Föreläsning_4.2
_del_b
den 24 september 2020 09:12

§3.4 [Exponential & Logarithmical Growth] page 185-189

e^x, $\underline{\text{poly } n} = a_n x^n + \dots$

poly $\times$ $\ln x =$

Pag 185 Theorem 4

If $\boxed{x>0}$, then $\boxed{\ln x \leq x-1}$

Proof

So, $g'(x) >$ et $g(x) = \ln(x) - (x-1)$ for $x>0$.

$g(1) = \underbrace{\ln(1)}_{0} - \underbrace{(1-1)}_{0} = 0$ $\qquad g(1)=0$

$g'(x) < 0$ $\qquad g'(x) = \frac{1}{x} - 1 \;\; \} < 0 \;?\; \frac{1}{x} - 1 > 0 \Rightarrow \frac{1}{x} > 1$

increasing on $(0,1)$ $\Rightarrow \boxed{1 > x > 0}$

$g(x) < g(1)$, $\forall x \in (0,1) \Rightarrow$

$\ln(x) - (x-1) < 0 \Rightarrow \ln(x) < (x-1)$

$g(1) = 0 \Rightarrow$

$\frac{1}{x} < 1$

$1 < x$ $\quad \boxed{x > 1}$

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Theo 5 $\qquad 0 \;\; \forall x \in (0,1) \Rightarrow g(x)$ is

If $a>0$

(a) $\lim\limits_{x \to +\infty}$ $\quad$, $\forall x \in (1,+\infty) \Rightarrow g(x)$ is decreasing on $(1,+\infty)$

$g(1) < g(x) \quad \forall x \in (1,+\infty)$

(*) (b) $\lim\limits_{x \to \infty}$ $\qquad 0 < \ln(x) - (x-1) \Rightarrow$

$(x-1) < \ln(x)$

$\ln(x) = x-1 \,\big|_{x=1}$ $\qquad$ So $\ln(x) \leq (x-1) \quad \forall x>0$

$\boxtimes$:)

(c) $\lim\limits_{x \to -\infty}$

(d) $\lim\limits_{x \to 0_+}$ $\qquad$ The growth properties of Exp & ln.

then

Proof. $\qquad \frac{x^a}{e^x} = 0$ $\quad$ (e^x ~~any~~ $\to \infty$ ~~poly~~ much faster than)

(i) we

Tryck: $\qquad \frac{\ln x}{x^a} = 0$ $\quad$ (polynomial $\to \infty$ much faster than $\ln x$.

Since

using $|x|^a e^x = 0$ // In a struggle between x^a & e^x, e^x wins. "

Now $\underbrace{}_{0}$ $x^a \ln x = 0$ between x^a & $\ln$, x^a wins!

start proving (b) $\lim_{x\to\infty} \frac{\ln x}{x^a} = 0$

Let $x > 1$, $a > 0$ $S = \frac{a}{2}$ $(a = 2s)$

$\ln(x^s) = s\ln x$, we have

Theo 4 that

$0 < s\ln x = \ln(x^s) \leq x^s - 1 \underset{*}{<} x^s$

$\frac{0}{\cancel{s}} < \frac{\cancel{s}\ln x}{\cancel{s}} < \frac{x^s}{s} \Rightarrow 0 < \ln x < \frac{x^s}{s}$

By the

$x^a = x^{2s}$

$\frac{0 < \ln x < x^s \cdot \frac{1}{s}}{x^{2s}} \Rightarrow 0 < \frac{\ln x}{x^a} < \frac{x^s}{s x^{2s}} = \frac{1}{s x^s}$

Proof (b) —

$0 < \frac{\ln x}{x^a} < \frac{1}{s x^s}$

$\downarrow x\to\infty$

0

Squeeze Theorem $\boxed{\lim_{x\to\infty} \frac{\ln x}{x^a} = 0}$

→ (d) Goal : $\boxed{\lim_{x\to 0_+} x^a \cdot \ln x = 0}$

using that $\boxed{\lim_{x\to\infty} \frac{\ln x}{x^a} = 0}$ Hypothesis.

(*) Change of variable

$x \to \frac{1}{t}$ As $x \to 0_+ \Rightarrow \frac{1}{t} \to 0_+ \Rightarrow t \to +\infty$

$x \to \frac{1}{t}$

$0 \quad 0$

As $x \to 0_+ \Rightarrow \frac{1}{t} \to 0_+ \Rightarrow t \to +\infty$

So $\boxed{\lim_{x \to 0_+} x^a \ln x} \overset{*}{=} \lim_{t \to \infty} \left(\frac{1}{t}\right)^a \ln\left(\frac{1}{t}\right) =$

$\lim_{t \to \infty} \frac{1}{t^a} \underbrace{\ln(t^{-1})}_{\ln(t^{-1})} = - \underbrace{\lim_{t \to \infty} \frac{\ln(t)}{t^a}}_{\text{done before (by (b))}} = - \overset{4}{0} = 0$