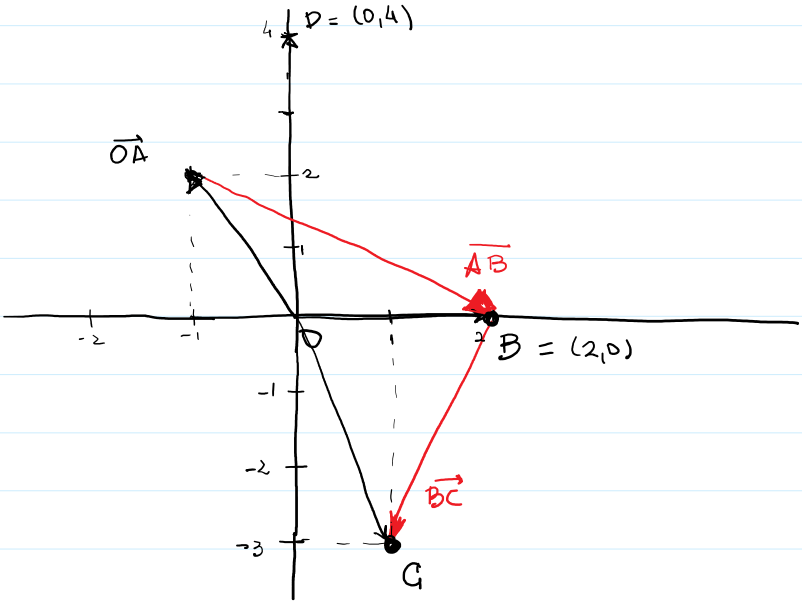
Övningsmöte 5.2

den 2 oktober 2020 10:00

Uppgifter Adams §10.2 1h, 2, 4, 5

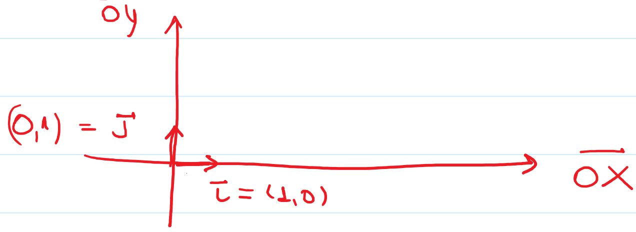


Övningsmöte 5.2 Adams §10.2

Ex: 1h, 2, 4, 5

Points in $\mathbb{R}^2$

1. $A = (-1,2)$
$B = (2,0)$
$C = (1,-3)$
$D = (0,4)$



h) $\dfrac{\vec{AB} + \vec{AC} + \vec{AD}}{3} = \vec{v}$ write as a linear combination of $\vec{i}$ & $\vec{j}$

Obs: $\vec{v}$ is a linear combination of $\{\vec{i}, \vec{j}\}$ if there exist constants a & b $(a, b \in \mathbb{R})$ such that

$$\vec{v} = a\vec{i} + b\vec{j}$$

$\vec{AB} = B - A = (2,0) - (-1,2) = (3,-2) = \vec{AB}$

$\vec{AC} = C - A = (1,-3) - (-1,2) = (2,-5) = \vec{AC}$

$\vec{AD} = D - A = (0,4) - (-1,2) = (1,2) = \vec{AD}$

$$\underbrace{\frac{(\vec{AB} + \vec{AC} + \vec{AD})}{3}} = \frac{(6,-5)}{3} = \left(\frac{6}{3}, \frac{-5}{3}\right) = \left(2, -\frac{5}{3}\right) =$$

$$= (2,0) + (0,-5/3) = 2(1,0) + -\frac{5}{3}(0,1) =$$

$$= \underbrace{2\vec{i} - \frac{5}{3}\vec{j}}$$

$a = 2$
$b = -5/3$

$$\vec{v} = \frac{(\vec{AB} + \vec{AC} + \vec{AD})}{3} = \underline{2\vec{i} + (-5/3)\vec{j}}.$$

? Can $\vec{i} = (1,0)$ be written as a linear combination of $\{\vec{AC}, \vec{AD}\}$?

