Övning 3.2

den 18 september 2020 09:54

A §2.8 Ex N°2 / N°8

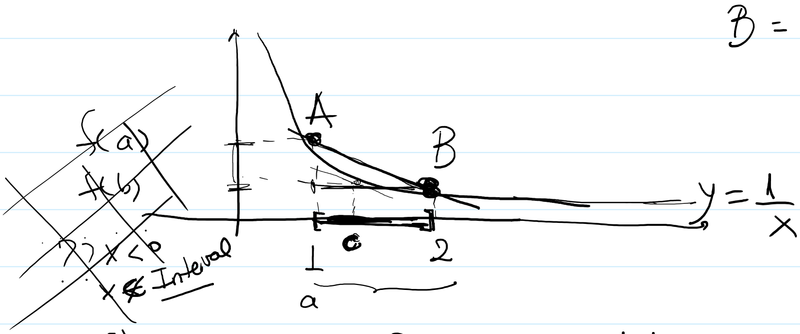
Ex 2 pag 144

[M-V-Theo: Find points in (a,b) where the Tangent line to $y=f(x)$ is // to the Chord line joining $(a, f(a))$ & $(b, f(b))$.)

2: $f(x) = 1/x$ on $[1, 2]$

$A = (a, f(a)) = (1, 1)$

$B = (b, f(b)) = (2, 1/2)$



$$\frac{1}{x} = x^{-1}$$

$$f'(x) = \frac{d}{dx}(x^{-1}) = -1x^{-1-1} = -\frac{1}{x^2}$$

Sekant: $$\frac{f(2)-f(1)}{2-1} = \frac{1/2 - 1}{2-1} = \frac{-1/2}{1} = -\frac{1}{2}$$

$$\cancel{-}\frac{1}{x^2} = \cancel{-}\frac{1}{2} \Longrightarrow 2 = x^2 \Longrightarrow x = +\sqrt{2} \leftarrow$$

$\cancel{x = -\sqrt{2}}$

So $c = +\sqrt{2}$

To confirm: $$f'(c) = -\frac{1}{(\sqrt{2})^2} = -\frac{1}{2}$$

$\square$.

N°8 Find the intervals where the function increases and when " " decreases.

$$f(x) = x^3 - 12x + 1$$

$$f'(x) = \frac{d}{dx}(f(x)) = \underbrace{3x^2 - 12}$$

$3x^2 - 12 = 0$

$3(\underbrace{x^2 - 4}) = 0$

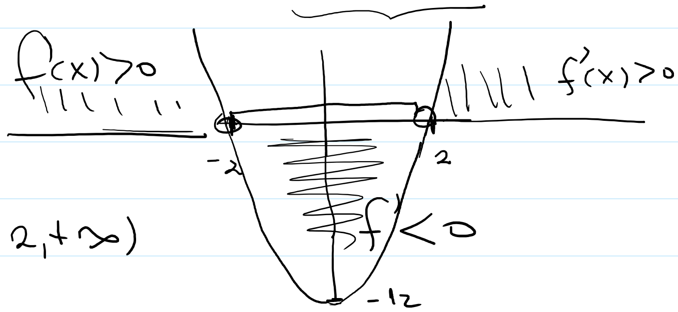
$f'(x) = \frac{d}{dx}(f(x)) = \underbrace{3x^2 - 12}$

$3(\underbrace{x^2-4}) = 0$

$3(\underbrace{x-2)(x+2}) = 0$

? $f'(x) > 0$?

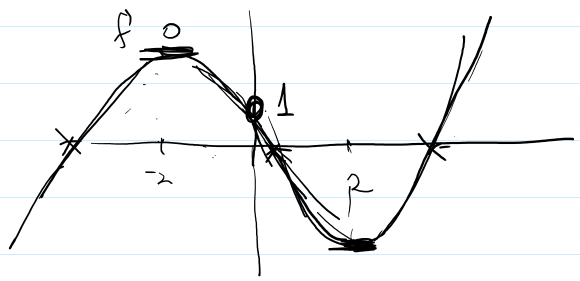
? $f'(x) < 0$?



