S1-pass 6 Løsningsforslag

1 a) $$\cosh x = \frac{e^{x}+e^{-x}}{2}$$

$$\frac{d}{dx}\left(\frac{e^{x}+e^{-x}}{2}\right) = \frac{e^{x}}{2} - \frac{e^{-x}}{2} = \frac{e^{x}-e^{-x}}{2} = \sinh x$$

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b) $$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$$

$$\cosh x \cosh y + \sinh x \sinh y = \frac{e^{x}+e^{-x}}{2} \cdot \frac{e^{y}+e^{-y}}{2} + \frac{e^{x}-e^{-x}}{2} \cdot \frac{e^{y}-e^{-y}}{2} =$$

$$= \frac{1}{4}\left(e^{x+y} + \cancel{e^{x-y}} + \cancel{e^{-x+y}} + e^{-x-y} + e^{x+y} - \cancel{e^{x-y}} - \cancel{e^{-x+y}} + e^{-x-y} \right) =$$

$$= \frac{1}{4}\left(2e^{x+y} + 2e^{-x-y}\right) = \frac{e^{x+y}+e^{-(x+y)}}{2} = \cosh(x+y) \quad \blacksquare$$

2. a) $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\cos x}{\frac{\pi}{2}-x} = \text{"}\dfrac{0}{0}\text{"} = [\text{l'Hopital}] = \lim\limits_{x\to\frac{\pi}{2}} \dfrac{-\sin x}{-1} = \underline{\underline{1}}$

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b) $\lim\limits_{x\to 0} \dfrac{\arcsin x}{\sin x} = \text{"}\dfrac{0}{0}\text{"} = [\text{l'Hôpital}] = \lim\limits_{x\to 0} \dfrac{1}{\sqrt{1-x^2}} \cdot \dfrac{1}{\cos x} = \underline{\underline{1}}$

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c) $\lim\limits_{x\to -\infty} \left(\sqrt{x^2+x} + x\right) = \text{"}\infty - \infty\text{"} = [\text{förläng med konjugat}] =$

$$= \lim_{x\to-\infty} \frac{\left(\sqrt{x^2+x}+x\right)\left(\sqrt{x^2+x}-x\right)}{\sqrt{x^2+x}-x} = \lim_{x\to-\infty} \frac{x^2+x-x^2}{\sqrt{x^2+x}-x} = \lim_{x\to-\infty} \frac{x}{\sqrt{x^2}\cdot\sqrt{1+\frac{1}{x}}-x} =$$

$$|x| = \begin{cases} x \text{ om } x>0 \\ -x \text{ om } x<0 \end{cases}$$

$$= \lim_{x\to-\infty} \frac{x}{|x|\sqrt{1+\frac{1}{x}}-x} = \lim_{x\to-\infty} \frac{x}{-x\sqrt{1+\frac{1}{x}}-x} = \lim_{x\to-\infty} -\frac{x}{x\left(\sqrt{1+\frac{1}{x}}+1\right)} =$$

$$= \lim_{x\to-\infty} -\frac{1}{\sqrt{1+\frac{1}{x}}+1} = -\frac{1}{\sqrt{1+0}+1} = \underline{\underline{-\frac{1}{2}}}$$

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d) $\lim\limits_{z\to 0} \dfrac{\sin(2z)+7z^2-2z}{z^2(z+1)^2} = \text{"}\dfrac{0}{0}\text{"} = [\text{l'Hopital}] = \left[z^2(z+1)^2 = z^4+2z^3+z^2\right] =$

$$= \lim_{z\to 0} \frac{2\cos(2z)+14z-2}{4z^3+6z^2+2z} = \text{"}\frac{0}{0}\text{"}\ [\text{l'Hopital}] = \lim_{z\to 0} \frac{-4\sin(2z)+14}{12z^2+12z+2} =$$

$$= \frac{0+14}{0+0+2} = \underline{\underline{7}}$$

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