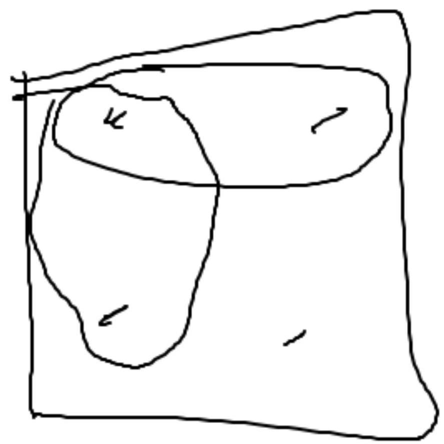
$\mathbb{X} = 4$	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
μ	$1/4$	$1/4$	$1/4$	$1/4$
ν	$1/2$	0	0	$1/2$

$$\mathcal{E} = \{ \{(0,0),(1,0)\}, \{(0,0),(0,1)\} \}$$

$\mu = \nu$ on $\mathcal{E}$. exercise/easy



$$\sigma(\mathcal{E}) = P(\mathbb{X})$$

$\mu \neq \nu$

Motivation

$X = (0,0)\ (1,0)\ (0,1)\ (1,1)$

$\mathbb{P}$. Y_1, Y_2 are indep each $\begin{array}{cc} 0 & 1/2 \\ 1 & 1/2 \end{array}$

$(\tilde{Y}_1, \tilde{Y}_2)$ $\quad \tilde{Y}_1 \ \begin{array}{cc} 0 & 1/2 \\ 1 & 1/2 \end{array} \quad \tilde{Y}_2 = \tilde{Y}_1$

Dist of $(Y_1, Y_2) = \mu$ $\quad$ Dist $(\tilde{Y}_1, \tilde{Y}_2) = \nu$

marg dist same but joint dist diff

$\{(0,0),(1,0)\} = \{Y_2 = 0\}$ $\quad \{\tilde{Y}_2 = 0\}$ $\square$

$A \subseteq [0,1]$ $\quad x \in A$

Def x is isolated in A if $\exists\, \varepsilon > 0$ such that $(x-\varepsilon, x+\varepsilon) \cap A = \{x\}$