? $(1,0) \overset{*}{=} a(2,-5) + b(1,2)$?

$\exists\, a, b$ such that this equality is satisfied?

$$\begin{cases} a.2 + b.1 = 1 \\ a.(-5) + b.2 = 0 \end{cases}$$

$$\begin{cases} 2a + 1b = 1 \\ -5a + 2b = 0 \end{cases}$$

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$\{\vec{i}, \vec{j}, \vec{k}\}$ vectors in $\mathbb{R}^3$

2

a) $\vec{u} + \vec{v}$

b) $\vec{u} - \vec{v}$

c) $2\vec{u} - 3\vec{v}$

d) $|\vec{u}|$, $|\vec{v}|$

$\vec{u} = \vec{i} - \vec{j} \overset{tok}{=} (1,-1,0)$

$\vec{v} = \vec{j} + 2\vec{k} = (0,1,2)$

a) $|u|$, $|v|$

a) $\vec{u}+\vec{v} = (1,-1,0) + (0,1,2) = (1+0, -1+1, 0+2) =$

$= (1, 0, 2)$

b) $\vec{u}-\vec{v} = (1,-1,0) + (0,-1,-2) = (1,-2,-2)$

c) $2\vec{u} - 3\vec{v} = (2,-2,0) + (0,-3,-6) = (2,-5,-6)$

d) $\|\vec{u}\| = \sqrt{(1)^2+(-1)^2+0^2} = \sqrt{2}$

d) $\|\vec{v}\| = \sqrt{0^2+1^2+2^2} = \sqrt{1+4} = \sqrt{5}$

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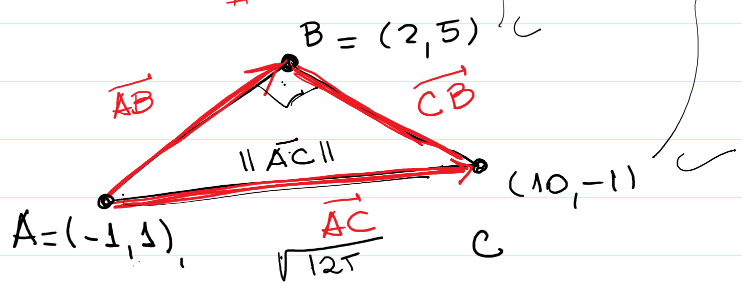
Use vectors to show that the Δ with vertices $\underbrace{(-1,1)}_{A}$, $\overbrace{(2,5)}^{B}$, $\overbrace{(10,-1)}^{C}$ is right-angled.



$\vec{AB} = B - A = (2,5) - \underbrace{(-1,1)}_{A} = (2+1, 5-1) = (3,4)$

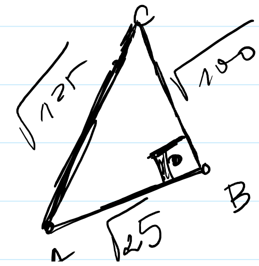
$\vec{AC} = (10,-1) - (-1,1) = (10+1, -1-1) = (11, -2)$

$\vec{CB} = B - C) = (2,5) - (10,-1) = (-8, 5+1) = (-8, 6)$

$\|\vec{AB}\| = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$

$\|\vec{AC}\| = \sqrt{(11)^2+(-2)^2} = \sqrt{121+4} = \sqrt{125} =$

$\|\vec{CB}\| = \sqrt{(-8)^2+(6)^2} = \sqrt{64+36} = \sqrt{100}$



$$(\sqrt{100})^2 + (\sqrt{25})^2 = (\sqrt{125})^2$$

$$100 + 25 = 125 \checkmark$$

So, since pitagoras theorem is valid,

△ has to have a right-angle

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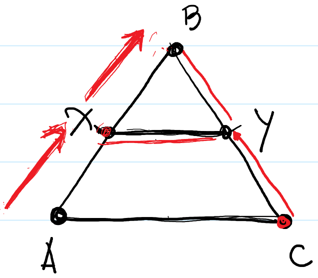
(?) " The line segment joining the midpoints of 2 sides of a △ is parallel to and half as long as the 3rd side ".

