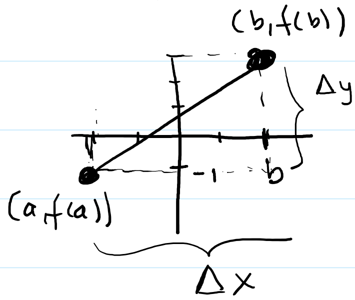
§ 2.8 Exercises

Illustrate the Mean Value Theorem
by finding any points on (a,b)
where the Tangent line to $y = f(x)$ is parallel
to the chord line joining $(a, f(a))$ & $(b, f(b))$

3.) $f(x) = x^3 - 3x + 1$ on $[-2,2]$

$f(-2) = (-2)^3 - 3(-2) + 1 = -8 + 6 + 1 = -8 + 7 = -1$
$(a, f(a)) = (-2, -1)$

$f(2) = (2)^3 - 3(2) + 1 = 8 - 6 + 1 = 9 - 6 = 3$
$(b, f(b)) = (2, 3)$



$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a} = \frac{3+1}{2-(-2)} = \frac{4}{4} = 1$$

$\circledast \ \frac{\Delta y}{\Delta x} = 1$ (inclination that we now are going to investigate)

The Mean-Value theorem states that

If $\left[\begin{array}{l} f \text{ is continuous on } [a,b] \\ \& \\ f \text{ is differentiable on } (a,b) \end{array}\right]$ then $\left[\begin{array}{l} \exists\, c \in (a,b) \text{ such that} \\ f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{\Delta y}{\Delta x} \circledast \end{array}\right.$

So $f(x) = x^3 - 3x + 1$ is a polynomial
therefore it is continuous on $[-2,2]$ &
differentiable on $(-2,2)$

$\frac{\Delta y}{\Delta x}$ we computed $= 1$

Now we need to find the value c such that $f'(c) = 1$.

i) Compute $f'(c)$

ii) Solve the equation $f'(c) = \frac{1}{2}$ for finding c.

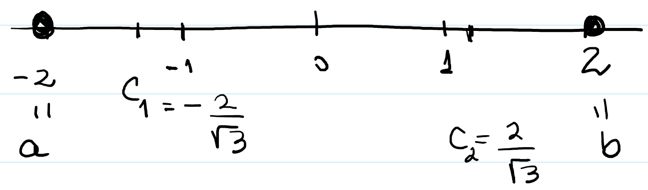
$$f(x) = x^3 - 3x + 1$$

$$f'(x) = 3x^2 - 3$$

$$f'(c) = 3c^2 - 3$$

$$f'(c) = 1 \Rightarrow 3c^2 - 3 = 1 \Rightarrow 3c^2 = 1 + 3 = 4$$

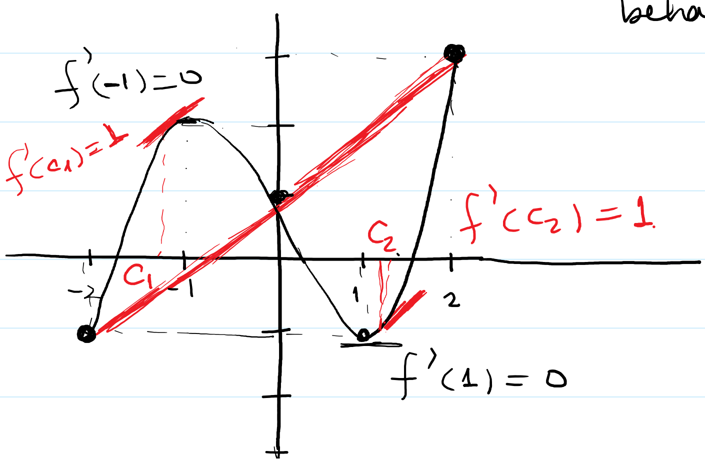
$$c^2 = \frac{4}{3}. \Rightarrow c_{1,2} = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}}$$



$f'(x) = 3x^2 - 3 = 0 \Rightarrow x^2 = \frac{3}{3} = 1 \Rightarrow x = \pm 1$ are points where the function f might change its behaviour

$$f(-1) = 3(-1) - 3 + 1$$

x	f(x)
-2	-1
-1	2
0	1
1	-1
2	3



$f'(1) = 0$