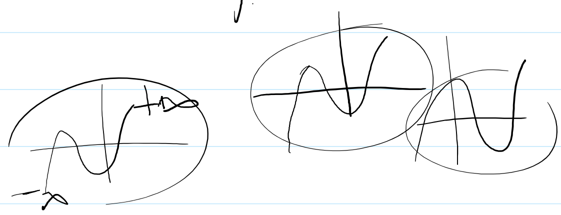
$f'(x) > 0$, when $x \in (-\infty, -2) \cup (2, +\infty)$

$f'(x) < 0$, when $x \in (-2, 2)$

$f'(x) = 0$ when $x = 2$, or $x = -2$



$f(x) = x^{3} - 12x + 1$



A 3.1 N10 & N 30

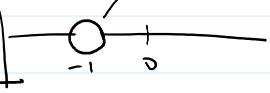
N10 . pag 171

1) Show that f is 1 : 1

2) Compute f^{-1}

3) Obtain D_f, $D_{f^{-1}}$, R_f, $R_{f^{-1}}$

N10 : $f(x) = \frac{x}{1+x}$ $1+x \neq 0 \Rightarrow \boxed{x \neq -1}$ $D_f = \mathbb{R} \setminus \{-1\}$ $-1 \notin D_f$



1) ? f. 1:1 ? $\underbrace{f(x_1) = f(x_2)} \Longrightarrow x_1 = x_2$ then f is 1:1.

$x_1, x_2 \in D_f$

$\left.\begin{array}{l} f(x_1) = \frac{x_1}{1+x_1} \\ f(x_2) = \frac{x_2}{} \end{array}\right\} =$

$$f(x_2) = \frac{x_2}{1+x_2}$$

$$\frac{x_1}{1+x_1} = \frac{x_2}{1+x_2} \Rightarrow x_1(1+x_2) = x_2(1+x_1) \Rightarrow$$

$$x_1 + \cancel{x_1 \cdot x_2} = x_2 + \cancel{x_2 \cdot x_1} \Longrightarrow$$

$$x_1 = x_2 \quad \checkmark$$

$\checkmark$ f is $1:1$.

$$f(\cancel{x}) = \frac{\cancel{x}}{1+\cancel{x}}$$

2) Compute f^{-1}.

$$y = f^{-1}(x) \Longleftrightarrow \boxed{f(y) = x}$$ starting point.

$$\frac{y}{1+y} = x \Longrightarrow$$

$$\underbrace{y = (1+y)x = x + xy}_{} \Longrightarrow$$

$$y - xy = x$$

$$y(1-x) = x \Rightarrow y = \frac{x}{1-x}$$

$1-x \neq 0 \quad x \neq 1 \qquad D_{f^{-1}} = \mathbb{R} \setminus \{1\}$

Check out:

$$x \overset{?}{=} \underbrace{f(y)} = f(f^{-1}(x)) \overset{?}{=} x\,?$$

Yes! $\boxed{f^{-1} = \frac{x}{1-x}}$

$$f(\blacksquare) = \frac{\blacksquare}{1+\blacksquare}$$

$$f(y) = \frac{\left(\frac{x}{1-x}\right)}{1 + \left(\frac{x}{1-x}\right)} = \frac{\frac{\cancel{x}}{1-x}}{\frac{(1-\cancel{x}) + \cancel{x}}{(1-x)}} =$$

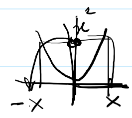
$$1 + \left(\frac{x}{1-x}\right) \qquad \frac{\quad}{(1-x)}$$

$$x = f(f^{-1}(x)) = f(y) = \frac{x}{\cancel{1-x}} \cdot \frac{(\cancel{1-x})}{1} = \frac{x}{1} = x$$

$$f(y) = \frac{\frac{x}{1-x}}{1 + \left(\frac{x}{1-x}\right)} = \frac{\frac{x}{1-x}}{\frac{1-\cancel{x}+\cancel{x}}{1-x}} = \frac{\frac{x}{1-x}}{\frac{1}{1-x}}$$

