1. a) $\bar{u} \cdot \bar{v} = (u_1, u_2, \ldots, u_n) \cdot (v_1, v_2, \ldots, v_n) = u_1v_1 + u_2v_2 + \ldots + u_nv_n$

b) $\bar{u} \cdot \bar{v} = 1 \cdot 2 + 4 \cdot 2 = 10$

c) Produkten av vektorernas längder där den ena längden projicerats i den andra vektorns riktning.

d) $\bar{v} \cdot \bar{v} = v_1^2 + v_2^2 + \ldots + v_n^2 = |\bar{v}|^2$

e) $\bar{u} \cdot \bar{v} = |\bar{u}| \cdot |\bar{v}| \cdot \cos\theta$, om $\bar{u}, \bar{v} \neq \vec{0}$ så gäller att $\cos\theta = 0 \Leftrightarrow \theta = 90°$, alltså är vektorerna vinkelräta mot varandra.

2. a) $(1,4,2)$ $(3,4,1)$ $(2,2,2)$

$$\bar{u} = (3,4,1)-(1,4,2) = (2,0,-1)$$
$$\bar{v} = (2,2,2)-(1,4,2) = (1,-2,0)$$

$$\bar{n} = \bar{u}\times\bar{v} = \begin{vmatrix} i & j & k \\ 2 & 0 & -1 \\ 1 & -2 & 0 \end{vmatrix} = (-2)i - (1)j + (-4)k = (-2,-1,-4)$$

omdefiniera $\bar{n}$ till $-\bar{n} \Rightarrow \bar{n} = (2,1,4)$.

Planets ekvation: $(r-r_0)\cdot\bar{n} = 0 \Rightarrow$ $\quad r_0 = (1,4,2)$

$$(x-1)\cdot 2 + (y-4)\cdot 1 + (z-2)4 = 0 \iff$$

$$\iff 2x+y+4z = 2+4+8 \iff 2x+y+4z = 14$$

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b) ekvationssystem

$$\begin{cases} 2x+y+4z=14 \\ 2x+2y+5z=4 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 2 & 1 & 4 & 14 \\ 2 & 2 & 5 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 1 & 4 & 14 \\ 0 & 1 & 1 & -10 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} 2 & 0 & 3 & 24 \\ 0 & 1 & 1 & -10 \end{array}\right] \Rightarrow \begin{array}{l} 2x = 24-3z \\ y = -10-z \end{array}$$

sätt $z=t$ vilket ger parametrisering:

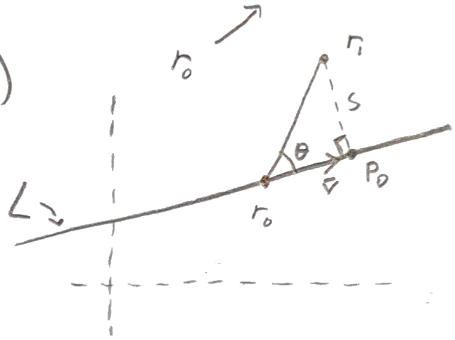
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -10 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3/2 \\ -1 \\ 1 \end{bmatrix} \quad \text{eller} \quad \frac{x-12}{-3/2} = \frac{y+10}{-1} = z$$

$$L = \begin{bmatrix} 12 \\ -10 \\ 0 \end{bmatrix} + t \begin{bmatrix} -3/2 \\ -1 \\ 1 \end{bmatrix} \leftarrow \text{kalla denna } \bar{v}.$$

r_0

$r_1 = (3,4,2)$

c)



Genom att utnyttja vektorprojektionen av $\overline{r_0 r_1}$ på $\bar{v}$ kan punkten P_0 erhållas genom $P_0 = r_0 + proj_{\bar{v}}(\overline{r_0 r_1})$ =>

$$P_0 = r_0 + \frac{\overline{r_0 r_1} \bullet \bar{v}}{|\bar{v}|^2} \cdot \bar{v} = (12,-10,0) + \frac{(3-12, 4+10, 2-0) \bullet (-\frac{3}{2}, -1, 1)}{\left(\frac{3}{2}^2 + 1^2 + 1^2\right)} \cdot \left(-\frac{3}{2}, -1, 1\right) \cdot$$

$$= (12,-10,0) + \frac{(-9, 14, 2) \bullet (-\frac{3}{2}, -1, 1)}{\frac{9+4+4}{4}} \cdot \left(-\frac{3}{2}, -1, 1\right) =$$

$$= (12,-10,0) + \frac{\frac{27}{2} - \frac{28}{2} + \frac{4}{2}}{\frac{17}{4}} \cdot \left(-\frac{3}{2}, -1, 1\right) = (12,-10,0) + \frac{6}{17}\left(-\frac{3}{2}, -1, 1\right) =$$

$$= (12,-10,0) + \left(-\frac{9}{17}, -\frac{6}{17}, \frac{6}{17}\right) = \left(\frac{204}{17} - \frac{9}{17}, -\frac{170}{17} - \frac{6}{17}, \frac{6}{17}\right) =$$

