Dugga 3 — lösn.

1) $f(x) = 2x + \ln x$ $\quad D_f: x > 0$

$f'(x) = 2 + \frac{1}{x} > 0$ på D_f

$\Rightarrow f$ strängt växande $\Rightarrow f^{-1}\ \exists$ ~~$\Leftrightarrow$~~

f inverterbar

Vi ser att $f(1) = 2$ så $f^{-1}(2) = 1$

Vidare: $(f^{-1})(f(x)) = x \Rightarrow$

$1 = \frac{d}{dx}(x) = \frac{d}{dx}(f^{-1}(f(x))) = \{$kedjeregeln$\} =$

$= (f^{-1})'(f(x)) \cdot f'(x) \Rightarrow (f^{-1})'(2) = (f^{-1})'(f(1)) =$

$= \frac{1}{f'(1)} = \frac{1}{3}$

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2) i) $\cos(\arctan 2)$ $\cos\alpha$ ~~[illegible]~~ $\alpha \overset{\text{def}}{\equiv} \arctan 2$

$\Leftrightarrow 2 = \tan\alpha, \ \frac{\pi}{4} < \alpha < \frac{\pi}{2}$ $\quad \sin\alpha = 2\cos\alpha \Rightarrow$

$\sin^2\alpha = 4\cos^2\alpha$ Vi söker $\cos\alpha$. Trig. ettan $\Rightarrow$

$1 - \cos^2\alpha = 4\cos^2\alpha \Rightarrow 5\cos^2\alpha = 1 \Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{5}}$

men då $\frac{\pi}{4} < \alpha < \frac{\pi}{2}$ så följer $\boxed{\cos\alpha = \frac{1}{\sqrt{5}}}$

Øving 3 – løsn. forsl. ②/②

2 iv) $\lim\limits_{x\to 0} \dfrac{2\sin x - \sin(2x)}{x - \sin x} = \text{"}\dfrac{0}{0}\text{"} = [\text{l'Hop}] =$

$= \lim\limits_{x\to 0} \dfrac{2\cos x - 2\cos(2x)}{1-\cos x} = \text{"}\dfrac{0}{0}\text{"} = [\text{l'Hop}] =$

$= \lim\limits_{x\to 0} \dfrac{-2\sin x + 4\sin(2x)}{\sin x} =$

$= \lim\limits_{x\to 0} \left(-2 + 8\dfrac{\cancel{\sin x}\cos x}{\cancel{\sin x}}\right) = 6 \quad \left[\text{eller } [\text{l'Hop}] \text{ igjen}\right]$

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3) $f(x) = \dfrac{e^{-x}}{\sqrt{x^2-x-2}} = \dfrac{e^{-x}}{\sqrt{(x+1)(x-2)}} \qquad D_f = (-\infty, -1) \cup (2, \infty)$

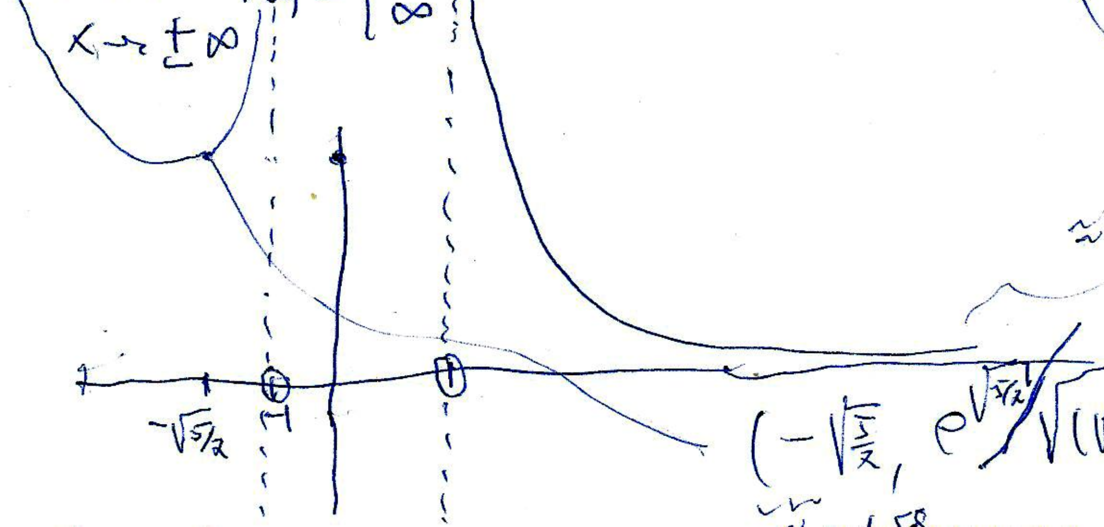
$\lim\limits_{x\to -1^-} f = \infty, \quad \lim\limits_{x\to 2^+} f(x) = \infty$

$f'(x) = \ldots = \dfrac{e^{-x}\left(-x^2+\frac{5}{2}\right)}{(x^2-x-2)^{3/2}}$

x	$-\infty$		$-\sqrt{5/2}$		-1		2		∞
f'	$-\infty$	$-$	0	$+$		▨		$-$	0
f	∞	↘	⇧	↗	∞	▨	∞	↘	0

$e^{\sqrt{5/2}} / \sqrt{\left(\sqrt{\tfrac{5}{2}}-1\right)\left(\sqrt{\tfrac{5}{2}}+2\right)}$

$\lim\limits_{x\to\pm\infty} f(x) = \begin{cases} 0 \\ \infty \end{cases}$



$\approx 3{,}37$

$\left(-\sqrt{\tfrac{5}{2}},\ e^{\sqrt{5/2}} / \sqrt{\left(\sqrt{\tfrac{5}{2}}-1\right)\left(\sqrt{\tfrac{5}{2}}+2\right)}\right)$

$\approx -1{,}58$