Lösningsförslag S1-pass 2 14-9-2020

1. i) $\cos(2x) = \cos^2 x - \sin^2 x$

$\iff \cos(2x) = 1 - \sin^2 x - \sin^2 x$

$\iff \cos(2x) = 1 - 2\sin^2 x$

$\iff 2\sin^2 x = 1 - \cos(2x)$

$\iff \sin^2 x = \dfrac{1-\cos(2x)}{2}$

ii) $\cos(2x) = \cos^2 x - \sin^2 x$

$\iff \cos(2x) = \cos^2 x - (1 - \cos^2 x)$

$\iff \cos(2x) = 2\cos^2 x - 1$

$\iff 2\cos^2 x = 1 + \cos(2x)$

$\iff \cos^2 x = \dfrac{1+\cos(2x)}{2}$

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b) $\dfrac{d}{dx} \sin^2 x = 2\sin x \cdot \cos x = \sin(2x)$

$\dfrac{d}{dx} \dfrac{1}{2} - \dfrac{\cos(2x)}{2} = -\dfrac{2}{2} \cdot (-\sin(2x)) = \sin(2x)$

$\dfrac{d}{dx} \cos^2 x = -2\cos x \sin x = -\sin(2x)$

$\dfrac{d}{dx} \dfrac{1}{2} + \dfrac{\cos(2x)}{2} = \dfrac{2}{2} \cdot (-\sin(2x)) = -\sin(2x)$

2. i) Vi ser att $x = n\pi$ $n = 0, \pm 1, \pm 2$ trivialt är en lösning ty $2\sin(2n\pi) = \sin(n\pi) = 0$

$$2\sin(2x) = \sin x \iff 4\cancel{\sin x}\cos x = \cancel{\sin x} \iff 4\cos x = 1$$

$$\cos x = \frac{1}{4}$$

Två fall:

1:
$$x = \arccos\left(\frac{1}{4}\right) + 2n\pi$$

2:
$$x = -\arccos\left(\frac{1}{4}\right) + 2n\pi$$

Alltså
$$\begin{cases} x_1 = n\pi \\ x_2 = \arccos\left(\frac{1}{4}\right) + 2n\pi \\ x_3 = -\arccos\left(\frac{1}{4}\right) + 2n\pi \end{cases} \quad n = 0, \pm 1, \pm 2$$

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ii)

$$\cos(4x) = \frac{1}{2}$$

Två fall:

1:
$$4x = \frac{\pi}{3} + 2n\pi$$

$$x = \frac{\pi}{12} + \frac{n\pi}{2} \quad n = 0, \pm 1, \pm 2$$

2
$$4x = -\frac{\pi}{3} + 2n\pi$$

$$x = -\frac{\pi}{12} + \frac{n\pi}{2} \quad n = 0, \pm 1, \pm 2$$

$$\begin{cases} x_1 = \frac{\pi}{12} + \frac{n\pi}{2} \\ x_2 = -\frac{\pi}{12} + \frac{n\pi}{2} \end{cases} \quad n = 0, \pm 1, \pm 2$$

iii) $-\sqrt{2}\sin x + \sqrt{2}\cos x = \sqrt{3} \iff$

$\iff -\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{\sqrt{3}}{2} \iff$

$\iff \cos\varphi \sin x + \sin\varphi \cos x = \frac{\sqrt{3}}{2}$ där $\cos\varphi = -\frac{1}{\sqrt{2}} \Rightarrow \frac{3\pi}{4}, \frac{5\pi}{4}$

$\sin\varphi = \frac{1}{\sqrt{2}} \Rightarrow \frac{\pi}{4}, \frac{3\pi}{4}$

kända vinklar

additionsformel sinus $\Rightarrow$

$\cos\frac{3\pi}{4}\sin x + \sin\frac{3\pi}{4}\cos x = \frac{\sqrt{3}}{2} \iff$

$\iff \sin\left(x + \frac{3\pi}{4}\right) = \frac{\sqrt{3}}{2} \Rightarrow$

Två fall:

1: $x + \frac{3\pi}{4} = \frac{\pi}{3} + 2n\pi$

$x = \frac{4\pi}{12} - \frac{9\pi}{12} + 2n\pi$

$x = -\frac{5\pi}{12} + 2n\pi$

2: $x + \frac{3\pi}{4} = \pi - \frac{\pi}{3} + 2n\pi$

$x = \frac{2\pi}{3} - \frac{3\pi}{4} + 2n\pi$

$x = \frac{8\pi}{12} - \frac{9\pi}{12} + 2n\pi$

$x = -\frac{\pi}{12} + 2n\pi$

Lösningar sammanfaller ej

$$\begin{cases} x_1 = -\frac{5\pi}{12} + 2n\pi \\ x_2 = -\frac{\pi}{12} + 2n\pi \end{cases} \quad n = 0, \pm 1, \pm 2, \ldots$$

3. Systemet ger följande totalmatris som kan förenklas med radoperationer:

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 1 & 2 & 1 & 0 \\ h & 1 & 1 & g \end{array}\right] \begin{array}{c} -1 \\ -h \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 0 & 1 & -2 & -1 \\ 0 & 1-h & 1-3h & g-h \end{array}\right] -(1-h) \sim$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 3-5h & 1+g-2h \end{array}\right]$$

Genom att undersöka sista raden med olika värden för g och h finner vi att:

a) entydig lösning då $3-5h \neq 0 \Leftrightarrow h \neq \frac{3}{5}$

b) oändligt många lösningar då $h = \frac{3}{5}$ och $g = \frac{1}{5}$ (nollrad)

c) ingen lösning då $h = \frac{3}{5}$ och $g \neq \frac{1}{5}$

4.

a) i) $|P_1 - P_2| = \sqrt{(1-6)^2 + (2-4)^2 + (3-2)^2} = \sqrt{25+4+1} = \underline{\underline{\sqrt{30}}}$ l.e.

ii) $|P_2 - P_3| = \sqrt{(6-3)^2 + (4-3)^2 + (2-3)^2} = \sqrt{9+1+1} = \underline{\underline{\sqrt{11}}}$ l.e.

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b) $5 \cdot \overline{P_1P_2} - 2\,\overline{P_1P_3} = 5(6-1, 4-2, 2-3) - 2(3-1, 3-2, 3-3) =$

$= 5(5, 2, -1) - 2(2, 1, 0) = (25, 10, -5) - (4, 2, 0) =$

$= \underline{\underline{(21, 8, -5)}}$