Övningsmöte

Övningsmötte_1.2

den 17 augusti 2020 14:15

A1.3 / Exercises 4, 12, 30

A1.4 / Exercises 2, 6, 10, 16

4. $\lim\limits_{x \to -\infty} \dfrac{x^2-2}{x-x^2}$

obs

$x - x^2 = 0$

$+(x^2 - x) = 0$

$x(x-1) = 0$

$x = 0$

$x = 1$

$D_f = \mathbb{R} \setminus \{0, 1\}$

1) $f(x) = \dfrac{x^2-2}{-(x^2-x)} = \dfrac{(x-\sqrt{2})(x+\sqrt{2})}{-(x)(x-1)}$???

2) $f(x) = \dfrac{\cancel{x^2}\left(1 - \frac{2}{x^2}\right)}{-\cancel{x^2}\left(1 - \frac{1}{x}\right)}$ $\quad -x^2 + x$

$f(x) = -\dfrac{(1 - 2/x^2)}{(1 - 1/x)}$

$\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} -\dfrac{(1 - 2/x^2)}{(1 - 1/x)} =$ $\quad (2/x^2 \to 0_+,\ 1/x \to 0_-)$

$= -\dfrac{[1]}{[1]} = -1$

$$\boxed{\lim_{x \to -\infty} f(x) = -1}$$

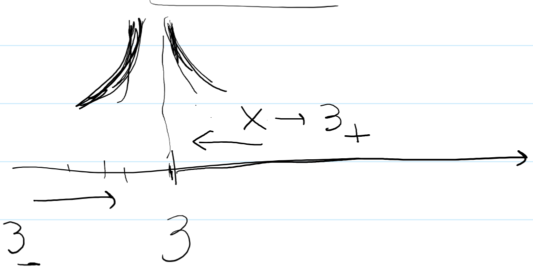
Ex 12 $\quad \lim\limits_{x \to 3} \dfrac{1}{(3-x)^2}$

obs $(3-x)^2 = 0$

$+3 - x = 0 \quad \boxed{3 = x}$

om $x = 3 \Rightarrow \boxed{\text{nämnare} = 0}$???

$D_f = \mathbb{R} \setminus \{3\}$



1) $\lim\limits_{x \to 3_+} \dfrac{1}{(3-x)^2} = \dfrac{1}{0_+} \longrightarrow +\infty$

2) $\lim\limits_{x \to +3_-} \dfrac{1}{(3-x)^2} = \dfrac{1}{0_+} \longrightarrow +\infty$

$$\lim_{x \to 3} f(x) = +\infty$$

Ex 30 $\quad \lim_{x \to +\infty} \left(\sqrt{x^2+2x} - \sqrt{x^2-2x}\right)$

1) $\left(\sqrt{\infty+\infty} - \sqrt{\infty-\infty}\right)$??? Det funkar inte!

2) $$f(x) = \left(\sqrt{x^2+2x} - \sqrt{x^2-2x}\right) \frac{\left(\sqrt{x^2+2x} + \sqrt{x^2-2x}\right)}{\left(\sqrt{x^2+2x} + \sqrt{x^2-2x}\right)}$$

$$f(x) = \frac{(x^2+2x) + \cancel{\left(\sqrt{x^2+2x}\sqrt{x^2-2x}\right)} - \cancel{\sqrt{x^2-2x}\sqrt{x^2+2x}} - (x^2-2x)}{\sqrt{x^2+2x} + \sqrt{x^2-2x}}$$

$$f(x) = \frac{\cancel{x^2} + 2x - \cancel{x^2} + 2x}{\sqrt{x^2+2x} + \sqrt{x^2-2x}} = \frac{4x}{\sqrt{x^2+2x} + \sqrt{x^2-2x}}$$

$$f(x) = \frac{4x}{\sqrt{x^2\left(1+\frac{2}{x}\right)} + \sqrt{x^2\left(1-\frac{2}{x}\right)}}$$

