1, class rep

2, HW - discussion

3 white board

$g : [0,1] \longrightarrow [0,2]$

$g(x) = f(x) + x$ where $f : [0,1] \longrightarrow [0,1]$ Ca

T

g bijection, g cont and g^{-1} cont (g, g^{-1} are

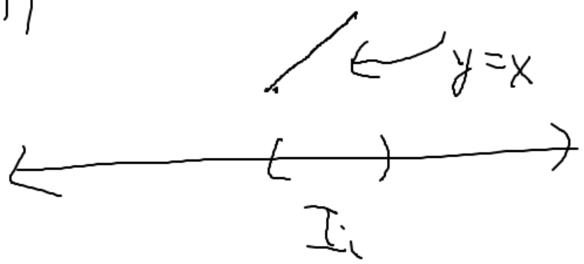
$m(g(C)) = 1$, ETS $m(g(C^c)) = 1$

$C^c = \bigcup_i I_i$ disjoint

$m(g(C^c)) = m(g(\bigcup_i I_i)) = m(\underbrace{\bigcup g(I_i)}_{\text{disjoint}})$ $[0,1)$

$\leq$ $(g(I_i))$

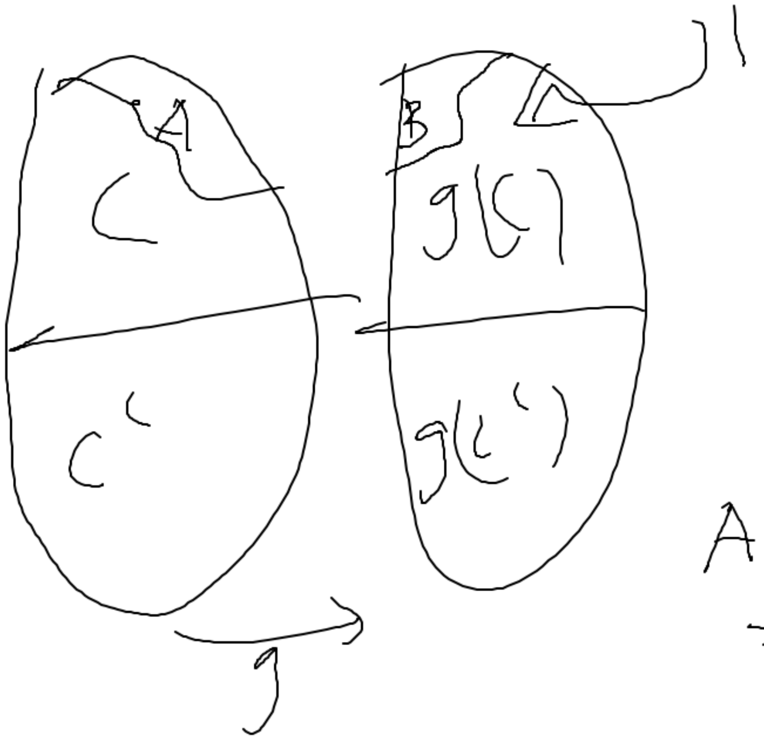
$m(g(I_i))$



$g(x) = x + f(x)$

$= m(I_i)$

$m(g(C)) = \sum_i m(g(I_i)) =$



Choose $B \subseteq g(C$

with B nonms

$\underline{A} = g^{-1}(B)$

A is msble since $A \subseteq C$

$\Rightarrow A$ msble $\rightarrow$ S

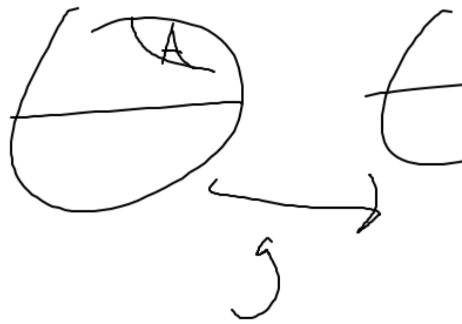
g^{-1} Borel msble   an

A cannot be Borel ~~beca~~ since if A were Borel, (g^{-1})

