A 10.4 Exerciser 2, 16 26 28 ⊗ I will do some extra exercises ...

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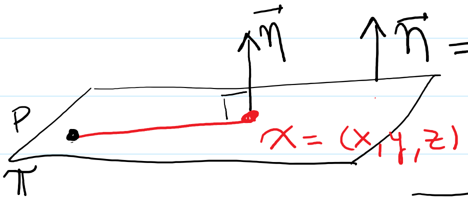
2)- 9) Find equations of the plane satisfying the given conditions

2) $P = (0,2,-3)$ $\Pi \perp \vec{n} = \underbrace{4\vec{i} - \vec{j} - 2k}$.

$\vec{n} = \underbrace{(4, -1, -2)}$



$\vec{PX} \perp \vec{n} \Rightarrow \boxed{\vec{PX} \cdot \vec{n} = 0}$ Plane equation

$\vec{PX} = X - P = (x,y,z) - (0,2,-3) = (x, y-2, z+3) \cdot (4, -1, -2) = 0$

Π: $4x - y + 2 - 2z - 6 = 0$

$4x - y - 2z = 6 - 2$

$\boxed{4x - y - 2z = 4}$

$P = (7, 4, 3)$ $P \in \Pi$? $4(7) - 4 - 2(3) \stackrel{?}{=} 4$ NO!

NO, P does not belong to Π.

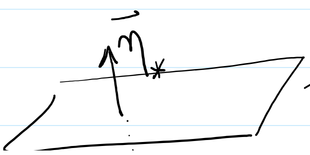
Give another Point $\in \Pi$.

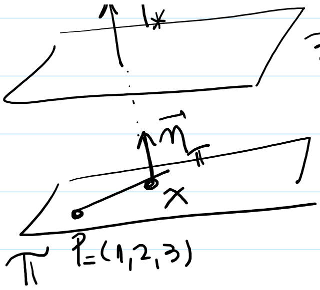
Q satisfies the equation of Π.

$Q = (0, 0, 2)$ $0 - 0 - 2z = 4$ $\boxed{z = -2}$

N°.4) Π passes through $P = (1,2,3)$ &

Π // to the plane $\underbrace{3x + y - 2z = 15}$





$3x + y - 2z = 15$

$\vec{n}_\pi \parallel n_* \Rightarrow \vec{n}_\pi = \vec{n}_*$

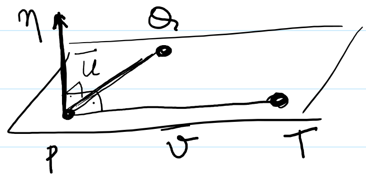
$\bar{n}_\pi = (3, 1, -2)$

$\overline{PX} \perp \bar{n}$

$\boxed{\overline{PX} \cdot \bar{n} = 0}$

Answer: $3x + y - 2z = -1$.

5) $P = (1,1,0) \quad Q = (2,0,2) \quad T = (0, 3, 3)$



$\bar{u} = \overline{PQ}$

$\bar{v} = \overline{PT}$

$\bar{u} \times \bar{v} = \vec{n}$

$\vec{u} = \overline{PQ} = Q - P = (2,0,2) - (1,1,0) = (1,-1,2)$

$\vec{v} = \overline{PT} = T - P = (0,3,3) - (1,1,0) = (-1, 2, 3)$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{vmatrix} = \vec{i}\,(-3-4) - \vec{j}\,(3+2) + \vec{k}\,(2-1)$$

$$= \boxed{\vec{i}\,(-7) - 5\vec{j} + 1\vec{k}}$$

$\bar{n} = (-7, -5, 1)$

$M_{1\times1} = [a_{11}] \qquad |M_{1\times1}| = a_{11}$

$M_{2\times2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad |M_{2\times2}| = a_{11} \cdot a_{22} - a_{12}a_{21}$

$= (-1)^{1+1} a_{11} \, |[a_{22}]| + (-1)^{1+2} a_{12} \, |[a_{21}]|$

$= +a_{11}a_{22} - a_{12}a_{21}$

$\boxed{PX \cdot \tilde{n} = 0}$ with this you obtain the equation.

8) $\Pi = ?$ equation?

