Comment: exercise on pg 108 harder than anticipa
whiteboard showed a relationship betwee
signed measures and fcns of finite vari
this exercise asks for uniqueness

Folland

Folland pg 88. #4

Jordon Decomp thm: If μ is a signed me
then $\exists$ unique ~~μ^+~~ measures μ^+, μ^-

(1) $\mu = \mu^+ - \mu^-$

(2) $\mu^+ \perp \mu^-$

Does one have uniqueness if One o

No If m is any measure

$$\mu = \underbrace{(\mu^+ + m)} - \underbrace{(\mu^- + m)}.$$ Only u

Key step exercise ν signed measure $\nu =$

~~$\mu = \lambda - \nu$~~ λ, μ measure

Show if $\nu = \lambda - \mu$, $\lambda - \mu = \nu^+ -$

then $\lambda \geq \nu^+$, $\mu \geq \nu^-$

Pf Let (P, N) be the Hahn decomp of ν (

$$\underline{\lambda(E)} \geq \lambda(E \cap P) = \mu(E \cap P) + \nu^+(E \cap P) \geq \nu^+(E \cap P) = \underline{\nu^+(E)} \checkmark$$

$$\underline{\mu(E)} = \underbrace{\lambda(E) - \nu^+(E)}_{\geq 0} + \nu^-(E) \geq \underline{\nu^-(E)}.$$

To answer ? 2 slides ago we WT

if $\nu = \lambda - \mu$, then there is a meas

so that $\lambda = \nu^+ + m \quad \mu = \nu^- + m$

Pf $\lambda - \mu = \nu^+ - \nu^- \quad (= \nu$

$\lambda - \nu^+ = \mu - \nu^-$

$\lambda = \nu^+ + \underbrace{\lambda - \nu^+}_{\geq 0 \text{ measure}} \quad \mu = \nu^- +$

$=$

pg 100 25

$E \subseteq \mathbb{R}$ borel set   Density of E at x,

$$D_E(x) = \lim_{r \to 0} \frac{m(E \cap (x-r, x+r))}{2r}$$

if lim exists

proportion of $(x-r, x+r)$ which is in

$E = [0,1]$

$$D_E(x) = \begin{cases} 1 & x \in (0,1) \\ 1/2 & x = 0,1 \\ 0 & x \notin [0,1] \end{cases}$$

(a) Show $D_E(x) = 1$ for ae x in E

$D_E(x) = 0$ for ae x not in E

Pf special case of the differentiation

Let $f = I_E$ f is locally integrable

Diff Thm $\Rightarrow \lim_{r \to 0} A_r(f)(x) = f(x)$ ae

$\lim_{r \to 0} \frac{\int_{x-r}^{x+r} f}{2r} = \frac{\int_{x-r}^{x+r} f(t)\,dt}{2r} = \frac{m(E \cap (x-r, x+r))}{2r}$

$\Rightarrow D_E(x) = 1$ For ae x

(b) Find examples of E where things fail at some

Give an example of E and $x=0$, so that

$$\underline{\lim}_{r\to 0} \frac{m(E\cap(-r,r))}{2r} \le 1/4 \qquad \overline{\lim} \frac{m(E\cap(}{2r}$$

$\cancel{m(B(0, 1/2}$

$$\frac{m\left(E\cap\left(-\frac{1}{2^n}, \frac{1}{2^n}\right)\right)}{2/2^n} = 1/4$$

$\forall n$

$$\frac{m\left(E_n \left(-\frac{1.5}{2^{n+1}}, \frac{1.5}{2^{n+1}}\right)\right)}{3/2^{n+1}} = \frac{\frac{.5}{2^{n+1}} + \frac{.5}{2^{n+1}}}{3/2^{n+1}}$$

<u>Lim not exist</u>

Pg 109 4.1(a),

Point of example is to find a strictly increas

abs cont function ~~which~~ whose deriv is 0

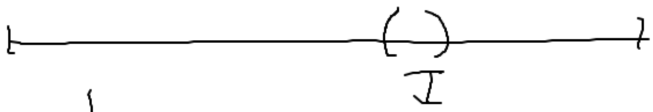
on a set of pos measure.

Cantor Ternary fcn ~~was~~ had a deriv = 0 a

but neither strictly increasing nor abs cont

Earlier exercise: $\exists$ a Borel set $A \subseteq [0,1]$ s.t.
for all intervals I, $0 < m(A \cap I) < m(I)$

ie $m(A \cap I) > 0$
$m(A^c \cap I) > 0$



$F(x) := m([0,x] \cap A)$ (1) F is abs. cont

(1) Pf Show Lipschitz, $|F(x+h) - F(x)| = m([x, x+h] \cap A) \leq$
Lip $\Rightarrow$ AC

(2) F is strictly increasing. ie $(F(x+h) > F(x))$

$F(x+h) - F(x) = m((x, x+h) \cap A) > 0$

$F' = 0$ on a set of pos msre

$$\frac{F(x+h) - F(x)}{h} = \frac{m((x, x+h) \cap A)}{h} \Rightarrow$$

$$= \frac{\int_x^{x+h} I_A \, dt}{h} \xrightarrow{\text{Diff. Thm}} I_A \text{ a.e } F$$

$\Rightarrow F' = 0$ a.e on A^c

$m(A^c) > 0 \Rightarrow F' = 0$ on a set of p

b. Finds a fcn G which is abs co

but not monotone on any interval

Comment Research pa

$\exists$ a fcn f which is diff at every

but not monotone on interva

f' not cont (f' cont, Impossible

$$L_p = \{ f : \Omega \to \mathbb{R} \ , \ \int |f|^p < \infty \}$$

$$\|f\| = \left(\int |f|^p \right)^{1/p}$$