Övning_4.2

den 25 september 2020 09:54

A 4.6 → Exercises 12, 34

A. 4.9 → Exercises 4, 21

Ex 12 $y = \dfrac{1}{4+x^2}$ >0 Sketch the graph of f

•) D_f, V_f $D_f = \mathbb{R}$ $V_f = \mathbb{R}_+ \setminus \{0\}$

•) critical points
singular points
Extreme points
↳ domain $(-\infty, +\infty)$

$\frac{1}{4} = 0{,}25$ $y(0) = \dfrac{1}{4+0} = \dfrac{1}{4}$

y', y''

•) inflection points

•) concativity ∪ up, ∩ down.

a) $y' = \dfrac{d}{dx}\left((x^2+4)^{-1}\right) = -1\,(x^2+4)^{-2}\cdot 2x$

$$\boxed{y' = \frac{-2x}{(x^2+4)^2}}$$

$\boxed{y'(0) = 0}$

$\boxed{y(0) = 0{,}25}$

$y'(x) = 0$? critical points ?

$\dfrac{-2x}{(x^2+4)^2} = 0 \Rightarrow -2x = 0 \Rightarrow \boxed{x = 0}$ $(x^2+4)^2 > 0$

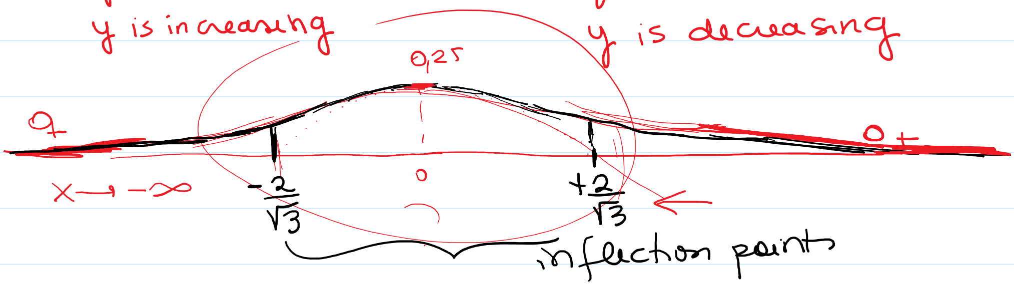
$y' > 0$		$y' < 0$
$x < 0$	$x = 0$	$x > 0$

$y'(x) > 0$ $y'(x) < 0$
y is increasing   y is decreasing

$y = \dfrac{1}{4+x^2}$

$\lim\limits_{x \to +\infty} \dfrac{1}{4+x^2} = 0_+$

$\lim\limits_{x \to -\infty} \dfrac{1}{4+x^2} = 0_+$



$y'' = \left(\dfrac{-2x}{(x^2+4)^2}\right)' = \dfrac{(-2)(x^2+4)^2 + (+2x)2(x^2+4)^1\cdot(2x)}{(x^2+4)^4} =$

$y'' = \dfrac{-2(x^2+4)^2 + 8x^2(x^2+4)^1}{(x^2+4)^4} = \dfrac{\cancel{(x^2+4)}\left(-2(x^2+4)+8x^2\right)}{(x^2+4)^{\cancel{4}}} =$

$(x^2+4)^4$ $\qquad\qquad$ $(x^2+4)^4$

$$y'' = \frac{-2x^2 - 8 + 8x^2}{(x^2+4)^3} = \frac{6x^2-8}{(x^2+4)^3}$$

$\boxed{y''(x) = 0}$

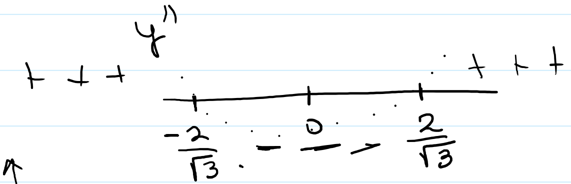
$$y''(x) = \frac{6x^2-8}{\underbrace{(x^2+4)^3}_{>0}} = 0$$