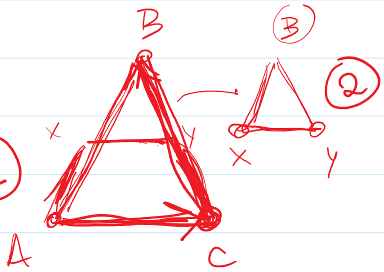


$\vec{AX} = \vec{XB}$ $\quad \vec{AB} = 2\vec{AX} \Rightarrow \boxed{\vec{AX} = \frac{1}{2}\vec{AB}}$

$\|\vec{AX}\| = \|\vec{XB}\| = \frac{1}{2}\|\vec{AB}\| \checkmark$

$\vec{CY} = \vec{YB}$ $\quad \vec{CB} = 2\vec{CY} \Rightarrow \boxed{\vec{CY} = \frac{1}{2}\vec{CB}}$

$\|\vec{CB}\| = 2\|\vec{CY}\|$



We want to show that

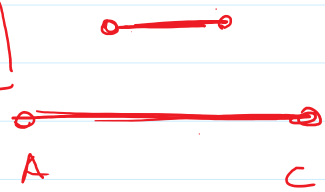
$$\boxed{\vec{XY} = \frac{1}{2}\vec{AC}} \qquad \|\vec{XY}\| = \frac{1}{2}\|\vec{AC}\|$$

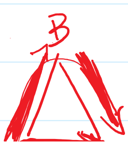
(1) $\vec{AC} + \vec{CB} = \vec{AB}$ (circuit)

$\vec{AC} + 2\vec{CY} = 2\vec{AX}$

$\vec{AC} = 2\vec{AX} - 2\vec{CY} = 2(\vec{AX} - \vec{CY})$

$\frac{\vec{AC}}{2} = \boxed{\vec{AX} - \vec{CY}}$ $\qquad \frac{\vec{AC}}{2} = \boxed{\vec{XY}}$

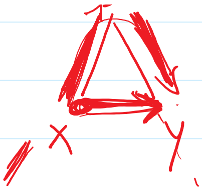


(2) 

$\vec{XB} + \vec{BY}$

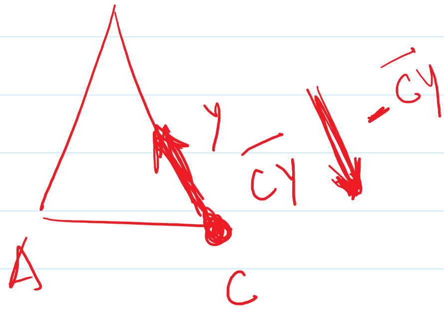
$\vec{XC} =$

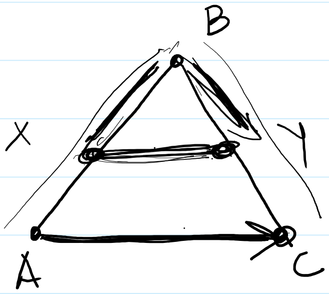
②

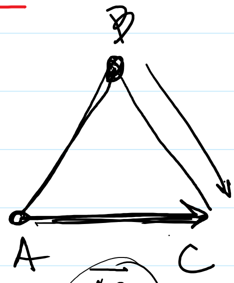


$\overrightarrow{AX} - \overrightarrow{CY}$

$$\boxed{\overrightarrow{XY}} = \boxed{\overrightarrow{AX} - \overrightarrow{CY}}$$







$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

$$\overrightarrow{XB} + \overrightarrow{BY} = \overrightarrow{XY}$$

$$\frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC}) = \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} =$$

$$\frac{1}{2}(\overrightarrow{AC}) = \overrightarrow{XY}$$