$$= \frac{x}{\cancel{(1-x)}} \cdot \frac{(\cancel{1-x})}{1} = \frac{x}{1}$$

$f(x) = x^2$



$$f(x_1) = f(x_2) \Rightarrow (x_1)^2 = (x_2)^2$$

$$\sqrt{(x_1)^2} = |x_1| < \begin{matrix} +x_1 \\ -x_1 \end{matrix}$$

$$\sqrt{(x_2)^2} = |x_2| \qquad x_1 < \begin{matrix} x_2 \\ -x_2 \end{matrix}$$

$$|x_1| = |x_2| \qquad -x_1 < \begin{matrix} x_2 \\ -x_2 \end{matrix}$$

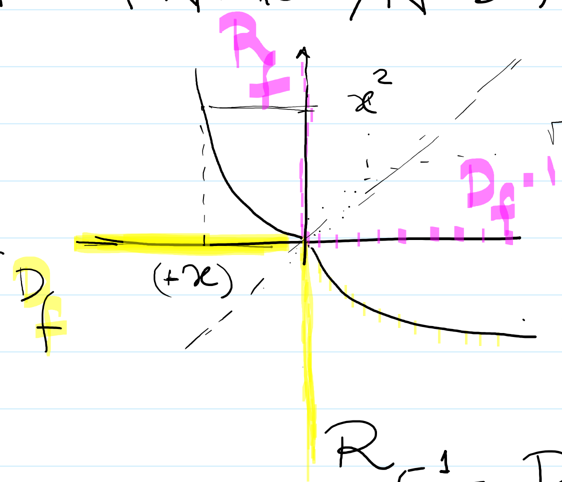
$$R_f = D_{f^{-1}} = \mathbb{R} \setminus \{1\}$$

$$D_f = R_{f^{-1}} = \mathbb{R} \setminus \{-1\}$$

A. 3.1 page 171 (N°. 10 / N°. 30)

A. 3.1 N°. 7

7) $f(x) = x^2 \quad x \leq 0$



$+x \leq 0+$

$(-x) \geq 0$

$$\boxed{y = -\sqrt{x}}$$ inverse

$D_f = (-\infty, 0]$

$R_f = [0, +\infty)$

$$R_{f^{-1}} = D_f = (-\infty, 0]$$

$$D_{f^{-1}} = R_f = [0, +\infty)$$

A. 3.1 (N30 is the recommended exercise.
But before we do N29)

1) Show that $f(x) = 4x^3$ has inverse

1) Show that $f(x) = \dfrac{4x^3}{x^2+1}$ has inverse

2) Find $(f^{-1})'(2)$

Resolution:

1) To show f is 1:1 we compute $f'(x)$ and verify its sign on an interval.

$(x^2+1)^{-1}$

$$f(x) = \frac{4x^3}{x^2+1} \qquad f'(x) = 4 \cdot 3x^2 \cdot \frac{1}{x^2+1} + 4x^3 \left((-1)(x^2+1)^{-2} \cdot 2x \right)$$

$$f'(x) = \frac{12x^2}{x^2+1} - \frac{8x^4}{(x^2+1)^2} =$$

$$f'(x) = \frac{12x^2(x^2+1) - 8x^4}{(x^2+1)^2} =$$

$$f'(x) = \frac{12x^4 + 12x^2 - 8x^4}{(x^2+1)^2} = \frac{4x^4+12x^2}{(x^2+1)^2} > 0$$

$$f'(x) > 0 \quad \forall x$$

$x^2+1 \neq 0$

$x^2 \neq -1$

$\boxed{x \neq \pm i}$

No Restriction on $\mathbb{R}$

Thus f is 1:1 (injektive)

2) Goal: $(f^{-1})'(2) \to \begin{cases} (f^{-1})' \\ (f^{-1})'(2) \end{cases}$

2.1) To find $(f^{-1})'$

$\boxed{y = f^{-1}(x)} \Longrightarrow f(y) = x$

Chain Rule

$$1 = \underline{dx} \qquad f'(y) \qquad \Rightarrow$$

$$(f^{-1}(x))' = y' = \frac{\quad}{f'(y)}$$

⊗ Obs we already have $f'(x)$

So, I just need to change $x \to y$ and put it in the denominator!

