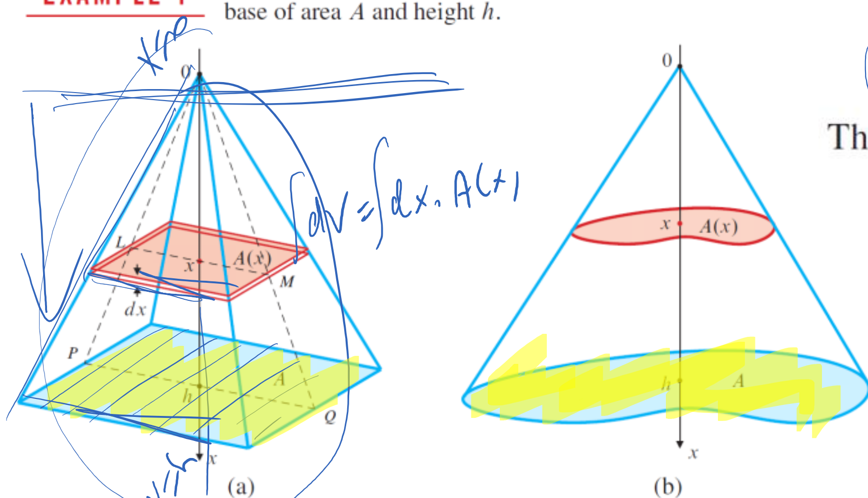
$$= -\pi \lim_{R\to\infty} \frac{1}{x}\bigg|_1^R = -\pi \lim_{R\to\infty} \left(\frac{1}{R} - 1\right) = \pi$$

It is interesting to note that this finite volume arises from rotating a region that itself has infinite area: $\int_1^{\infty} dx/x = \infty$. We have a paradox: it takes an infinite amount of paint to paint the region but only a finite amount to fill the horn obtained by rotating the region. (How can you resolve this paradox?)

7.2 More Volumes by Slicing



EXAMPLE 1 Verify the formula for the volume of a pyramid with rectangular base of area A and height h.



(a) (b)

$$A(x) = \left(\frac{x}{h}\right)^2 A.$$

$A(0) = 0$

$A(h) = \left(\frac{h}{h}\right)^2 A = A$

The volume of the pyramid is therefore

$$V = \int_0^h \left(\frac{x}{h}\right)^2 A\, dx = \frac{A}{h^2}\frac{x^3}{3}\bigg|_0^h = \frac{1}{3}Ah$$

$$= \frac{1}{h^2}A \int_0^h x^2\, dx$$

A similar argument, resulting in the same formula for the volume, holds for a cone, that is, a pyramid with a more general (curved) shape to its base, such as that in Figure 7.16(b). Although it is not as obvious as in the case of the pyramid, the cross-section at x still has area $(x/h)^2$ times that of the base. A proof of this volume formula for an arbitrary cone or pyramid can be found in Example 3 of Section 16.4.

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7.3 Arc Length and Surface Area

$$A = \int dA$$

$dl = ds$

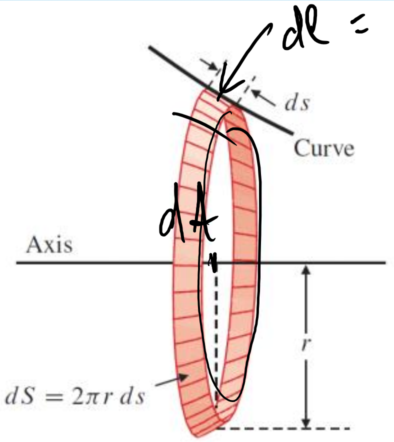
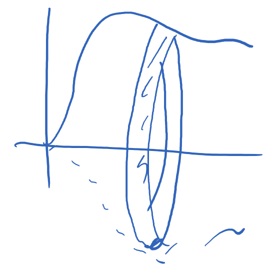