$$= \left(\frac{195}{17}, -\frac{176}{17}, \frac{6}{17}\right)$$

$$s = |P_0 - r_1| = \left|\left(\frac{195}{17} - \frac{51}{17}, -\frac{176}{17} - \frac{68}{17}, \frac{6}{17} - \frac{34}{17}\right)\right| = \left|\left(\frac{144}{17}, -\frac{244}{17}, -\frac{28}{17}\right)\right| =$$

$$= \frac{\sqrt{144^2 + 244^2 + 28^2}}{17} \approx 16,75 \text{ l.e.}$$

alternativt $\quad s = |r_1 - r_0| \cdot \sin\theta = \frac{|r_1 - r_0| \cdot |\bar{v}| \cdot \sin\theta}{|\bar{v}|} = \frac{|(r_1 - r_0) \times \bar{v}|}{|\bar{v}|} \Rightarrow$

$$\Rightarrow \frac{\left|(-9, 14, 2) \times \left(-\frac{3}{2}, -1, 1\right)\right|}{\sqrt{\frac{9}{4} + \frac{4}{4} + \frac{4}{4}}} = \frac{\left|\begin{vmatrix} i & j & k \\ -9 & 14 & 2 \\ -\frac{3}{2} & -1 & 1 \end{vmatrix}\right|}{\frac{\sqrt{17}}{2}} = \frac{\left|(14+2, -9+3, 9+21)\right|}{\frac{\sqrt{17}}{2}} =$$

$$= \frac{\sqrt{16^2 + 6^2 + 30^2}}{\frac{\sqrt{17}}{2}} = \frac{\sqrt{1192} \cdot 2}{\sqrt{17}} = \underline{\underline{16.75}} \text{ l.e.}$$

3. a)

$$\bar{u} = \left(\tfrac{1}{2}, \tfrac{3}{2}, \tfrac{5}{2}\right) - \left(4, -\tfrac{1}{2}, 0\right) = \left(-\tfrac{7}{2}, 2, \tfrac{5}{2}\right)$$

$$\bar{v} = \left(1, \tfrac{7}{2}, 1\right) - \left(4, -\tfrac{1}{2}, 0\right) = \left(-3, 4, 1\right)$$

$$\bar{n} = \bar{u} \times \bar{v} = \begin{vmatrix} \mathbb{i} & \mathbb{j} & \mathbb{k} \\ -7/2 & 2 & 5/2 \\ -3 & 4 & 1 \end{vmatrix} = (2-10)\mathbb{i} - \left(-\tfrac{7}{2} + \tfrac{15}{2}\right)\mathbb{j} + (-14+6)\mathbb{k} =$$

$$= (-8, -4, -8) \qquad \left(\bar{n} = \tfrac{\bar{n}}{-4}\right) \Rightarrow \bar{n} = (2,1,2)$$

planets ekvation $\qquad \bar{n} \bullet (r - r_0) = 0 \Rightarrow$

$$(x-4)\cdot 2 + \left(y + \tfrac{1}{2}\right)\cdot 1 + (z-0)2 = 0 \Leftrightarrow 2x + y + 2z = \tfrac{16}{2} - \tfrac{1}{2} \Leftrightarrow$$

$$\Leftrightarrow 4x + 2y + 4z = 15$$

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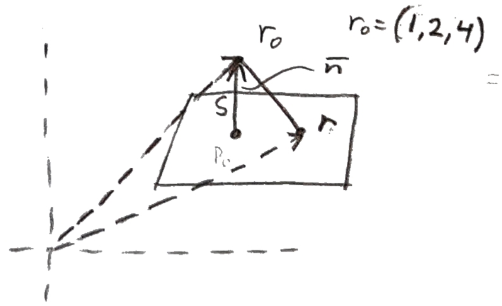
b) Alternativ I:

Från figur

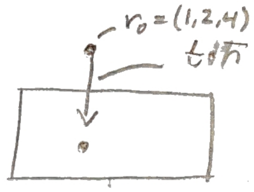


$$s = \left| \mathrm{proj}_{\bar{n}} (\overline{rr_0}) \right| = \left| \frac{\overline{rr_0} \bullet \bar{n}}{|\bar{n}|^2} \cdot \bar{n} \right| =$$

$$= \frac{|\overline{rr_0} \bullet \bar{n}|}{|\bar{n}|} \cdot \frac{\cancel{|\bar{n}|}}{\cancel{|\bar{n}|}} = \frac{|(1-x, 2-y, 4-z) \bullet \bar{n}|}{\sqrt{4^2 + 2^2 + 4^2}} =$$

$$= \frac{|4\cdot 1 + 2\cdot 2 + 4\cdot 4 - (4x + 2y + 4z)|}{\sqrt{36}} = \frac{|4 + 4 + 16 - 15|}{6} = \frac{|9|}{6} = \frac{9}{6} = \frac{3}{2} \text{ l.e.}$$

Alternativ II:



Om man adderar vektorn $t\cdot\bar{n}$ till r_0 kommer man för något $t \in \mathbb{R}$ hamna i planet, och genom att sätta in denna erhållna punkt $(r_0 + t\bar{n})$ i planets ekvation får man en ekvation som kan lösas för t.