&

$P_1 = (-2, 0, -1)$

Π passes through the line of intersection of the planes

$\{ 2x + 3y - z = 0 \qquad \Pi_1$

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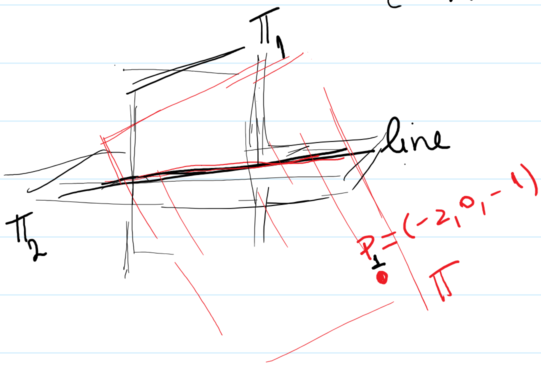
$P_1 = (-2,0,-1)$ of intersection of the planes $\begin{cases} 2x+3y-z=0 & \pi_1 \\ x-4y+2z=-5 & \pi_2 \end{cases}$



1) $\underline{\pi_1 \cap \pi_2}$

$$\begin{array}{r} (1) \\ (-2) \end{array} \left| \begin{array}{l} 2x+3y-z=0 \\ x-4y+2z=-5 \end{array} \right.$$

$$\begin{cases} 2x+3y-z=0 \\ -2x+8y-4z=10 \end{cases}$$

$$11y - 5z = 10$$

$$11y = 10 + 5z$$

$$\boxed{y = \frac{10}{11} + \frac{5}{11}z} \quad (A)$$

Substitute (A) in eq (1) of the system

$$2x + 3\left(\frac{10}{11} + \frac{5}{11}z\right) - z = 0$$

$$2x + \frac{30}{11} + \frac{15}{11}z - z = 0 \Rightarrow 2x = -\frac{4z}{11} - \frac{30}{11}$$

$$\boxed{x = -\frac{2z}{11} - \frac{15}{11}}$$

$$\boxed{y = \frac{5}{11}z + \frac{10}{11}}$$

$$x = -\frac{15}{11} - \frac{2}{11}\cancel{z}$$

$$y = \frac{10}{11} + \frac{5}{11}\cancel{z}$$

$$z = 0 + \cancel{z}, \quad \lambda \in \mathbb{R}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} -15/11 \\ 10/11 \\ 0 \end{pmatrix}}_{P_2 \text{ of the line}} + \lambda \underbrace{\begin{pmatrix} -2/11 \\ 5/11 \\ 1 \end{pmatrix}}_{\vec{v} \text{ of line}}$$

direction vector.

Now

$P_1 \in \pi$

$P_2 = \left(-\frac{15}{11}, \frac{10}{11}, 0\right) \in \pi$

$\vec{v} = \left(-\frac{2}{11}, \frac{5}{11}, 1\right)$ $\quad \vec{v}_* = (-2,5,11) \quad \vec{v}_* \parallel \vec{v} \quad \vec{v}_* = 11\vec{v}$

$\vec{w}_* = (7,10,11)$

$\vec{w} = \overrightarrow{P_1P_2} = P_2 - P_1 = \left(-\frac{15}{11}, \frac{10}{11}, 0\right) - (-2,0,-1) =$

$$\vec{w} = \overline{P_1P_2} = P_2 - P_1 = \left(-\frac{15}{11}, \frac{10}{11}, 0\right) - (-2, 0, -1) = \qquad \vec{w}_* = 11\,\vec{w}$$

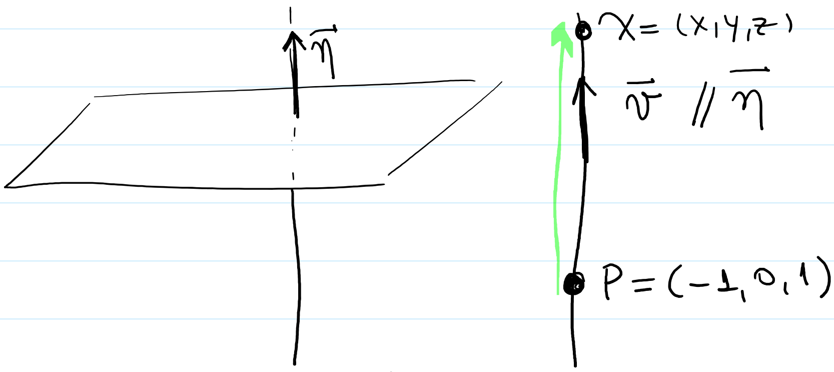
$$= \left(-\frac{15}{11} + 2, \frac{10}{11} - 0, +1\right) = \qquad \vec{w}_* \parallel \vec{w}$$

$$\vec{w} = \left(\frac{-15+22}{11}, \frac{10}{11}, 1\right) = \left(\frac{7}{11}, \frac{10}{11}, 1\right)$$

$$\bar{\eta}_\pi = \vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{7}{11} & \frac{10}{11} & 1 \\ -\frac{2}{11} & \frac{5}{11} & 1 \end{vmatrix}$$

with all these, you can compute π equation...