Figure 7.26 The circular band generated by rotating arc length element ds about the axis



$$d\cancel{S}A = 2\pi r\, ds$$

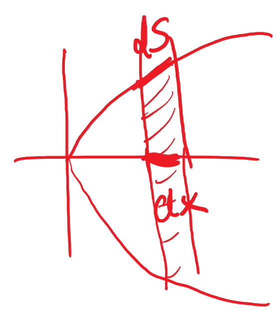
Area of a surface of revolution

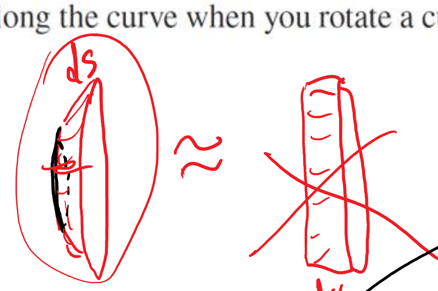
If $f'(x)$ is continuous on $[a, b]$ and the curve $y = f(x)$ is rotated about the x-axis, the area of the surface of revolution so generated is

$$\boxed{S = 2\pi \int_{x=a}^{x=b} |y|\, ds = 2\pi \int_a^b |f(x)| \underline{\sqrt{1 + (f'(x))^2}}\, dx.}$$

$$A = \int dA = \int 2\pi f(x) \cdot ds = 2\pi \int f(x)\, ds$$

Remark Students sometimes wonder whether such complicated formulas are actually necessary. Why not just use $dS = 2\pi|y|\,dx$ for the area element when $y = f(x)$ is rotated about the x-axis instead of the more complicated area element $dS = 2\pi|y|\,ds$? After all, we are regarding dx and ds as both being infinitely small, and we certainly used dx for the width of the disk-shaped volume element when we rotated the region under $y = f(x)$ about the x-axis to generate a solid of revolution. The reason is somewhat subtle. For small thickness Δx, the volume of a slice of the solid of revolution is only approximately $\pi y^2 \Delta x$, but the error is *small compared to the volume of this slice*. On the other hand, if we use $2\pi|y|\,\Delta x$ as an approximation to the area of a thin band of the surface of revolution corresponding to an x interval of width Δx, the error is *not small compared to the area of that band*. If, for instance, the curve $y = f(x)$ has slope 1 at x, then the width of the band is really $\Delta s = \sqrt{2}\,\Delta x$, so that the area of the band is $\Delta S = 2\pi\sqrt{2}|y|\,\Delta x$, not just $2\pi|y|\,\Delta x$. Always use the appropriate arc length element along the curve when you rotate a curve to find the area of a surface of revolution.

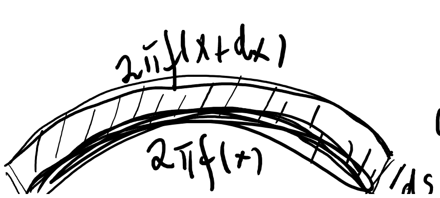




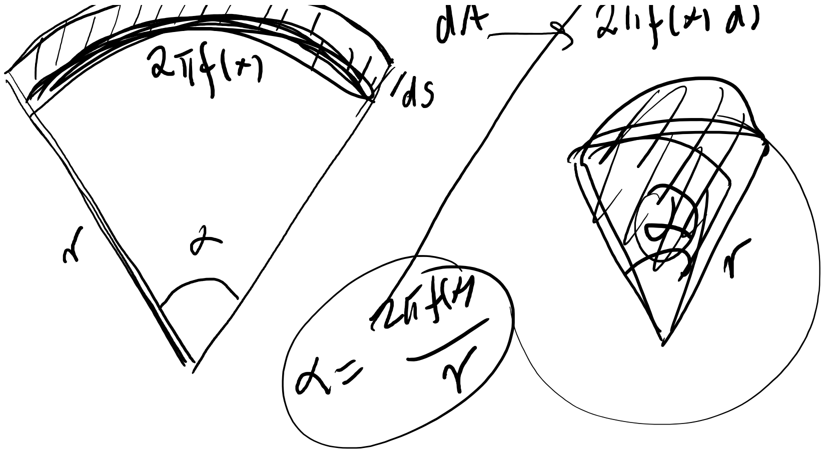
$$dV = 2r\,dr = \frac{2\pi f(x)}{r} \cdot r\,dr$$

