Föreläsning 6.2_distances

den 5 oktober 2020 15:30

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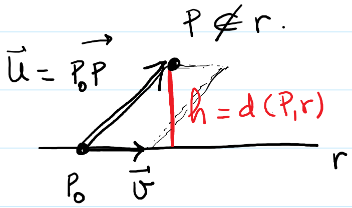
Mål: distance
- $d(P,r)$ Point / line
- $d(r,s)$ line / line
- $d(P,\Pi)$ Point / Plane
- $d(r,\Pi)$ line / Plane
- $d(\Pi_1,\Pi_2)$ Plane / Plane

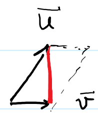
distance
- $d(P,C)$ P / circle
- $d(r,C)$ r / Circle
- $d(\Pi,C)$ Π / Circle.

1) $d(P,r)$, $P \notin r$. (case $P \in r$, $d(P,r)=0$)

$P \notin r$.



we can compute

$|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin\theta$ = Area of 

= base $*$ height

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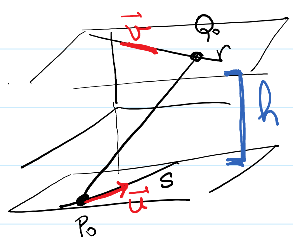
$$\boxed{d(P,r) = \frac{|\overrightarrow{P_0P} \times \vec{v}|}{|\vec{v}|}} = \left(\underbrace{\frac{|\vec{u}|\cancel{|\vec{v}|}\sin\theta}{\cancel{|\vec{v}|}} = |\vec{u}|\sin\theta}_{\text{this is not that useful}} \right)$$

2)

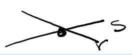


$d(r,s)$, $r \cap s = \emptyset$ no intersection

Remember: given 3 vectors $\{\overrightarrow{P_0Q_0}, \vec{u}, \vec{v}\}$ defining a paralelepiped, it is possible to define its Volume considering $|\overrightarrow{P_0Q_0} \cdot (\vec{u} \times \vec{v})|$

obs

$r \cap s$ = Point $\Rightarrow$ $d(r,s) = 0$



Since Volume = Area$_{\square}$ $\cdot$ height

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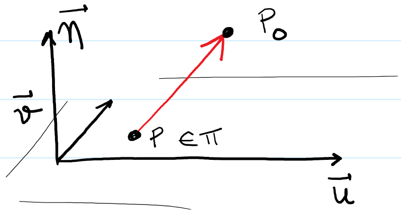
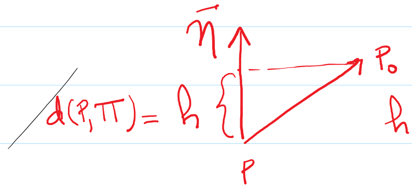
$$\boxed{d(r,s) = \text{height} = \frac{\text{Volume}}{\text{Area}_{\square}} = \frac{|\overrightarrow{P_0Q_0} \cdot \vec{u} \times \vec{v}|}{|\vec{u} \times \vec{v}|}}$$

here $\vec{u}$ & $\vec{v}$ are the direction vectors of the lines r & s.

and $\overrightarrow{P_0Q_0}$ is the third vector, constructed with the variation of one point on the line r and the other on s.

3) $d(P_0, \Pi)$ $\quad P_0 \notin \Pi$. $\quad$ (Case $P_0 \in \Pi$, $d(P_0,\Pi)=0$)





$h = |\operatorname{proj}_{\vec{n}} \vec{PP_0}| =$

$h = |\vec{PP_0}_{\vec{n}}|$

$$h = \left| \frac{\vec{PP_0} \cdot \vec{n}}{\cancel{\vec{n} \cdot \vec{n}}} \right| \cancel{\|\vec{n}\|}$$

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Example 7

$$\boxed{h = \frac{|\vec{PP_0} \cdot \vec{n}|}{|\vec{n}|}}$$

Now we can simplify this expression.

$$\vec{PP_0} = P_0 - P = (x_0\, y_0\, z_0) - (x, y, z) = (x_0 - x, y_0 - y, z_0 - z)$$

$$(P_0 - P) \cdot (a,b,c) = a(x_0 - x) + b(y_0 - y) + c(z_0 - z) =$$

$$= ax_0 + by_0 + cz_0 - (ax + by + cz) \stackrel{*}{=}$$

$*$ Since $P \in \Pi$,
$ax + by + cz = d$

$$= ax_0 + by_0 + cz_0 - d$$

$$\text{So } h = \frac{|(P_0 - P) \cdot (a,b,c)|}{|\vec{n}|} = \frac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2 + b^2 + c^2}}$$