$$f(x) = \frac{4x}{\sqrt{x^2}\left(\sqrt{1+\frac{2}{x}}\right) + \sqrt{x^2}\left(\sqrt{1-\frac{2}{x}}\right)}$$

$$\sqrt{x^2} = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases} \longleftarrow (*)$$

$$f(x) \overset{(*)}{=} \frac{4\cancel{x}}{\cancel{x}\left(\sqrt{1+\frac{2}{x}} + \sqrt{1-\frac{2}{x}}\right)} = \frac{4}{\left(\sqrt{1+\frac{2}{x}} + \sqrt{1-\frac{2}{x}}\right)}$$

$$\lim f(x) = \lim_{x \to \infty} \frac{4}{\ldots} = \frac{4}{\ldots} =$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{4}{\left(\sqrt{1+\frac{2}{x}} + \sqrt{1-\frac{2}{x}}\right)} = \frac{4}{\sqrt{1}+\sqrt{1}} = \frac{4}{2} = 2$$

$\frac{2}{x} \to 0$, $\sqrt{1+\frac{2}{x}} \to 1$, $\sqrt{1-\frac{2}{x}} \to 1$

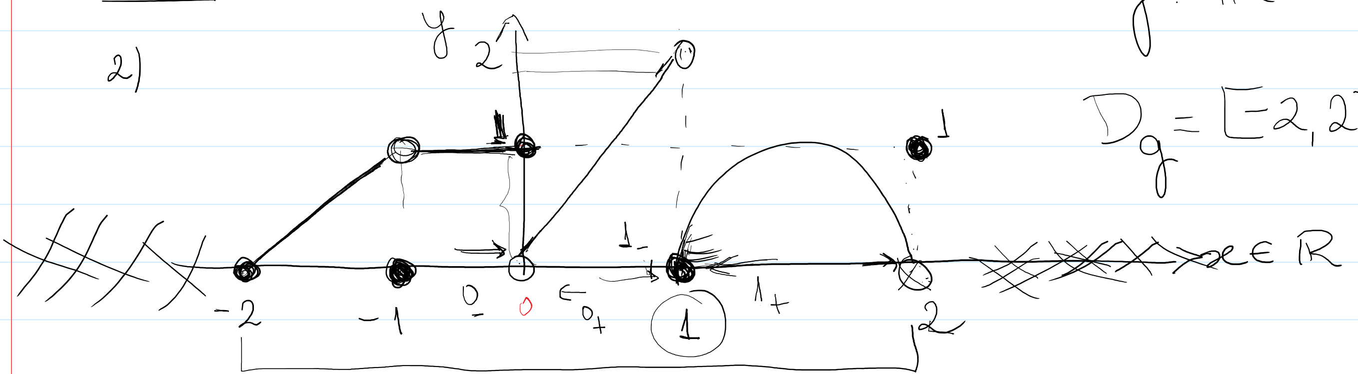
$$\lim_{\substack{x\to\infty \\ (+\infty)}} f(x) = 2$$

§1.4 Exercises 2/6/10/16

$g: \mathbb{R} \to \mathbb{R}$

$D_g = [-2, 2]$

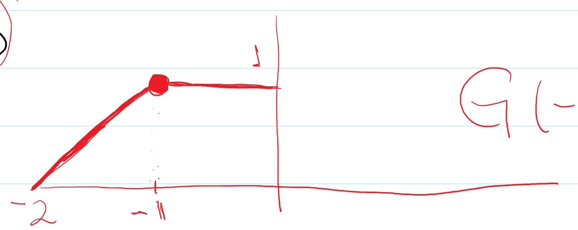
2)



1) $\lim_{x\to -1} g(x) = \lim_{x\to -1_+} g(x) = \lim_{x\to -1_-} g(x) = 1 = L$

$g(-1) = 0$

$G(x) =$



$G(-1) = 1 = L$

2) $x \to 0$

$\lim_{x\to 0_-} g(x) = 1$

$\lim_{x\to 0_+} g(x) = 0$

$\neq$ Så kan man inte fixa det.