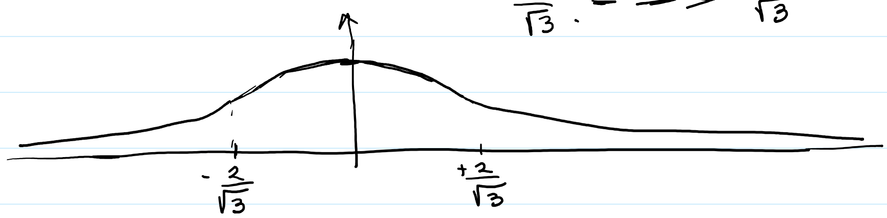
$6x^2 - 8 = 0 \Rightarrow$

$x^2 = \frac{8}{6} \Rightarrow x_{1,2} = \pm\sqrt{\frac{8}{6}}$

$x_{1,2} = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}}$

$\frac{2\sqrt{3}}{3}$





x		$-2/\sqrt{3}$		0		$2/\sqrt{3}$	
$y = f(x)$	+ +	+	+	+	+	+	+ + + +
$y' = f'(x)$	+ +	+	+	0	−	−	− − − − −
$y'' = f''(x)$	+ + +	0	− −	−	− − −	0	+ + + + + + +

inflection point $\qquad$ inflection point

$f'(x_0) = 0 \wedge f''(x_0) < 0 \Longrightarrow$

$x_0 =$ local maximum

$\square$

$$y'\left(-\frac{2}{\sqrt{3}}\right) = \left(\frac{-2x}{(x^2+4)^2}\right)\Bigg|_{x=-\frac{2}{\sqrt{3}}} = -2\left(-\frac{2}{\sqrt{3}}\right) = +\frac{4}{\sqrt{3}}$$

$(x^3)' = (3x^2)'$

$(3x^2)' = (6x)$

$y = x^3$

$y' = 3x^2$

$y'' = 6x$

Theorem 10

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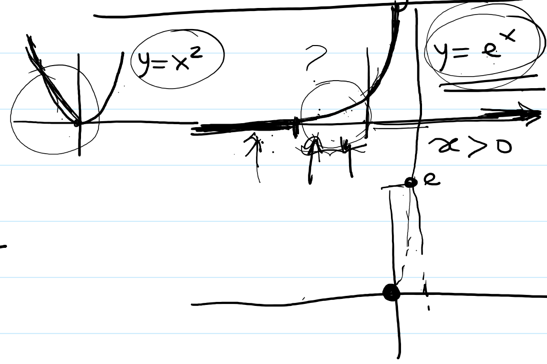
1) $f'(x)=0 \wedge f''(x_0) < 0 \Rightarrow$ Local Max at x_0

2) $f'(x_0)=0 \wedge f''(x_0) > 0 \Rightarrow$ Local minimum at x_0

3) $f'(x_0)=0 \wedge f''(x_0)=0 \Rightarrow$???

Ex 34 page 255 Sketch the graph

$y = x^2 e^x$



$e^x \to \infty$ faster than any $x^n \to \infty$

$y(0) = 0^2 e^0 = 0.1$

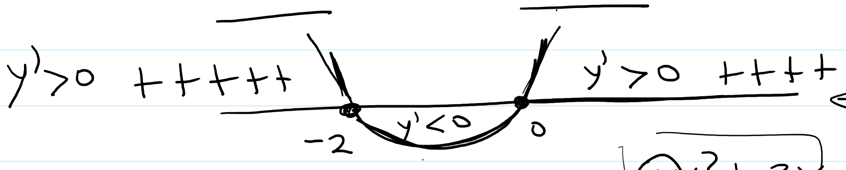
$y(1) = 1^2 e^1 = e$

$y'(x) = (2x)e^x + (x^2)e^x$

$\boxed{y'(x) = e^x(x^2+2x)} \Rightarrow y'(x) = 0 \quad \underbrace{e^x}_{\neq 0}\underbrace{(x^2+2x)}_{=0\,?} = 0$