$$dV = d\left(\frac{2r^2}{2}\right) = \frac{2}{2} d(r^2)$$

$$= \frac{2}{2} 2r\,dr$$



$dA \to 2\pi f(x)\, ds$



$= \frac{\alpha}{2} 2r\, dr$

$Y = \frac{\alpha r^2}{2}$

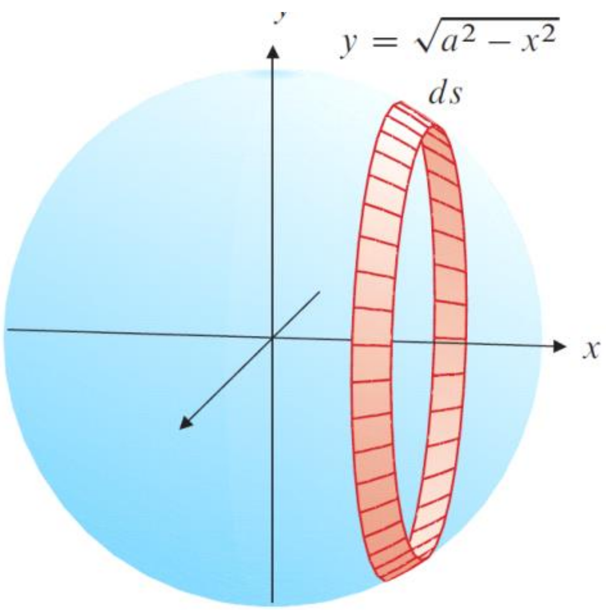
$\alpha = 2\pi$

$\frac{2\pi}{2} \cdot r^2 = \pi r^2$

EXAMPLE 5 **(Surface area of a sphere)** Find the area of the surface of a sphere of radius a.

Solution Such a sphere can be generated by rotating the semicircle with equation $y = \sqrt{a^2 - x^2}$, $(-a \le x \le a)$, about the x-axis. (See Figure 7.27.) Since

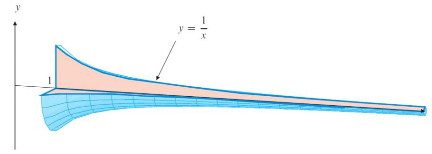
Figure 7.27 An area element on a sphere



$$\frac{dy}{dx} = -\frac{x}{\sqrt{a^2 - x^2}} = -\frac{x}{y}$$

$$\begin{aligned} S &= 2\pi \int_{-a}^{a} y \sqrt{1 + \left(\frac{x}{y}\right)^2}\, dx \\ &= 4\pi \int_0^a \sqrt{y^2 + x^2}\, dx \\ &= 4\pi \int_0^a \sqrt{a^2}\, dx = 4\pi a x \Big|_0^a = 4\pi a^2 \end{aligned}$$

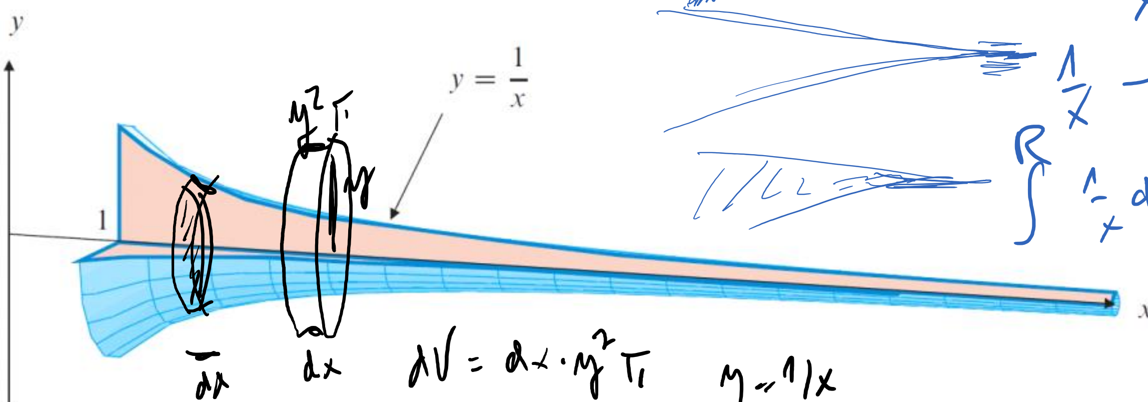
39. A hollow container in the shape of an infinitely long horn is generated by rotating the curve $y = 1/x$, $(1 \le x < \infty)$, about the x-axis.



