Lösningsförslag SI-Pass 1 11-9-2020

1. Kvadratkomplettering och pq-formeln

a) i) $2x^2 + 8x - 10 = 0$

$x^2 + 4x - 5 = 0$

$(x+2)^2 - 4 - 5 = 0$

$(x+2)^2 = 9$

$x + 2 = \pm 3$

$x = -2 \pm 3 \Rightarrow \begin{cases} x_1 = -5 \\ x_2 = 1 \end{cases}$

ii) $2x^2 + 8x - 10 = 0$

$x^2 + 4x - 5 = 0$

$x = -2 \pm \sqrt{(-2)^2 + 5}$

$x = -2 \pm \sqrt{4+5}$

$x = -2 \pm 3 \Rightarrow \begin{cases} x_1 = -5 \\ x_2 = 1 \end{cases}$

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b)

$x^2 + px + q = 0$

$\left(x + \frac{p}{2}\right)^2 - \left(\frac{p}{2}\right)^2 + q = 0$

$\left(x + \frac{p}{2}\right)^2 = \left(\frac{p}{2}\right)^2 - q$

$x + \frac{p}{2} = \pm\sqrt{\left(\frac{p}{2}\right)^2 - q}$

$x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$

2. Komplexa tal och faktorisering

a) $u = i+1$
$v = 1-2i$

i) $u+v = i+1+1-2i = 2-i$

$|u+v| = \sqrt{2^2+(-1)^2} = \sqrt{5}$

$\arg(u+v) = \arctan\left(\frac{-1}{2}\right)$

$u+v = \sqrt{5}\left(\cos(\arctan(-\frac{1}{2})) + i\sin(\arctan(-\frac{1}{2}))\right)$

ii) $u-v = 1+i-(1-2i) = 0+3i = 3i$

$|u-v| = \sqrt{3^2} = 3$

$\arg(u-v) = \frac{\pi}{2}$

$u-v = 3\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

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b) $3x^2+6x+4 = -x^3 \iff x^3+3x^2+6x+4 = 0$ gissa $x=-1$ rot $\Rightarrow$

$\Rightarrow (x+1)$ faktor

$$\begin{array}{r|l} x^3+3x^2+6x+4 & x+1 \\ \hline -x^2(x+1) & x^2+2x+4 \\ \hline 2x^2+6x+4 & \\ -2x(x+1) & \\ \hline 4x+4 & \\ -4(x+1) & \\ \hline 0 & \end{array}$$

$x^3+3x^2+6x+4 = 0 \iff (x+1)(x^2+2x+4) = 0$

$x^2+2x+4 = 0$

$x = -1 \pm \sqrt{1-4}$ $\Rightarrow$ x^2+2x+4 saknar reella rötter

$$\begin{cases} x_1 = -1 \\ x^3+3x^2+6x+4 = (x+1)(x^2+2x+4) \end{cases}$$

3. Ekvationer med absolutbelopp

i) $|x-2| \leq -1$ saknar lösning ty $|a| \geq 0 \quad \forall a \in \mathbb{R}$

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ii) $|x-2| = 1$

$$|x-2| = \begin{cases} x-2 & \text{då } x \geq 2 \\ 2-x & \text{då } x < 2 \end{cases}$$

$x \geq 2$:

$x - 2 = 1$

$x = 3$

$3 \geq 2$ så OK!

$x < 2$

$2 - x = 1$

$x = 1$

$1 < 2$ så OK

Svar: $\begin{cases} x_1 = 3 \\ x_2 = 1 \end{cases}$

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iii) $|x-k| = z$

$$|x-k| = \begin{cases} x-k & \text{då } x \geq k \\ k-x & \text{då } x \leq k \end{cases}$$

$x \geq k$:

$x - k = z$

$x = z + k$

OK om $z \geq 0$

$x \leq k$:

$k - x = z$

$x = k - z$

OK om $z \geq 0$

$|x-k| = z$

har lösningar: $\begin{cases} x_1 = k+z \\ x_2 = k-z \end{cases}$ då $z \geq 0$