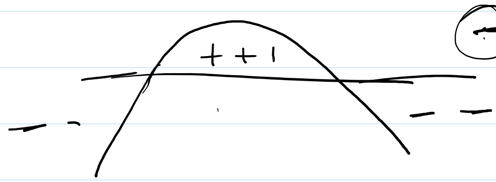
$(x^2+2x) = 0 \Rightarrow x(x+2) = 0$

$\underline{x=0}$ or $\underline{x=-2}$



$\boxed{1x^2+2x}$

$-x^2-2x$



$y'' = \frac{d}{dx}\Big(\underbrace{e^x}_{f}\underbrace{(x^2+2x)}_{g}\Big) = f'\cdot g + f\cdot g'$

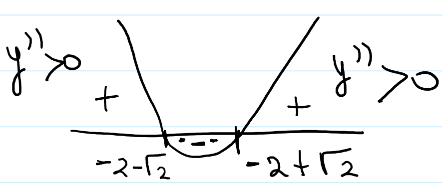
$= e^x(x^2+2x) + e^x(2x+2)$

$y'' = e^x(x^2+4x+2)$

$y''(x) = 0 \Rightarrow \underbrace{e^x}\underbrace{(1x^2+4x+2)} = 0 \Rightarrow 1x^2+4x+2 = 0$

$x_{1,2} = \dfrac{-4 \pm \sqrt{16-4.1.2}}{2.1} =$

$x_{1,2} = \dfrac{-4 \pm \sqrt{8}}{2}$ $\begin{cases} \dfrac{-\cancel{4}+\cancel{2}\sqrt{2}}{\cancel{2}} \\ \dfrac{-\cancel{4}-\cancel{2}\sqrt{2}}{\cancel{2}} \end{cases}$



$x_{1,2} = -2 + \sqrt{2}$ (

$y'' < 0$

$$x_{1,2} \begin{cases} -2+\sqrt{2} \\ -2-\sqrt{2} \end{cases} \Big\}\ \text{inflection points}$$

x	-4		$-2-\sqrt{2}$		-2		$-2+\sqrt{2}$		0	1	2
$f(x)$	+		+		+	+	+	+	0	+	+
$f'(x)$	+	+	+	+	0	−	−	−	0	+	+ +
$f''(x)$	+	+ +	0	− −	−	− −	0	+ + +	+	+ +	+ + + +