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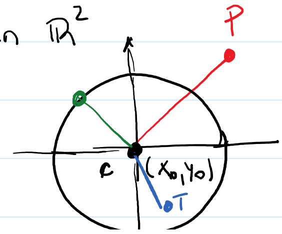
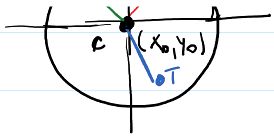
In $\mathbb{R}^2$



$C: (x - x_0)^2 + (y - y_0)^2 = r^2$ $\quad$ circle

$d((x_0,y_0), Q) = r \Rightarrow Q \in C$



$c = (x_0, y_0)$ the center of the Circle C.

$d((x_0,y_0), Q) = r \Rightarrow Q \in C$

$d((x_0,y_0), T) < r \Rightarrow T$ is interior to C

$d((x_0,y_0), P) > r \Rightarrow P$ is exterior to C

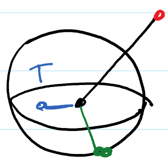
In $\mathbb{R}^3$ $\quad C = S$ = sphere : $(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2$

$c = (x_0, y_0, z_0)$ the center of the Sphere S.

$d(P, c) = r \Rightarrow P \in S$

$d(P, c) < r \Rightarrow P \in \text{int}(S) =$ interior of S

$d(P, c) > r \Rightarrow P \in \text{ext}(S) =$ exterior of S



P, $d(P,c) > r$

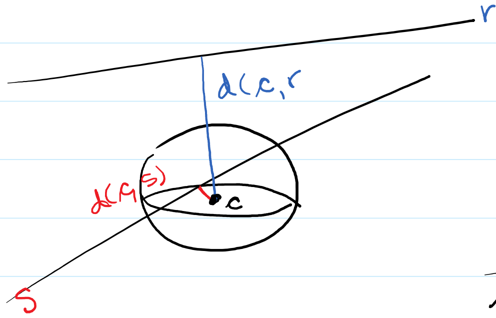
T, $d(T,c) < r$

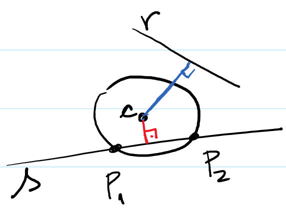
Q, $d(Q,c) = r$

$d(r, S) = d(c, r)$

$d(r, C) = d(c, r)$

being c = center of S
or
c = center of C

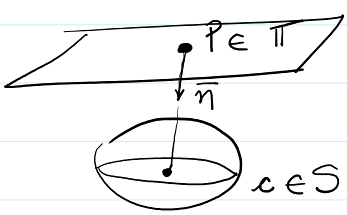




Case $d(s, C) < r$
than $s \cap C = P_1, P_2$

$d(\Pi, S)$

1)

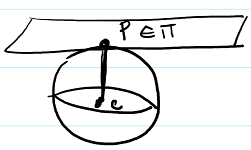


$c \in S$

$d(c, \Pi) > r \Rightarrow$

$\Pi \cap S = \emptyset$

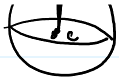
2)



$d(c, \Pi) = r \Rightarrow$

Π is tangent to S

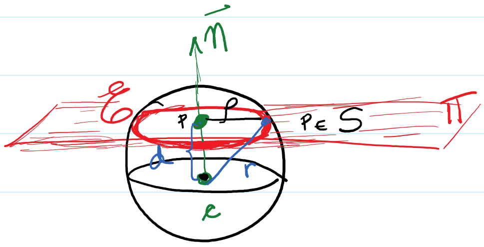
$S \cap \Pi = P$ Tangency



Π is tangent to S

and $S \cap \Pi = P$ = Tangency point.

3)



$\Pi \cap S = \mathcal{E}$ = sphere of center $P \in \Pi$ & radius $\rho < r$

$d = d(\Pi, c)$

r = radius of S

ρ = radius of de Sector $\mathcal{E}$