no pt in $\mathbb{Q} \cap [0,1]$ is isolated

0 not isolated in $\{0, 1/2, 1/3, \ldots$

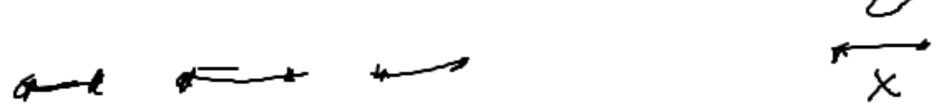
No pt in Cantor set is isolat

Let $x \in C$ assume $(x-\varepsilon, x+\varepsilon) \cap C = $

Level n interv

There are no~~t~~ atoms In

nth Level

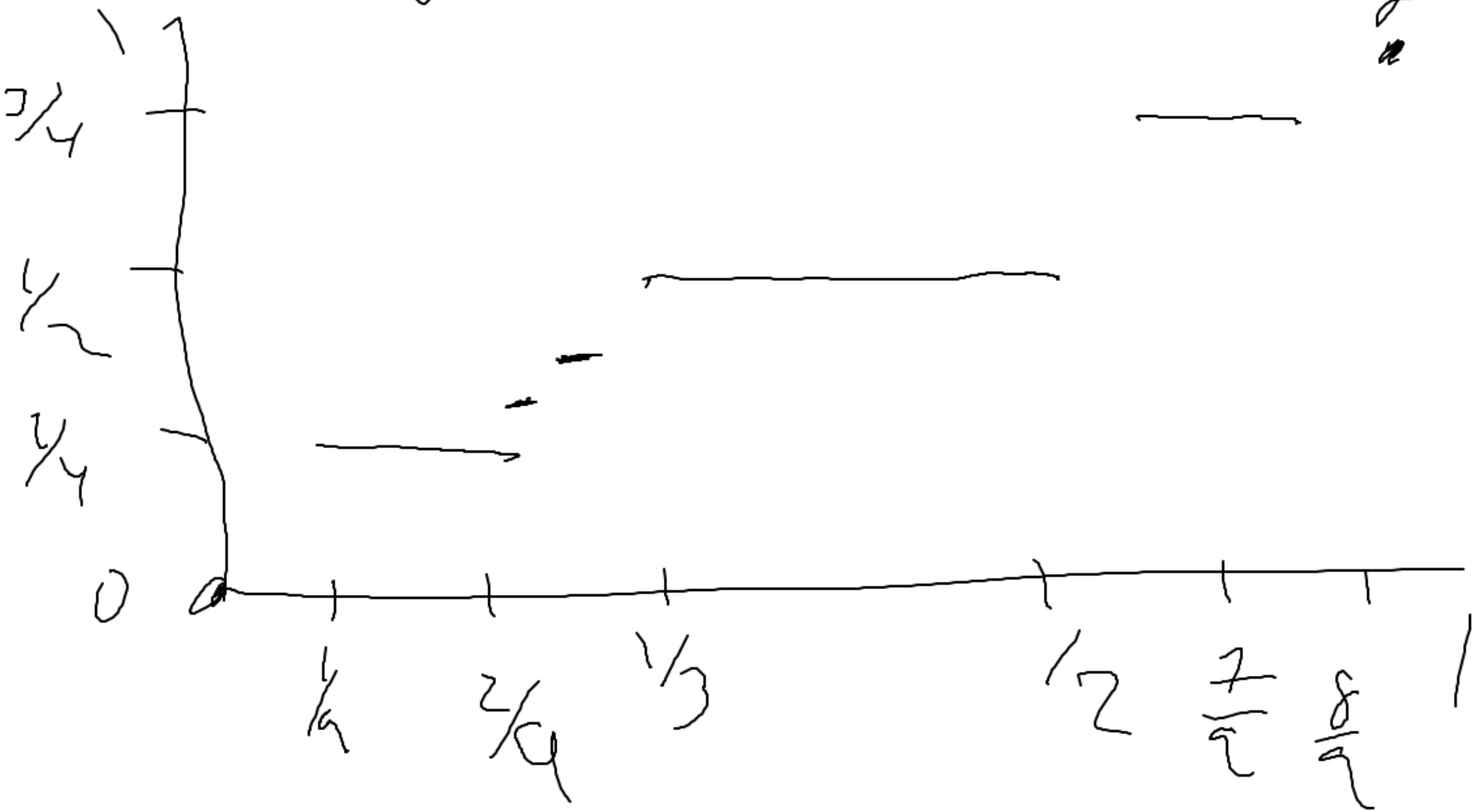


$x \in C$

2^n

Length $(1/3)^n$ weight $(1/2)^n$ for each $\mu_C(\{x\}) \le \mu_C$

Devil's ~~stair~~ staircase F coming from usually F defined directly.



μ msre on $[0,1]$ which $(\mathbb{Q} = \{q_1, q$

$\mu\{q_1\} = 1/2$, $\mu(\mathbb{Q}^c) = 0 \Rightarrow \mu$ atom

F_μ discont at every rational + ~~not~~

$\exists$? F inc which jumps at uncble points? NO $\Longleftrightarrow \nexists$ a finite Borel msr on $[0,1]$ with un

~~# of jum~~

# of atom

$\text{Jump at}(x) := \lim_{h \searrow x} F(h) - \lim_{h \nearrow x} F(h)$ $F(0$

$U_n = \{x : \text{Jump}(x) \geq 1/n\}$ $|U_n| < \infty$

$\bigcup_{n=1}^{\infty} U_n$ ctble but $\bigcup_n U_n =$

pg 7 alg not σ-alg

$X = \mathbb{N}$ $\quad \mathcal{Q} = \{\text{finite sets, cofinite sets}\}$

$\mathcal{Q}$ alg easy $\quad \{1\}, \{3\}, \{5\}, \{7\} \cdots \quad \bigcup$

μ

$|X| = 1, \Rightarrow \mathcal{S}_1 = \mathcal{F}_2$

$|X| \geq 2 \Rightarrow \mathcal{S}_2 \setminus \mathcal{S}_1$ Pf take $x \in X$ $\{x\} \in$

$\mathcal{S}_3 \overset{?}{=} \mathcal{S}_2$ X is ctble, then $\mathcal{S}_2 = \mathcal{S}_3$

X unctble $\overset{?}{\Longrightarrow} \mathcal{S}_2 \neq \mathcal{S}_3$

This is the same as finding $A \subseteq$

with A unctble, A^c unctble

Eg $[0,2]$ $A = [0,1)$

need

of ch

Recall $\mathcal{B} := \sigma(\text{open sets})$

$\mathcal{B} = \sigma(\text{open sets}) = \sigma(\text{open intervals})$

$\supseteq$ each open interval is $\subseteq \sigma(\text{op$

Since RHS is a σ-alg $\sigma(\text{intervals})$

$\subseteq$ Fact every open set in $\mathbb{R}$ is a <u>ctble</u> disjoint union of interval