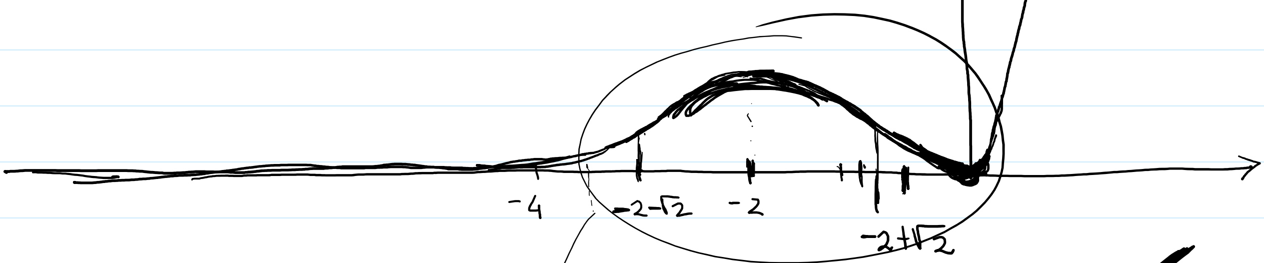
inflection point

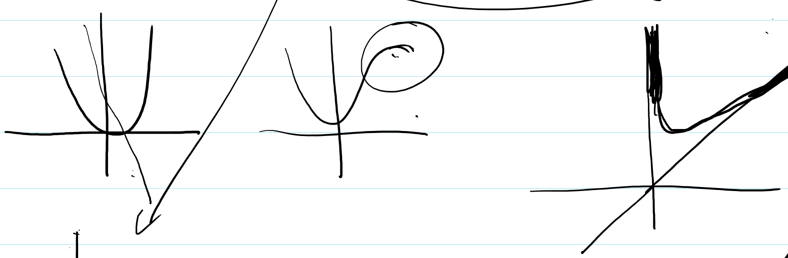
Local max point

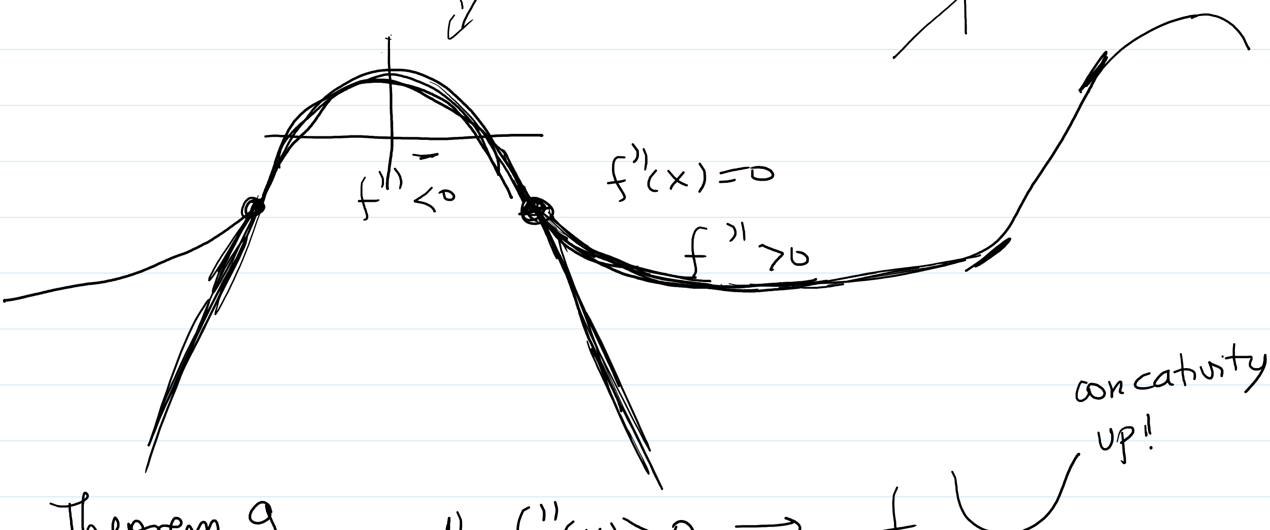
Critical point

inflection point

$-2+$







concativity up!

Theorem 9.

1) $f''(x) > 0 \Longrightarrow f$ ∪

concativity down

2) $f''(x) < 0 \Longrightarrow f$ ∩

3) Inflection point $\Longrightarrow f''(x) = 0$

Theo. 10

1) $f'(x_0)=0 \wedge f''(x_0)<0 \Rightarrow f(x_0)$ local Max Value

2) $f'(x_0)=0 \wedge f''(x_0)>0 \Rightarrow f(x_0)$ local Minvalue

3) $f'(x_0)=0 \wedge f'(x_0)=0$???

~~§ 4.9~~

Ex 4 & Ex 21 page 274

4) Find the linearization of f in $x_0=$ given

$f(x)=\sqrt{3+x^2}$ about $\boxed{x_0=1}$

$f(1)=\sqrt{3+(1)^2}$

$f(1)=\sqrt{4}$

$\boxed{f(1)=2}$

$f(x)=(3+x^2)^{1/2}$

$$f'(x)=\frac{1}{\cancel{2}}\underbrace{(3+x^2)}^{\frac{1}{2}-1}\cdot \cancel{2}x = \frac{x}{\sqrt{3+x^2}}$$

$f'(1)=\frac{1}{\sqrt{3+1}}$

$\frac{1}{\sqrt{4}}$

$\frac{1}{2}$

$x_0=1$ Tangent point

$T(x)=f(1)+f'(1)(x-1)$

$$\boxed{y(x)=2+\frac{1}{2}(x-1)}$$

Ex 21 is coming on Monday. :) Happy weekend!