Ex 16 page 599 : Find the equation of the line (vector, parametric & standard forms) passing through $P = (-1, 0, 1)$ & perpendicular to $\pi: 2x - y + 7z = 12$



$\vec{\eta} = (2, -1, 7)$

1) vector form: $\overrightarrow{PX} = \lambda \vec{v}$, $\lambda \in \mathbb{R}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix} \Rightarrow \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}}_{P} + \lambda \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix}$$

2) Parametric form

$$\begin{cases} x = -1 + \lambda 2 \\ y = 0 + \lambda(-1) \\ z = 1 + \lambda(7) \end{cases} \qquad \begin{cases} x = -1 + 2\lambda \\ y = 0 - 1\lambda \\ z = 1 + 7\lambda \end{cases}$$

3) Standard form:

$$x+1 = 2\lambda \Rightarrow \lambda = \frac{x-(-1)}{2}$$

$$y-0 = -1\lambda \Rightarrow \lambda = \frac{y-(0)}{-1} \qquad \lambda = -y$$

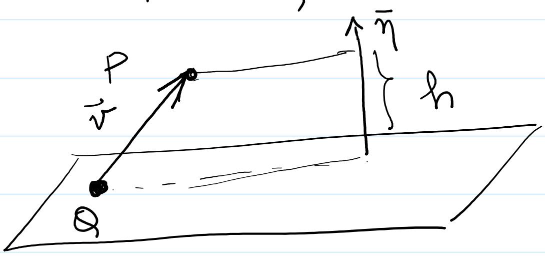
$$z-1 = 7\lambda \Rightarrow \lambda = \frac{z-1}{7}$$

$$\frac{x-(-1)}{2} = \frac{y-0}{(-1)} = \frac{z-1}{7}$$

Ex 26 page 600 Distances :

$P = (0,0,0)$, $\Pi: \ x+2y+3z = 4$ $\qquad P \notin \Pi$

$0+2(0)+3(0) \neq 4$



$$h = \left\| \operatorname{proj}_{\vec{\eta}} \vec{QP} \right\| = \left\| \vec{QP}_{\vec{\eta}} \right\|$$

$$h = \left| \frac{\vec{QP}\cdot\vec{\eta}}{\vec{\eta}\cdot\vec{\eta}} \right| \|\vec{\eta}\| = \frac{|\vec{QP}\cdot\vec{\eta}|}{\|\vec{\eta}\|^{\cancel{2}}} \cdot \cancel{\|\vec{\eta}\|} =$$

$$h = \frac{|\vec{QP}\cdot\vec{\eta}|}{\|\vec{\eta}\|}$$

$Q = (4,0,0)$ $\qquad \vec{QP} = P - Q = (0,0,0) - (4,0,0) = (-4,0,0)$

$(Q = \ \ ,0,0)$

$$|\vec{QP}\cdot\vec{\eta}| = |(-4,0,0)(1,2,3)| = |-4+0+0| = |-4| = 4$$

$\vec{\eta} = (1,2,3)$

$\|\vec{\eta}\| = \sqrt{1^2+4+9} = \sqrt{14}$ $\qquad \boxed{h = \frac{4}{\sqrt{14}}}$

28 $\quad d(P, \text{line})$

$P = (0,0,0)$ $\qquad \text{line} = \Pi_1 \cap \Pi_2$

$\pi_1: \; x + y + z = 0$

$\pi_2: \; 2x - y - 5z = 1$

(1)
$$+\begin{cases} x + y + z = 0 \\ 2x - y - 5z = 1 \end{cases}$$
$$3x \quad \cancel{\;\;} \quad -4z = 1 \implies 3x = 1 + 4z$$

$$\boxed{x = \frac{1}{3} + \frac{4}{3}z} \quad (A)$$

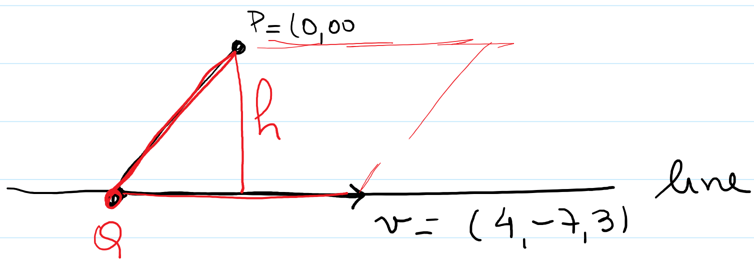
Substitute (A) in eq (1)

$$\left(\frac{1}{3} + \frac{4}{3}z\right) + y + z = 0 \qquad \boxed{y = -\frac{1}{3} - \frac{7}{3}z}$$

