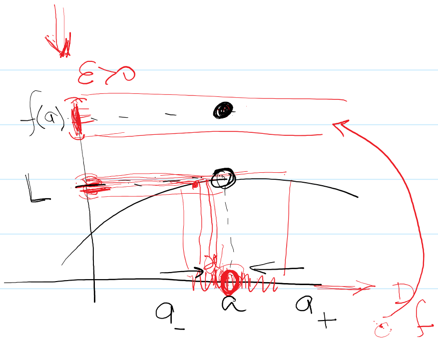
Mål Adams Bok §2.1 & §2.2 Derivata.



Från föra Föreläsning:

Uppgift: §1.5 Adams:

Använd formella Definition för att visa

$$\lim_{x\to 1/2} \frac{(1-4x^2)}{(1-2x)} = 2$$

$x \neq 1/2$

$1-2x=0 \Rightarrow 2x=1$, $x=1/2$

Definition
$$\lim_{x\to a} f(x) = L \Leftrightarrow \forall \varepsilon>0,\ \exists \delta>0 \ /\ x\in D_f \wedge |x-a|<\delta \Rightarrow |f(x)-L|<\varepsilon$$

Lösning. $\boxed{\text{Låt } \varepsilon>0}$

Vi har $1-4x^2 = (1-2x)(1+2x)$

Så $f(x) = \dfrac{1-4x^2}{1-2x} = \dfrac{(1-\cancel{2x})(1+2x)}{(1-\cancel{2x})} = (1+2x)$, då $\boxed{x\neq 1/2}$

$D_f = \mathbb{R}\setminus\{1/2\}$

Om $|x-1/2|<\delta$ (vi väljer δ senare),

då $|f(x)-2| = |1+2x-2| = |2x-1| = 2|x-1/2| < 2\delta$

$|2(x-1/2)|$

Så välj $\delta = \varepsilon/2$

Då är

$$|f(x)-2| = 2|x-1/2| < 2\delta = \cancel{2}\left(\frac{\varepsilon}{\cancel{2}}\right) = \varepsilon \quad \square$$

$|f(x)-L|$ $\qquad$ $< 2\delta$

$|f(x)-L|<\varepsilon$

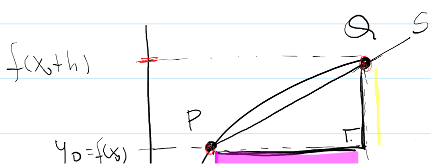
Kapitel 2 - Differentiation (Adams Bok sidan 95 - 105)

Def: 1 Suppose that the function f is continuous at $x=x_0$ & that $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = m$ exists

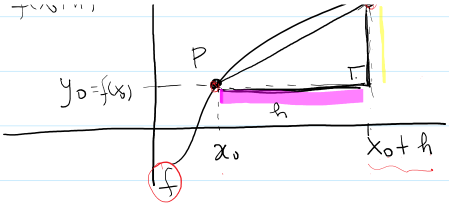
$\lim\limits_{x\to x_0} f(x) = f(x_0)$

Then the straight line having slope m & passing through the point $P=(x_0, f(x_0))$ is called the Tangent line to the graph of $y=f(x)$ at P.

Tangent equation: $\boxed{y = m(x-x_0)+y_0}$

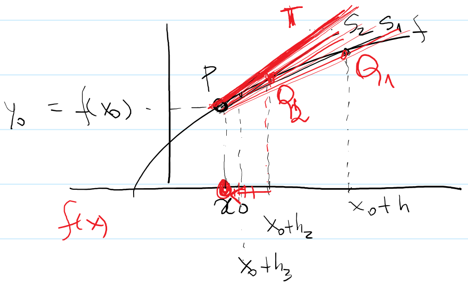


$$\frac{f(x_0+h)-f(x_0)}{(x_0+h)-x_0} = \frac{f(x_0+h)-f(x_0)}{h}$$



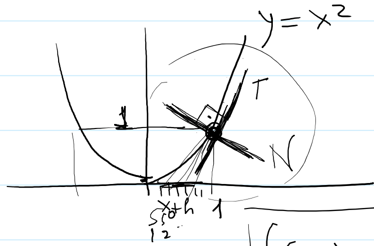
$x \in D_f$

$$\frac{f(x_0+h)-f(x_0)}{(x_0+h)-x_0} = \underbrace{\frac{f(x_0+h)-f(x_0)}{h}}_{\text{Newton's Quotient.}}$$



$$\lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h}$$

Page 97 Find eq of Tangent line to the curve $y = x^2$
at the point $P = (1, 1)$



$\boxed{f(x) = x^2}$

Lösning: $y = m(x-1) + 1$

$$m = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$$f(x_0) = (1)^2 = 1$$

$$f(x_0+h) = (1+h)^2 = 1^2 + 2 \cdot 1 \cdot h + h^2$$

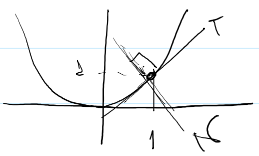
$$\lim_{h \to 0} \frac{(\cancel{1} + 2h + h^2) \cancel{-1}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(2+h)}{\cancel{h}} = \lim_{h \to 0} (2+h) = 2 + 0 = \underline{\underline{2}} = m$$