3) $x \to 1$

$\lim_{x\to 1_-} g(x) = 2$

$\lim_{x\to 1_+} g(x) = 0$

$\neq$

4) $x \to 2$

$\lim_{x\to 2_-} g(x) = 0$

$\neq$

$$x \to 2_-$$

$g(2) = 1$ $\neq$

$$G(x) \begin{cases} = g(x) & \forall x \in (1,2) \\ = 0, & x = 2 \end{cases}$$

6) $\mathrm{sig}(x) = \dfrac{x}{|x|}$? $x = 0$?

$x = 0 \Rightarrow |x| = 0$ men $\mathrm{sig}(x)$ är inte väl definerad på $x=0$.

$\mathrm{sig}(0) = \dfrac{0}{|0|}$?? Problem

$D_{sig} = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, +\infty)$

Så kan man inte säga att funktion är kontinuerlig i punkten 0

→ funktion är inte väl definerad där.

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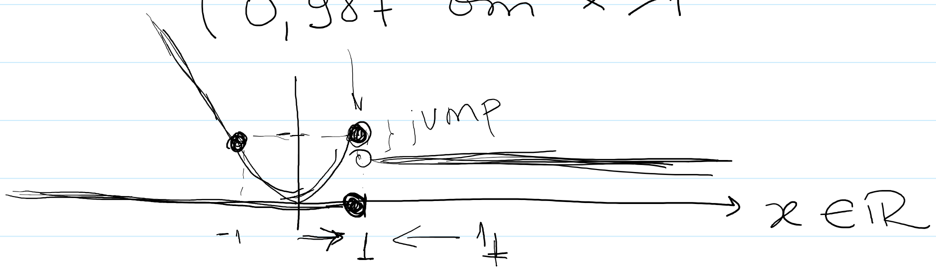
Förlåt Det är n°. 10

$$f(x) = \begin{cases} x^2 & \text{om } x \leq 1 \\ 0,987 & \text{om } x > 1 \end{cases}$$



$\forall x \in (-\infty, 1)$ $f(x)$ är kontinuerlig i x

vid $x = 1$ $f(x)$ är vänsterkontinuerlig $\neq$

$\forall x \in (1, +\infty)$ f(x) är kontinuerlig i x

vid $x=1$ f(x) är högerkontinuerlig $\nexists$

$\boxed{\text{f är diskontinuerig i } \underline{x = 1}}$

Ex 16 $f(x) = \frac{x^2 - 2}{x^4 - 4}$ $\lim_{x \to \sqrt{2}} f(x) = ?$

1) Direkt (?) $\frac{(\sqrt{2})^2 - 2}{(\sqrt{2})^4 - 4} = \frac{2-2}{(\sqrt{2}\cdot\sqrt{2})(\sqrt{2}\sqrt{2}) - 4} = \frac{0}{(4-4)} = \frac{0}{0}$???

Problem 1 $\sqrt{2} \notin D_f$

2) $x^4 - 4 = \underbrace{(x^2-2)(x^2+2)} = x^4 + \cancel{2x^2} - \cancel{2x^2} - 4 = x^4 - 4$

$f(x) = \frac{\cancel{(x^2-2)}}{\cancel{(x^2-2)}(x^2+2)} = \frac{1}{(x^2+2)}$

$\lim_{x \to \sqrt{2}} f(x) = \lim_{x \to \sqrt{2}} \frac{1}{x^2+2} = \frac{1}{[\sqrt{2}]^2+2} = \frac{1}{2+2} = \frac{1}{4}$

$= 0{,}25$

$$\boxed{\lim_{x \to \sqrt{2}} f(x) = 1/4}$$

$\forall x$ = for varje x som tillhör D_f

$\forall x \in D_f$