Quiz-1-answer

den 14 september 2020 12:21

Fråga: ∘) Hitta min/max värde av funktioner med hjälp av Min/Max-satsen.

∘) Förklara när är det möjligt att använda satsen.

a) $f(x) = (x-1)^2, \quad x \in [0,1]$

b) $g(x) = (x-1)^2, \quad x \in (0,1)$

c) $h(x) = \begin{cases} (x-1)^2, & x \in [0,1) \quad x \neq 1 \\ 2, & x = 1. \end{cases}$

Lösning:

min-Max Satsen:

Hypothesis

$\boxed{\begin{array}{l} f:[a,b] \longrightarrow \mathbb{R} \\ f \text{ kontinuerlig i } \underbrace{[a,b]}_{\text{avstängd}} \end{array}} \Longrightarrow \left[\begin{array}{l} \exists\, p,q \in [a,b] / \\ \forall x \in [a,b] \\ \underbrace{f(p)}_{\min} \leq f(x) \leq \underbrace{f(q)}_{\text{Max}} \end{array} \right]$

One explanation about this Theorem:

1) The theorem tells us that the min and the max values of f exist on $I = [a,b]$, when

Hypothesis

1) f is continuous at I.
2) $I = [a,b] \equiv$ closed finite interval

2) This theorem does not give us a method to estimate the minimum or the maximum value of f. (This comes next with help of derivatives)

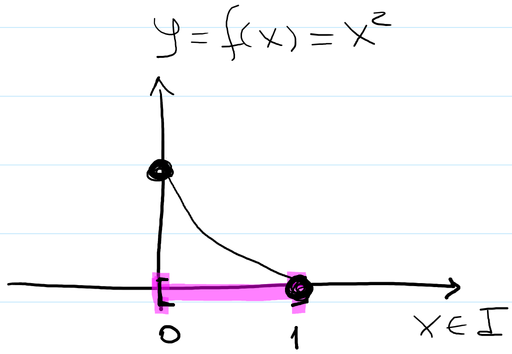
3) This is called an existence Theorem

4) $p \in I = [a,b]$: point on the domain where f achieves its minimum value

$f(p)$ = min value of f

$q \in I = [a,b]$ point on the domain where f achieves its maximum value.

$f(q)$ = Max value of f



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a) $f : [0,1] \to \mathbb{R}$

$x \longrightarrow f(x) = (x-1)^2$

f is defined on I,
I is a finite and Closed Interval (Hypothesis 1)

$f(x) = (x-1)^2 = x^2 - 2x + 1$ is a polynomial which is continuous on $\mathbb{R}$ and therefore is continuous at any subset of $\mathbb{R}$.
In particular f is continuous on $I = [a,b]$.
Hypothesis 2

So, by the min-Max theorem,
There exist $p, q \in [0,1]$ such that

$$\min = f(p) \leq f(x) \leq f(q) = \mathrm{Max}$$

So now, because we can evaluate the function and can verify by its graf, then we can obtain

$q = 0 \Rightarrow f(q) = f(0) = 1$ Max of f

$p = 1 \Rightarrow f(p) = f(1) = 0$ min of f

Obs. Be carefull when you draw the graph of a function
It depends on the domain.

→ Many of you consider $f(x)$ on intervals containing $[0,1]$,
→ Many of you did not care about scales,
→ Many of you did not care about the position of the function's zero.
These details are important.

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b) $f(x) = (x-1)^2$, $x \in (0,1)$

Now we have the same function
Which is continuous $\forall x \in \mathbb{R}$

BUT its domain now is an open interval

In this case, the hypothesis of the Theorem are not fully satisfied. (1 is, the other no), therefore we can not conclude anything supported by the min-max Theorem.

So $\exists \lim_{x\to 0} f(x) = 1$
$\exists \lim f(x) = 0$
But these 2 values do not belong to Im (f)

$\exists \lim_{x \to 1} f(x) = 0$ $\Big]$ do not belong to $Im(f)$

$Im(f) = \{ y \in \mathbb{R} / \exists x \in (0,1) \wedge f(x) = y \}$

↳ this is the image of the function.

The set of all function values.

But $M = \max f(x) = 1 \notin Im(f)$
because $0 \notin I = (0,1)$

$m = \min f(x) = 0 \notin Im(f)$
because $1 \notin I = (0,1)$

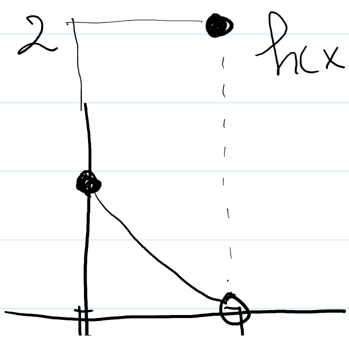
So, the function has no max nor min values on $(0,1)$

c) $h(x) = \begin{cases} (x-1)^2 & x \neq 1, \ x \in [0,1) \\ 2 & , x = 1 \end{cases}$

$h(x)$ is defined on $I = [0,1]$ = closed interval

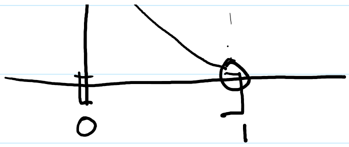
OK! Hypotesis (1/2) satisfied.

But $h(x)$ is NOT continuous on I.



$h(x)$ has a jump discontinuity

So, we again can not conclude anything based



conclude anything based on the Min-Max theorem, because now the continuity of the function on $I = [0,1]$ is lost.

Never the less,

by the graph of $f(x)$ and because the expression of the function is easy to be evaluated we still can find that

there exist $q \in [0,1]$, $q = 1$, such that
$$2 = f(q) \geq f(x), \quad \forall x \in [0,1]$$

So, there exist a Maximum value of f on the interval $[0,1]$.

But, there is no min value of f on $[0,1]$.

$$\lim_{x \to 1} f(x) = 0, \quad \text{but } f(1) \neq 0.$$

$\square$

Thank you! Alice