(a) Find the volume of the container.

(b) Show that the container has infinite surface area.

(c) How do you explain the "paradox" that the container can be filled with a finite volume of paint but requires an infinite amount of paint to cover its surface?



$dV = dx \cdot y^2 \pi$ $\quad y = 1/x$

$x \to \infty$

$\frac{1}{x} \to 0$

$\int^R \frac{1}{x}\,dx \to \infty$

(a) Find the volume of the container.

$$V = \int_0^V dV = \int_1^\infty dx \cdot \frac{1}{x^2} \cdot \pi = \pi \int_1^\infty \frac{dx}{x^2} = \pi \lim_{R\to\infty} \int_1^R \frac{dx}{x^2}$$

$$-\frac{1}{x}\Big|_1^R$$

$$= \pi \lim_{R\to\infty} \left(-\frac{1}{R} + \frac{1}{1}\right) = \pi \cdot 1 = \pi$$

(b) Show that the container has infinite surface area.

dx (ds) $\quad ds = \sqrt{1+y'^2}\,dx$

$$A = \int_1^\infty 2\pi y \cdot \sqrt{1+y'^2}\,dx = \lim_{R\to\infty} \int_1^R 2\pi \frac{1}{x} \sqrt{1 + \frac{1}{x^4}}\,dx$$

$y = 1/x \quad y' = -1/x^2$

$$= 2\pi \lim_{R\to\infty} \int_1^R \sqrt{1 + \frac{1}{x^4}} \cdot \frac{dx}{x} = 2\pi\,\infty = \infty$$

$$\overbrace{\sqrt{1 + \frac{1}{x^4}}}^{>1} \cdot \frac{1}{x} > \frac{1}{x}$$

$\sqrt{1.01}$

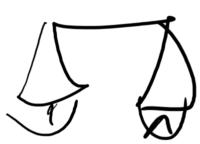
(1) $\frac{1}{x^4} > 0 \quad \Big| +1$

(2) $1+\frac{1}{x^4} > 1 \quad \Big| \sqrt{}$

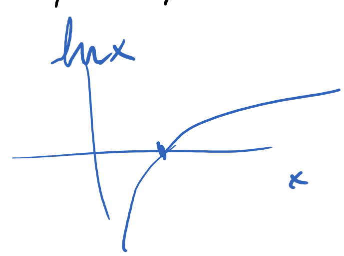
(3) $\sqrt{1+\frac{1}{x^4}} > 1 \quad \Big| \cdot \frac{1}{x}$

(4) $\sqrt{1+\frac{1}{x^4}} \cdot \frac{1}{x} > \frac{1}{x} \quad \Big| \int_1^R dx$

(5) $\int_1^R \sqrt{1+\frac{1}{x^4}} \cdot \frac{1}{x}\, dx > \int_1^R \frac{1}{x}\, dx = \ln|x| \Big|_1^R$



$\int \lim$



$= \ln R - \ln 1$

$= \ln R$

(6) $\int_1^R \sqrt{1+\frac{1}{x^4}}\, \frac{1}{x}\, dx > \ln R \quad \Big| \lim_{R\to\infty}$

(7) $\lim_{R\to\infty} \int_1^R \sqrt{1+\frac{1}{x^4}}\, \frac{dx}{x} > \boxed{\lim_{R\to\infty} \ln R} = \infty$

38. The curve $y = \ln x$, $(0 < x \leq 1)$, is rotated about the y-axis. Find the area of the horn-shaped surface so generated.

frågor

den 8 oktober 2020 10:15

$$\int \underbrace{x^2}_{u} \sin x \, dx =$$

Om det är en enklare integral som till exempel xcos(x) kan man då endast referera till integral table eller krävs redovisning för full poäng? dV

Har inte kommit dit i boken ännu, men är inte tabellerna nödvändiga för att lösa vissa integraler?

$$\int \sqrt{x^2 + a^2} \, dx =$$

Menar du då att så länge man visar sina steg är det okej?

så vissa standard integraler är okej och vissa är inte?
länst bak och fram i boken?
på pärmen typ

Hinner du visa de exemplet som du skrev upp? Integral(x^2 *sin(x) dx) hur du hänvisar etc. och hur man ska svara

$= \{$