SI-pass 3

1.) a) $$\lim_{x\to 1} \frac{x^2-1}{x^2+2x-3} = \lim_{x\to 1} \frac{(x+1)\cancel{(x-1)}}{(x+3)\cancel{(x-1)}} =$$

$$= \frac{2}{4} = \frac{1}{2}$$

$$x^2+2x-3 \Rightarrow (x+1)^2-4 = x^2+2x+1-4 = x^2+2x-3$$

b) $$\lim_{x\to 0} \frac{\ln(1+3x)}{x} = \infty \Rightarrow x \to 0 \text{ snabbare än } \ln(1+3x)$$

c) $$\lim_{x\to\infty} \frac{2^x+\ln x}{3.2^x-\ln x} = \lim_{x\to\infty} \frac{2^x}{3.2^x} = 0$$

2.) a) $u = \hat{i} - 3\hat{j} - 2\hat{k}$, $V = 3\hat{i} + 2\hat{j} - \hat{k}$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -2 \\ 3 & 2 & -1 \end{bmatrix} = \hat{i}(3+4) - \hat{j}(-1+6) + \hat{k}(2+9) =$$

$$= \hat{i}\,7 - \hat{j}\,5 + \hat{k}\,11$$

b) $$\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

$$\vec{u}\times\vec{v} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ -1 & 1 & -1 \end{bmatrix} = \hat{i}(1+1) - \hat{j}(-1-1) + \hat{k}(1-1) = \hat{i}\,2 + \hat{j}\,2$$

$$\text{Area} = \frac{|\vec{u}\times\vec{v}|}{2} = \frac{\sqrt{2^2+2^2}}{2} = \frac{\sqrt{8}}{2} = \sqrt{2}$$

3.1)

$$\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$$

$$\vec{n} = \vec{u} \times \vec{v} = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & k \\ 1 & -1 & -1 \\ -2 & -1 & 1 \end{bmatrix} = -2\hat{\imath} + \hat{\jmath} - 3\hat{k}$$

Planets ekv.:

$$-2(x-1) + (y-1) - 3(z-0) = 0$$

Punkt $(1,1,0)$

$$\Rightarrow \boxed{2x - y + 3z = 1}$$

3.2)

$$-2(x-1) + (y-2) - 3(z-3) = 0$$

$$\Rightarrow \quad -2x + \cancel{2} + y - \cancel{2} - 3z + 9 = 0$$

$$\Rightarrow \quad \boxed{2x - y + 3z = 9}$$

4. Veckans Quack!

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

För intervallet $0 < x < \frac{\pi}{2}$ kan det visas att:

$$\sin x < x < \tan x$$

Division med $\sin x$ ger:

$$1 < \frac{x}{\sin x} < \frac{\tan x}{\sin x} \qquad \left(\tan x = \frac{\sin x}{\cos x} \right)$$

$$1 < \frac{x}{\sin x} < \frac{1}{\cos x}$$

$$\lim_{x \to 0} \frac{1}{\cos x} = \frac{1}{1} = 1$$

Instängningssatsen ger då:

$$\lim_{x \to 0} \frac{x}{\sin x} = 1 \Rightarrow \boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1}$$ V.S.V.