Section 4.9 page 274

Ex 21. About linearization. (I did this Exercise during the Lecture)

Instructions: Use a linearization to approximate $\sin(3.14)$

Resolution: $f(x) = \sin(x)$

$x_* = 3.14$

$a = \pi$ (point where I am going to compute the Tangent line)

$$f(x) = \sin x \longrightarrow f(\pi) = \sin(\pi) = 0$$

$$f'(x) = \cos x \qquad f'(\pi) = \cos(\pi) = -1$$

$$f''(x) = -\sin x \qquad f''(\pi) = -\sin(\pi) = -0 = 0$$

$$f'''(x) = -\cos x \qquad f'''(\pi) = -\cos(\pi) = -(-1) = +1.$$

So, for point in the vicinity of a we can approximate the function $f(x) = \sin(x)$ by its Tangent line : But $x \in (a-\delta, a+\delta)$ $\delta > 0$.

$$L(x) = T(x) = f(a) + f'(a)(x-a)$$

$$L(x) = \sin(\pi) + \cos(\pi)(x - \pi)$$

$$L(x) = -(x-\pi) = (\pi - x)$$

$$\boxed{\sin(3{,}14) \approx L(3{,}14) = (\pi - 3{,}14)} = 0{,}001592653\ldots$$

Now, we have to investigate the error involved in this approximation.

By theorem 11 page 272,

$$E(x) = \frac{f''(s)}{2}(x-a)^2, \qquad s \in (x,a) = (3{,}14;\ \pi)$$

But here
$$f''(x) = -\sin(x)$$
$$f''(\pi) = -\sin(\pi) = 0 \quad \therefore$$
$$f''(t) \approx 0 \quad \forall t \in (x,\pi)$$

So, one suggestion based on
The Taylor expansion & the error in the Taylor expansion is to consider:

$$T(x) = f(\pi) + \frac{f'(\pi)}{1!}(x-\pi) + \frac{f''(\pi)}{2!}(x-\pi)^2$$

$$E_3(x) = \frac{f'''(s)}{3!}(x-a)^3 \qquad s \in (x,a)$$

&

$$|E(x)| \leq K\frac{(\quad)^3}{3!} \qquad K = \max_{t \in (x,a)} |f'''(t)|$$

$$|f'''(t)| = |-\cos(t)| \leq 1$$
$$x \in (3{,}14;\ \pi)$$

So
$$|E(x)| \leq 1 \cdot \frac{(\pi - 3{,}14)^3}{3!} \cong 6{,}733 \times 10^{-10}$$

$\square$