$z = \lambda$ free parameter
free variable

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} 1/3 \\ -1/3 \\ 0 \end{pmatrix}}_{} + \lambda \begin{pmatrix} 4/3 \\ -7/3 \\ 1 \end{pmatrix} \qquad \vec{v} = \begin{pmatrix} 4/3 \\ -7/3 \\ 1 \end{pmatrix} \; \underline{\text{or}} \; \vec{v} = \begin{pmatrix} 4 \\ -7 \\ 3 \end{pmatrix}$$

$Q = \underline{\text{Point}} = \left(\frac{1}{3}, -\frac{1}{3}, 0\right)$



Area = ||base|| × $\overset{\text{distance}}{\overline{\text{height}}}$

height = $\dfrac{\text{Area}}{\|\text{base}\|}$

base $= \|\vec{v}\| = \sqrt{(4)^2 + (7)^2 + (3)^2}$

$$\text{Area} = \|\vec{QP} \times \vec{v}\| = \left\| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -7 & 3 \\ \frac{1}{3} & -\frac{1}{3} & 0 \end{vmatrix} \right\|$$

$$\vec{QP} \times \vec{v} = \vec{i}\,(1) - \vec{j}\,(-1) + \vec{k}\left(-\frac{4}{3} + \frac{7}{3}\right) =$$

$$\vec{i}\,(1) + 1\vec{j} + 1\vec{k}$$

$\|\vec{QP} \times \vec{v}\| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$$\text{height} = \frac{\text{Area}}{\|\text{base}\|} = \frac{\sqrt{3}}{\sqrt{4^2+7^2+3^2}}$$

Ex 29: Find the distance between 2 lines

line 1: $\begin{cases} x + 2y = 3 & (1) \\ y + 2z = 3 & (*) \end{cases}$

$\boxed{y = 3 - 2z}$ substitute on eq (1)

$x + 2(3 - 2z) = 3$

$x + 6 - 4z = 3$

$x = -6 + 3 + 4z$

$\boxed{x = -3 + 4z}$

$x = -3 + 4z$

$y = 3 - 2z$

$z = \quad z$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} -3 \\ 3 \\ 0 \end{pmatrix}}_{P_1} + \lambda \underbrace{\begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}}_{\vec{v}}$$

line 2: $\begin{cases} x + y + z = 6 \\ +x - 2z = -5 \end{cases}$

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$y + 3z = 11$

$\boxed{y = 11 - 3z}$

$\boxed{x = -5 + 2z}$

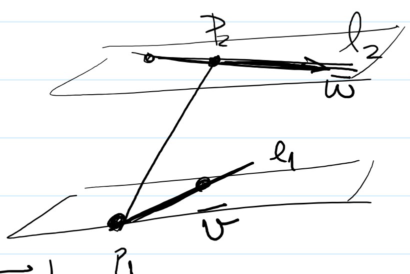
$x = -5 + 2z$

$y = 11 - 3z$

$z = 0 + z$ $\quad$ $z = \lambda \in \mathbb{R}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} -5 \\ 11 \\ 0 \end{pmatrix}}_{P_2} + \lambda \underbrace{\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}}_{\vec{w}}$$

$$P_2 - P_1 = \begin{pmatrix} -5+3 \\ 11-3 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 8 \\ 0 \end{pmatrix}$$



$$\frac{\text{parallelogram Volume}}{\text{Area}} = \text{height}$$

$$\text{Volume} = \frac{|\vec{P_1P_2} \cdot \vec{v} \times \vec{w}|}{\|\vec{v} \times \vec{w}\|}$$

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -2 & 1 \\ 2 & -3 & 1 \end{vmatrix} = \vec{i}(-2+3) - \vec{j}(4-2) + \vec{k}(-12+4)$$

$$= (1, -2, -8) = \vec{v} \times \vec{w}$$

$$\begin{vmatrix} 2 & -3 & 1 \end{vmatrix} = (1, -2, -8) = \bar{v} \times \bar{w}$$

$$\text{Area} = \|\bar{v} \times \bar{w}\| = \sqrt{1^2 + (-2)^2 + (-8)^2} = \sqrt{1+4+64} = \sqrt{69}$$

$$\overrightarrow{P_1P_2} = (-2, 8, 0)$$

$$\overrightarrow{P_1P_2} \cdot (\bar{v} \times \bar{w}) = (-2, 8, 0) \cdot (1, -2, -8) = -2 - 16 + 0 = -18$$

$$|\overrightarrow{P_1P_2} \cdot (\vec{v} \times \vec{w})| = |-18| = \text{volume.}$$

$$d(\text{line}_1, \text{line}_2) = \frac{18}{\sqrt{69}}$$