

7. Dugga

$$1) i) \frac{2+3i}{3-4i} \cdot \frac{3+4i}{3+4i} = \frac{(2+3i)(3+4i)}{3^2+(-4)^2} = \frac{6-12+i(8+9)}{25} = \frac{-6+17i}{25}$$

$$ii) e^{-i\pi/6} = \cos(-\frac{\pi}{6}) + i \sin(-\frac{\pi}{6}) = \cos(\frac{\pi}{6}) - i \sin(\frac{\pi}{6}) = \frac{\sqrt{3}}{2} - \frac{i}{2}$$

$$2) 3x^2 - 9x + 1 = 3(x^2 + 2(\frac{-3}{2})x) + 1 = 3(x - \frac{3}{2})^2 - \frac{9}{4} + 1 = 3(x - \frac{3}{2})^2 - \frac{23}{4} \geq -\frac{23}{4} = \text{minsta värde}$$

$$3) a) \left(\begin{array}{ccc|c} 2 & 1 & a & 1 \\ a & 1 & 0 & 1 \\ 1 & 1 & a & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & a & 1 \\ 2 & 1 & a & 1 \\ a & 1 & 0 & 1 \end{array} \right) \xrightarrow{\begin{smallmatrix} -2 \\ -a \end{smallmatrix}} \left(\begin{array}{ccc|c} 1 & 1 & a & 1 \\ 0 & -1 & -a & -1 \\ 0 & 1-a & -a^2 & 1-a \end{array} \right) \xrightarrow{1-a} \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & a & 1 \\ 0 & 1 & a & 1 \\ 0 & 0 & -a & 0 \end{array} \right) \therefore a \neq 0 \text{ ger unik lösning; } a = 0 \text{ ger oändligt många lösningar; inget } a \text{ ger ingen lösning.}$$

$$b) v_1 \cdot v_2 = \frac{1}{5}(3, -4) \cdot \frac{1}{5}(4, 3) = \frac{1}{25}(3 \cdot 4 - 4 \cdot 3) = 0 \text{ så } v_i \text{ ortogonala}$$

$$\|v_1\|^2 = \frac{1}{25}(9+16) = 1 \text{ så } v_i \text{ normerade} \therefore \text{ON-bas}$$

$$\therefore (2, -1) = \alpha v_1 + \beta v_2 \Rightarrow v_1 \cdot (2, -1) = v_1 \cdot (\alpha v_1 + \beta v_2) = \alpha v_1 \cdot v_1 + \beta v_1 \cdot v_2 = \alpha \|v_1\|^2 + \beta \cdot 0 = \alpha$$

$$\therefore \alpha = v_1 \cdot (2, -1) = \frac{1}{5}(3, -4) \cdot (2, -1) = \frac{1}{5}(6+4) = 2$$

$$\text{P.S.S. } \beta = v_2 \cdot (2, -1) = \dots = 1$$