Lösningsförslag SI-pass 4 200928

1. a) $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

b) i) $\frac{d}{dx} (\sin(2x) + x^2)^5 = 5(\sin(2x) + x^2)^4 \cdot (2\cos(2x) + 2x) =$

$= 10(\sin(2x) + x^2)^4 \cdot (\cos(2x) + x)$

ii) $\frac{d}{dy} e^{y^2} = e^{y^2} \cdot 2y = 2ye^{y^2}$

2. a) $\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$

$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

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b) i) $\frac{d}{dx}\left(2\sqrt{x} + \frac{3}{x}\right)\left(3\sqrt{x} - \frac{2}{x}\right) = \left(\frac{1}{\sqrt{x}} - \frac{3}{x^2}\right)\left(3\sqrt{x} - \frac{2}{x}\right) +$

$+ \left(2\sqrt{x} + \frac{3}{x}\right)\left(\frac{3}{2\sqrt{x}} + \frac{2}{x^2}\right) = 3 - \frac{2}{x^{3/2}} - \frac{9}{x^{3/2}} + \frac{6}{x^3} + 3 + \frac{4}{x^{3/2}} + \frac{9}{2x^{3/2}} + \frac{6}{x^3} =$

$= 6 + \frac{5}{2x^{3/2}} + \frac{12}{x^3}$

ii) $\frac{d}{dy}\left(\frac{\cos(y)}{1-\sin(y)}\right) = \frac{-\sin(y)(1-\sin(y)) - \cos(y)\cdot(-\cos(y))}{(1-\sin(y))^2} =$

$= \frac{-\sin(y) + \sin^2(y) + \cos^2(y)}{(1-\sin(y))^2} = \frac{1-\sin(y)}{(1-\sin(y))^2} = \frac{1}{1-\sin(y)}$

3. a) $\lim_{x\to 1} \frac{x^2-1}{x^2+2x-3} = \left[\begin{array}{c} x=1 \text{ nollställe} \\ \text{till båda polynomen} \end{array}\right] = \lim_{x\to 1} \frac{(x-1)(x+1)}{(x-1)(x+3)} = \lim_{x\to 1} \frac{x+1}{x+3} =$

$= \frac{1+1}{1+3} = \frac{2}{4} = \frac{1}{2}$

b) $\lim_{x\to 2^+} \frac{x}{(2-x)^3} = \left[2-x \to 0^- \text{ då } x\to 2^+\right] = -\infty$

c) $\lim_{x\to -3} \frac{\sqrt{2x+22}-4}{x+3} = \frac{"0"}{0} = [\text{förlängning med konjugatet}] =$

$= \lim_{x\to -3} \frac{\sqrt{2x+22}-4 \cdot \sqrt{2x+22}+4}{(x+3)(\sqrt{2x+22}+4)} = \lim_{x\to -3} \frac{2x+22-16}{(x+3)(\sqrt{2x+22}+4)} =$

$= \lim_{x\to -3} \frac{2(x+3)}{(x+3)(\sqrt{2x+22}+4)} = \lim_{x\to -3} \frac{2}{\sqrt{2x+22}+4} = \frac{2}{\sqrt{16}+4} = \frac{2}{8} = \frac{1}{4}$

d) Finn $\lim_{x\to 3} \frac{x^3-2x^2-2x-3}{x^3-4x^2+4x-3} = \frac{"0"}{0} = \left[\begin{array}{c} (x-3) \text{ faktor i} \\ \text{båda polynomen} \end{array}\right] \Rightarrow$

$\lim_{x\to 3} \frac{(x-3)(x^2+x+1)}{(x-3)(x^2-x+1)} = \lim_{x\to 3} \frac{(x^2+x+1)}{(x^2-x+1)} = \frac{9+3+1}{9-3+1} = \frac{13}{7}$

$a = \frac{13}{7}$

4. a) $\lim_{x\to a} f(x) = L \Leftrightarrow \forall \varepsilon > 0 \; \exists \, \delta(\varepsilon)$ sådant att om

$$0 < |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon$$

b) $\lim_{x\to 2} x^2 = 4 \Leftrightarrow$

$\forall \varepsilon > 0 \; \exists \, \delta(\varepsilon)$ sådant att om $0 < |x-2| < \delta \Rightarrow |x^2 - 4| < \varepsilon$.

$|x^2-4| = |x-2||x+2|$ Säg $\delta \leq 1 \Leftrightarrow |x-2| < 1 \Rightarrow$

$1 < x < 3 \Rightarrow$

$3 < |x+2| < 5$

alltså $|x-2||x+2| < \delta |x+2| \leq 5\delta$

Så om $\delta = \min\left(1, \frac{\varepsilon}{5}\right)$ gäller att

$|x^2-4| = |x-2||x+2| < \delta |x+2| < \delta \cdot 5 \leq \varepsilon$

Därmed är $\lim_{x\to 2} x^2 = 4$. //

5. a) Se Satz 9 i kapitel 1.4

b) Se satss 11 i kapitel 2.8

c) Se satz 14 i kapitel 2.8