Find F, G $\quad$ F msble $\quad$ G cont

$F \circ G$ not msble

$F = I_A \qquad G = g^{-1}$



$F \circ G = I_A \circ g^{-1} = I_B$ not msble since B is not

1, class rep

2, HW - discussion

3 white board

$g: [0,1] \longrightarrow [0,2]$

$g(x) = f(x) + x$ where $f: [0,1] \longrightarrow [0,1]$ Ca

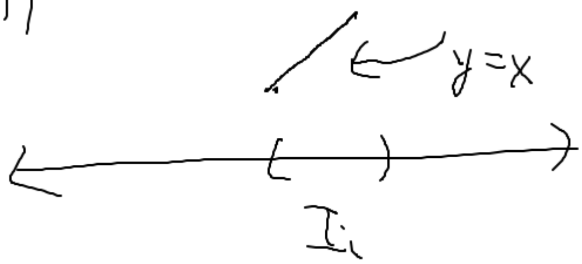
g bijection, g cont and g^{-1} cont (g, g^{-1} are

$m(g(C)) = 1$, ETS $m(g(C^c)) = 1$

$C^c = \bigcup_i I_i$ disjoint

$m(g(C^c)) = m(g(\bigcup_i I_i)) = m(\underbrace{\bigcup g(I_i)}_{\text{disjoint}})$ [0,1]

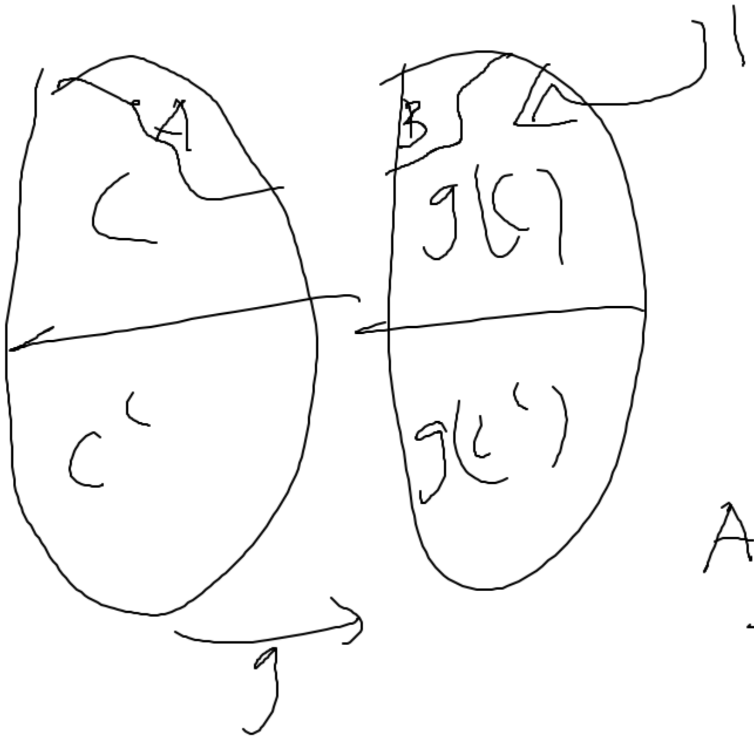
$m(g(I_i))$



$g(x) = x + f(x)$

$= m(I_i)$

$m(g(C)) = \sum_i m(g(I_i)) =$



Choose $B \subseteq g(C$
with B nonm

$\underline{A} = g^{-1}(B)$

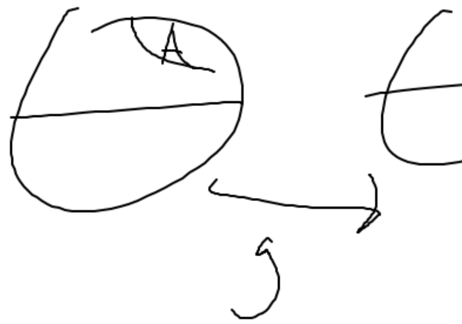
A is msble since $A \subseteq C$

$\Rightarrow A$ msble

g^{-1} Borel msble an

A cannot be Borel ~~beach~~ since if A were Borel, (g^{-1})

Find F, G F msble G cont



$F \circ G$ not msble

$F = I_A$ $G = g^{-1}$

$F \circ G = I_A \circ g^{-1} = I_B$ not msble since B is not

Are the groups $\mathbb{R}$ and $\mathbb{R} \times \mathbb{R}$ isomorphic?

$\exists ?$ a bijection f from $\mathbb{R}$ to $\mathbb{R} \times \mathbb{R}$

st $f(x+y) = f(x) + f(y)$