In $\mathbb{R}^n$, tru

but disjo

not

Then open sets $\subseteq \sigma($open interval$)$

$\sigma($open sets$) \subseteq \sigma($open interval$)$

$\sigma($closed sets$) = \sigma($open sets$)$

all closed sets $\subseteq \sigma($open sets$)$

$\Rightarrow \sigma($closed$) \subseteq \sigma($open sets$)$

U open $= \quad U = \bigcup_{i=1}^{\infty} I_i$

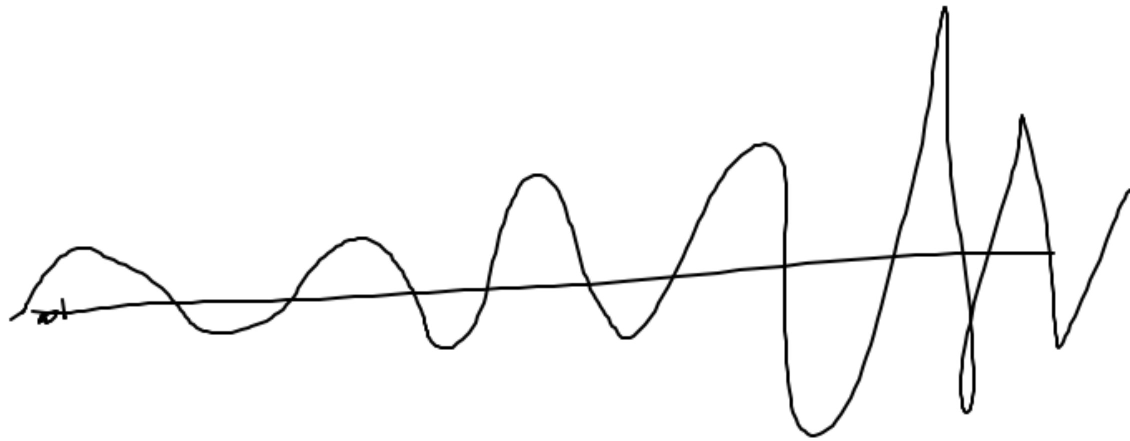
( )( )   ( )   ( )   ( ) ( ).

more complicated $U = \bigcup I_i$

Could be $\forall$ [illegible]

a side B_t 1-d. B M.

$\{t : B_t \neq 0\}$



open set

If people want feedback on Home you could send 15 to me.

Concise

Clear

(b) $(\mathbb{R}, \mathcal{B}$, Leb msre $m)$

$E_n = [n, \infty) \qquad E_1 \supseteq E_2 \supseteq E_3 \qquad \bigcap (E_i$

$0 = m\left(\bigcap_{n=1}^{\infty} E_n\right) \neq \lim_{i \to \infty} m(E_i)$

(a) $m(X) < \infty \qquad E_1 \supseteq E_2 \supseteq \cdots \Rightarrow m(\bigcap$

$G_i = E_i^c \qquad G_1 \subseteq G_2 \subseteq G_3 \cdots$

$\Rightarrow \quad m(\bigcup G_i) = \lim_{n\to\infty} m(G_n)$

$m((\bigcap E_i)^c) = \lim m(G_n)$

$m(X) - m(\bigcap E_i) = m(X) - \lim m$

a. $\mathbb{R}$ σ finite $\mathbb{R} = \bigcup_{n \in \mathbb{Z}} [n, n+1]$

(b) assume $X = \bigcup_{i=1}^{\infty} A_i$, $\mu(A_i) < \infty$ $\forall$

not necess disjoint

$B_1 = A_1$ $B_2 = A_2 \setminus A_1$ $B_3 = A_3 \setminus (A_1 \cup A$

$- - -$ $B_n = A_n \setminus (\bigcup_{i=1}^{n-1}$

$x \in B_n$, if n is first i st $x \in$

$([0,1], \mathcal{P}[0,1], \underset{\mu}{\text{counting measure}})$

$\mu(A) = \text{card of } A$

not σ-finite, $\quad [0,1] = \underbrace{\bigcup_{i=1}^{\infty} A_i}_{\text{ctble}} \quad \mu(A$

$(X, \mathcal{F}, \mu)$ σ-finite $\Rightarrow$ $\mathcal{C}$ = atoms ctble

Pf $X = \bigcup_{i=1}^{\infty} A_i$ $\mu(A_i) < \infty$

$A_i \cap \mathcal{C}$ [illegible]

$= \bigcup_{k=1}^{\infty} F$ [illegible]

$\Rightarrow A_i \cap \mathcal{C}$ [illegible]

$\mathcal{C} = \bigcup_{i=1}^{\infty} (\mathcal{C} \cap A_i)$. Need only show

$\mathcal{C} \cap A_i$ ctble

$F_k = \{x \in \mathcal{C} \cap A_i : \mu(\{x\})$ [illegible]

F_k finite

$\mu(A_i \cap \mathcal{C}) \geq$ [illegible]

If F_k infinite, $\geq \infty$

Recall: If μ, ν are measure on $(\mathbb{X}, \sigma(\mathcal{D}))$ where $\mathcal{D}$ is a π-system and $\mu(A) = \nu(A)\ \forall A$

$\mu(\mathbb{X}) = \nu(\mathbb{X}) \implies \mu = \nu$

If μ and ν agree on some collection of sets $\mathcal{E}$,

then μ, ν defined on $(\mathbb{X}, \sigma(\mathcal{E}))$ are the same