$$y' \quad \frac{\quad}{\left(\frac{\quad}{\quad}\right)} \quad \cdots$$

$$\frac{4y^4 + 12y^2}{(y^2+1)^2}$$

$$\boxed{y' = \frac{(y^2+1)^2}{4y^2(y^2+3)}}$$

$$\| $$

$$(f^{-1}(x))'$$

2.2) $\underline{(f^{-1})'(2)}$

[We need to find k such that $f(k) = 2$]

$$\boxed{X = \underbrace{f(k) = 2} \Longrightarrow \underbrace{y = f^{-1}(2) = k}} \quad (*)$$

$$f(k) = 2 \Rightarrow \frac{4k^3}{k^2+1} = 2 \Rightarrow 4k^3 = 2(k^2+1)$$

$$\boxed{\underbrace{4k^3}_{4} = \underbrace{2k^2+2}_{4}}$$

$k = 1$ ✓

$\underline{\text{So}}$: $\quad x = f(1) = 2 \Rightarrow \underbrace{y = f^{-1}(2) = 1}$

So.

$$(f^{-1})'(2) = \quad (y^2+1)^2 \quad | \qquad (1^2+1)^2$$

$$(f^{-1})'(2) = \left.\frac{(y^2+1)^2}{4y^2(y^2+3)}\right|_{y=1} = \frac{(1^2+1)^2}{4(1)^2(1+3)} = \frac{\cancel{4}}{\cancel{4}\cdot 4} = \frac{1}{4}$$

N°30 Find $(f^{-1})'(-2)$

$$f(x) = x\sqrt{3+x^2}$$

$$\boxed{y = f^{-1}(x)} \longrightarrow x = f(y)$$

↓ chain Rule

$$1 = \frac{dx}{dx} = f'(y)\cdot \boxed{y'}$$

$$(f^{-1})' = y' = \frac{1}{f'(y)}$$

$$\sqrt{3+x^2} = (3+x^2)^{1/2}$$

$$f(x) = x\sqrt{3+x^2}$$

$$f'(x) = 1\cdot\sqrt{3+x^2} + x\left(\frac{1}{\cancel{2}}(3+x^2)^{-1/2}\cdot \cancel{2}x\right)$$

$$f'(x) = \frac{\sqrt{3+x^2}}{1} + \frac{x^2}{\sqrt{3+x^2}} = \frac{(3+x^2)+x^2}{\sqrt{3+x^2}} =$$

$$f'(x) = \frac{2x^2+3}{\sqrt{3+x^2}} \Longrightarrow f'(y) = \frac{2y^2+3}{\sqrt{3+y^2}}$$

$$\boxed{y' = \frac{1}{f'(y)} = \frac{\sqrt{3+y^2}}{3+2y^2}}$$

$$y' = (f^{-1})'$$

Goal: $(f^{-1})'(-2) = ?$

Goal: $(f^{-1})'(-2) = ?$

$$\boxed{x = \underbrace{f(k) = -2}\ \Rightarrow\ y = f^{-1}(-2) = k}$$

$f(k) = -2$

$k\sqrt{3+k^2} = -2$

$k^2(3+k^2) = 4$

$k^4 + 3k^2 - 4 = 0$

$k^2 = t \rightarrow t^2 + 3t - 4 = 0$

$$t_{1,2} = \frac{-3 \pm \sqrt{9 + 4 \cdot 1 \,(+4)}}{2}$$

$$t_{1,2} = \frac{-3 \pm 5}{2} \quad \begin{cases} \frac{2}{2} = 1 \\ -\frac{8}{2} = -4 \end{cases}$$

$k^2 = t \Rightarrow k^2 = 1 \Rightarrow k = \begin{cases} 1 \\ -1 \end{cases}$

$k^2 = -4 \Rightarrow \cancel{k = \begin{matrix} +2i \\ -2i \end{matrix}} \notin \mathbb{R}$

$f(1) = 1\sqrt{3+1} = \sqrt{4} = 2$

$f(-1) = -1\sqrt{3+1} = -\sqrt{4} = -2$

The exercise wants (-2) $\therefore$ we need to choose $\underline{k=-1}$

$$\boxed{k = -1}$$

$$(f^{-1})'(-2) = \left.\frac{1}{f'(y)}\right|_{y=-1} = \frac{\sqrt{3+1}}{3+2} = \frac{2}{5} \quad \square$$