Tangent : $\boxed{y = 2(x-1) + 1}$ on $P = (1, 1)$

Normal line :

$$\boxed{\text{Slope of the normal} = \frac{-1}{\text{Slope of the Tangent}}}$$



$$N: \; y = \left(-\frac{1}{2}\right)(x-1) + 1$$

$$y = -\frac{1}{2}x + \frac{1}{2} + 1$$

$$\boxed{y = -\frac{1}{2}x + \frac{3}{2}}$$

Ex: Tangent line ? is it possible to compute for $f(x) = \sqrt[3]{x}$ $P = (0,0)$.

$$m = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$$f(x) = \sqrt[3]{x}$$

$$f(x_0) = f(0) = \sqrt[3]{0} = 0$$

$$f(x_0+h) = f(0+h) = f(h) = \sqrt[3]{h} = h^{1/3}$$

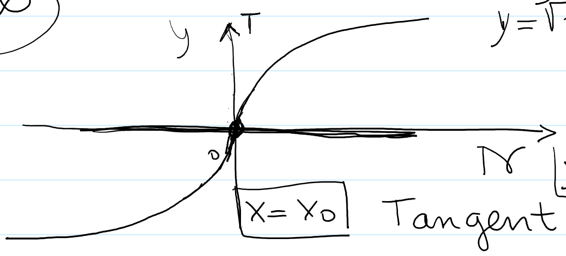
$$m = \lim_{h \to 0} \frac{\sqrt[3]{h} - 0}{h} = \lim_{h \to 0} \frac{h^{1/3}}{h} = \lim_{h \to 0} h^{1/3 - 1} =$$

$$= \lim_{h \to 0} h^{-2/3} = \lim_{h \to 0} \frac{1}{\sqrt[3]{(h^2)}} = \lim_{h \to 0} \frac{1}{h^{2/3}} \quad \left[\frac{1}{0_+}\right] = +\infty$$

$$- - - - \quad + + + +$$

$$0_- \to \quad 0 \quad \leftarrow 0_+$$

$$m = +\infty$$



$y = \sqrt[3]{x}$

$\boxed{x = x_0}$ Tangent

$\boxed{y = y_0}$

$m = +\infty$

$$\boxed{\cancel{y = m(x - x_0) + y_0}}$$

Tangent : $\boxed{x = 0}$ y-axis

$$m_{Normal} = \left(-\frac{1}{\infty}\right) = 0$$

Normal $\boxed{\begin{matrix} y = 0 \\ x\text{-axis} \end{matrix}}$

<u>Ex3</u> $f(x) = x^{2/3}$

Can we compute the Tangent of the curve $y = f(x)$ at $P = (0,0)$?

$$m = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$f(x) = x^{2/3}$

$f(x_0) = f(0) = 0^{2/3} = 0$

$f(x_0+h) = f(0+h) = f(h) = h^{2/3}$

$$m = \lim_{h\to 0} \frac{h^{2/3}}{h} = \lim_{h\to 0} h^{2/3 - 1} = \lim_{h\to 0} h^{-1/3} = \lim_{h\to 0} \frac{1}{h^{1/3}}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt[3]{h}}$$

$$\xrightarrow{\hspace{1cm}} 0 \xleftarrow{\hspace{1cm}}$$

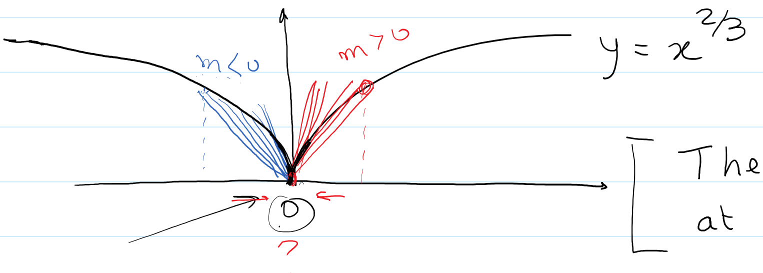
0_- , $h<0$ $\qquad$ 0_+ , $h>0$

$$\lim_{h\to 0_+} \frac{1}{\sqrt[3]{h}} = \left(\frac{1}{0_+}\right) = +\infty$$

But $$\lim_{h\to 0_-} \frac{1}{\sqrt[3]{h}} = \left(\frac{1}{0_-}\right) = -\infty$$

$\nexists \lim_{h\to 0} \frac{1}{\sqrt[3]{h}}$ $\therefore$ $\nexists\, m \equiv$ slope of Tangent $\Longrightarrow$

$\nexists$ Tangent



The function is not "smooth" at $x = 0$.