$\bar{n} = (4,2,4) \Rightarrow \quad r_0 + t\cdot\bar{n} = (1+4t, 2+2t, 4+4t) \Rightarrow$

$$4(1+4t) + 2(2+2t) + 4(4+4t) = 15 \Leftrightarrow$$

$$\Leftrightarrow 4 + 16t + 4 + 4t + 16 + 16t = 15 \Leftrightarrow$$

$$\Leftrightarrow 36t = 15 - 16 - 8 \Leftrightarrow 36t = -9$$

$$12t = -3$$

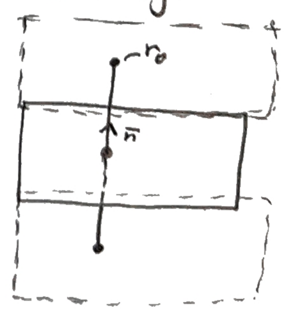
$$t = -\frac{1}{4}$$

Längden från punkten till planet ges nu av $|t\cdot\bar{n}| =$

$$= \left|\left(-\tfrac{1}{4}\cdot 4, -\tfrac{1}{4}\cdot 2, -\tfrac{1}{4}\cdot 4\right)\right| = \left|\left(-1, -\tfrac{1}{2}, -1\right)\right| = \sqrt{1 + \tfrac{1}{4} + 1} = \sqrt{\tfrac{9}{4}} = \frac{3}{2} \text{ l.e.}$$

Med ledning av alternativ II kan vi föra följande resonemang:

c) Det ena planet innehåller punkten $r_0\,(1,2,4)$ och alla planen måste ha samma normal. Ekvationen för det första planet ges då av:



$$4(x-1) + 2(y-2) + 4(z-4) = 0 \Leftrightarrow 4x + 2y + 4z = 24$$

Eftersom vektorn från r_0 till det ursprungliga planet var $-\frac{1}{4}\bar{n}$, måste punkten på lika stort avstånd på andra sidan planet ges av $r_0 + 2\cdot\left(-\frac{1}{4}\right)\bar{n} = r_0 - \frac{1}{2}\bar{n} = (1,2,4) - (2,1,2) = (-1,1,2)$.

och vi sätter in denna punkt i planets ekvation. (Alltså samma normal)

$$4(x+1)+2(y-1)+4(z-2)=0 \Leftrightarrow 4x+2y+4z=\underline{\underline{6}}$$

Alltså de två planen ges av:

$4x+2y+4z=\underline{\underline{24}}$

$4x+2y+4z=\underline{\underline{6}}$

4 a) $\bar{u} \times (\bar{v} \times \bar{w}) = \bar{u} \times \begin{vmatrix} i & j & k \\ 3 & 2 & -1 \\ 0 & 1 & 1 \end{vmatrix} = \bar{u} \times (2+1, -3, 3) =$

$$= \begin{vmatrix} i & j & k \\ 1 & -3 & -2 \\ 3 & -3 & 3 \end{vmatrix} = (-9-6, -3-6, -3+9) = \underline{\underline{(-15, -9, 6)}}$$

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b) $(\bar{u} \times \bar{v}) \times \bar{w} = \begin{vmatrix} i & j & k \\ 1 & -3 & -2 \\ 3 & 2 & -1 \end{vmatrix} \times \bar{w} = (3+4, 1-6, 2+9) \times \bar{w} =$

$$= \begin{vmatrix} i & j & k \\ 7 & -5 & 11 \\ 0 & 1 & 1 \end{vmatrix} = (-5+11, -7, 7) = \underline{\underline{(6, -7, 7)}}$$

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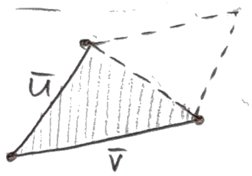
c)



area $= \dfrac{|\bar{u} \times \bar{v}|}{2}$

$\bar{u} = (0,2,1) - (1,1,0) = (-1,1,1)$

$\bar{v} = (2,0,1) - (1,1,0) = (1,-1,1)$

$$A = \frac{\left| \begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} \right|}{2} = \frac{|(1+1, 1+1, 0)|}{2} = \frac{\sqrt{2^2+2^2}}{2} = \frac{\sqrt{4 \cdot 2}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$\underline{\underline{\text{a.e}}}$

---

d)

Volymen för parallellepipeden av tre vektorer $\bar{u}, \bar{v}$ o $\bar{w}$ ges av $|\bar{u} \bullet (\bar{v} \times \bar{w})| \Rightarrow$

$$V = \left| (1,1,1) \bullet \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 1 & 0 & 5 \end{vmatrix} \right| = |(1,1,1) \bullet (10, -5+3, -2)| = |10-2-2| =$$

$$= |6| = 6 \ \underline{\underline{\text{v.e.}}}$$