Def 3 The slope of curve C at a point P is the slope of the Tangent line to C at P

If such a tangent line exist.

$y = f(x) \Rightarrow m$ of a function at the point x_0

$$\lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

Ex

Ex Find the slope of the curve $y = \left(\dfrac{x}{3x+2}\right)$ at $\underline{x_0 = -2}$

$$\lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h} \qquad \underline{x_0 = -2}$$

$$\frac{-2}{3(-2)+2} \quad \frac{-2}{-6+2} = \frac{-2}{-4} = \frac{1}{2}$$

$$\lim_{h\to 0} \frac{f(-2+h) - f(-2)}{h} = \lim_{h\to 0} \frac{\left(\dfrac{-2+h}{3(-2+h)+2}\right) - \left(\dfrac{1}{2}\right)}{h}$$

$$\lim_{h\to 0} \frac{\left(\dfrac{-2+h}{-\cancel{6}+3h+\cancel{2}}\right) - \dfrac{1}{2}}{h} =$$

$$\lim_{h\to 0} \frac{\dfrac{-2+h}{-4+3h} - \dfrac{1}{2}}{h} = \lim_{h\to 0} \frac{\dfrac{2(-2+h) - 1(-4+3h)}{2(-4+3h)}}{h} =$$

$$\lim_{h\to 0} \frac{\cancel{-4}+2h+\cancel{4}-3h}{h(-8+6h)} = \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}(-8+6h)} =$$

$$\lim_{h\to 0} \frac{-1}{-8+\cancel{6h}^{\,0}} = \frac{-1}{-8} = \boxed{+\frac{1}{8}}$$

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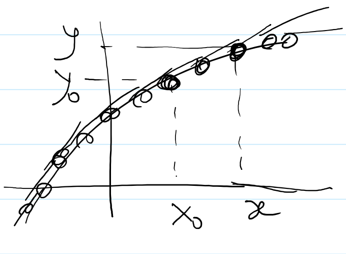
§2.2

Def 4 The Derivative of a function f is ANOTHER function f' (f prime) by

$$\underline{f'(x)} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$



Leibniz Notation

If $y = f(x)$ $\quad y =$ dependent variable
$\quad x =$ independent variable $\qquad \underline{x \in D_f}$

$$D_x y = y' = \frac{dy}{dx} = \underbrace{\frac{d}{dx}\big(f(x)\big)}_{*} = f'(x) = D_x f(x) = D f(x)$$

$\frac{d}{dx}\cdot \equiv$ Differential Operator

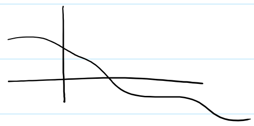
$$\frac{d}{dx}\cdot : \mathcal{F} \to \mathcal{F}$$

$\mathcal{F}$ = set of Differentiable functions

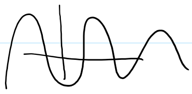
$$\underbrace{f(x)} \longrightarrow \frac{d}{dx}(f(x)) = f'(x)$$

$f(x)$ $\xrightarrow{\frac{d}{d}(f)}$ f'

$x \in D_f$



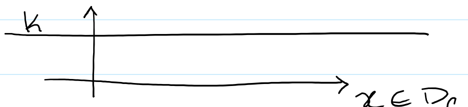
$g(x)$ $\xrightarrow{\frac{d}{dx}(g)}$ g'



$t(x)$ $\xrightarrow{\frac{d}{dx}(t)}$ t'

Some Important Derivatives:

$$\boxed{f(x) = k,\ k \equiv cte} \longrightarrow f'(x) = 0$$



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\boxed{\cancel{k} - k}}{h} = \frac{0}{h} = 0$$

$h \neq 0$ $\quad$ $h \neq 0$

$$f(x+h) = k$$
$$f(x) = k$$

$$f(x) = ax + b \qquad f'(x) = \lim_{h \to 0} \frac{a(x+h) + b - (ax+b)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{ax} + ah + \cancel{b} - \cancel{ax} - \cancel{b}}{h} = \lim_{h \to 0} a\frac{\cancel{h}}{\cancel{h}} = \underline{\underline{a}}$$

$$f(x) = ax + b \longrightarrow f'(x) = a$$

$$\boxed{f(x) = x^2},\quad f(x) = x^3, \ldots \quad f(x) = x^n$$

$f(x) = x^2 \longrightarrow f'(x) = ?$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} =$$

$$\boxed{\lim_{h \to 0} 2x + \cancel{h}^{\to 0} = \underline{\underline{2x}}}$$