Föreläsning_8.1
den 19 oktober 2020 07:28

L' Hopital Rule

Theorem 3 - 1st L' Hopital Rule page 231

Suppose f & g diff on (a,b)

$g'(x) \neq 0$ on (a,b)

$$\lim_{x\to a_+} f(x) = 0 = \lim_{x\to a_+} g(x)$$

$$\lim_{x\to a_+} \frac{f'(x)}{g'(x)} = L \quad \begin{cases} |L| < \infty \\ L = +\infty \\ L = -\infty \end{cases}$$

$\left[\frac{0}{0}\right]$??? L'Hopital

Then $$\lim_{x\to a_+} \frac{f(x)}{g(x)} = L$$

Examples:

$$\lim_{x\to 1_+} \frac{x}{\ln x} = \frac{1}{0_+} = +\infty$$

$$\frac{x}{0} \to \frac{x}{0_+}, \quad \frac{x}{0} \to \frac{x}{0_-}$$

Here : No L'Hopital !

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(d) $\lim_{x\to 0_+} x^a \ln x = 0$

Example 2.

$$\lim_{x\to (\frac{\pi}{2})_-} \frac{2x-\pi}{\cos^2 x} = \underbrace{\left[\frac{2(\frac{\pi}{2}) - \pi}{\cos^2(\frac{\pi}{2})}\right] = \left[\frac{0}{0}\right]}_{\text{indetermination } \left[\frac{0}{0}\right]}$$

L'Hopital Rule is a good strategy.

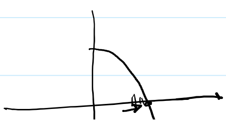
$$f(x) = 2x - \pi \to f'(x) = 2$$

$$g(x) = \cos^2 x \to g'(x) = 2\cos(x)\cdot(\underbrace{-\sin x}) = -2\cos x \cdot \sin x$$

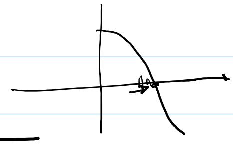
$$\lim_{x\to(\frac{\pi}{2})_-} \frac{f'(x)}{g'(x)} = \frac{\cancel{2}}{-\cancel{2}\underbrace{\cos(\frac{\pi}{2})_-}_{+}\underbrace{\sin(\frac{\pi}{2})}_{1}} = \frac{-1}{\cos(\frac{\pi}{2})} = \left[\frac{-1}{0}\right] = -\infty$$

By the L'Hopital Rule

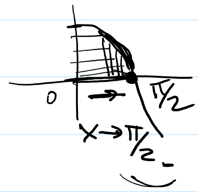
$$\lim_{x\to(\frac{\pi}{2})_-} \frac{f(x)}{g(x)} = \lim_{x\to(\frac{\pi}{2})_-} \frac{f'(x)}{g'(x)} = -\infty$$



$x \to (\pi/2)_- \quad \overline{g(x)} \qquad x \to (\pi/2)_- \quad \overline{g'(x)}$



page $\underline{233}$ The 2^{nd} L'Hopital Rule



Suppose

f & g diff on (a,b)

$g'(x) \neq 0$ on (a,b)

i) $\lim_{x \to a_+} g(x) = \pm\infty$

ii) $\lim_{x \to a_+} f'(x)/g'(x) = L$ $\quad \begin{cases} |L| < \infty \\ L = +\infty \\ L = -\infty \end{cases}$

Then $\lim_{x \to a_+} \frac{f(x)}{g(x)} = L$

Obs. $\left[\frac{a}{\infty}\right]$ this is not an indetermination if $a \in \mathbb{R}$

$\frac{a>0}{+\infty} > 0_+ = 0$

you can solve this without L'Hopital

Obs $\left[\infty/\infty\right]$ 2^{nd} L'Hopital Rule is a good option.

Ex $\lim_{x \to +\infty} \frac{x^2}{e^x} = \left[\frac{+\infty}{+\infty}\right]$??? $\quad$ 2^{nd} L'Hopital Rule

$f(x) = x^2 \longrightarrow f'(x) = 2x$

$g(x) = e^x \qquad\quad g'(x) = e^x$

$\lim_{x \to +\infty} \frac{2x}{e^x} = \left[\frac{\infty}{\infty}\right]$?? 2^{nd} L'Hopital Rule

$f''(x) = 2$

$g''(x) = e^x$

$$\lim_{x \to +\infty} \frac{f''(x)}{g''(x)} = \lim_{x \to \infty} \frac{2}{e^x} = \frac{+2}{+\infty} \longrightarrow 0_+$$

$$\lim_{x \to +\infty} \frac{f(x)}{g(x)} = \lim_{x \to +\infty} \frac{f'(x)}{g'(x)} = \lim_{x \to +\infty} \frac{f''(x)}{g''(x)} = 0_+$$

$$f(x) = \frac{x^2 - 4}{x^2 - 1}$$

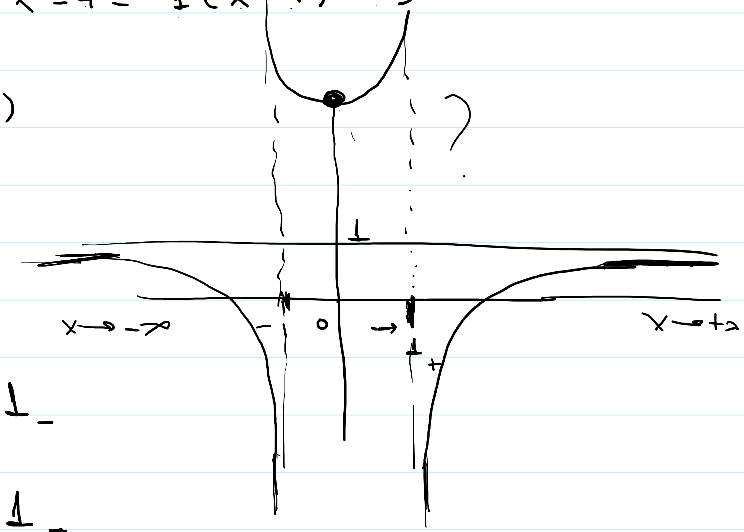
$$\begin{array}{l|l} x^2 - 4 & x^2 - 1 \\ \hline -(x^2-1) & 1 \\ \hline -3 & \end{array}$$

$$f(0) = \frac{0^2 - 4}{0 - 1} = +4$$

$$x^2 - 4 = 1\,(x^2 - 1) - 3$$

$$\frac{x^2 - 4}{x^2 - 1} = 1 \, \frac{\cancel{(x^2 - 1)}}{\cancel{(x^2 - 1)}} - \frac{3}{(x^2 - 1)}$$

$$\frac{x^2 - 4}{x^2 - 1} = 1 - \frac{3}{(x^2 - 1)}$$



$$\lim_{x \to +\infty} \frac{x^2 - 4}{x^2 - 1} = 1 - (0_+) = 1_-$$

$$\lim_{x \to -\infty} \frac{x^2 - 4}{x^2 - 1} = 1 - (0_+) = 1_-$$

$$f(x) = \frac{x^2 - 4}{x^2 - 1} \qquad x^2 - 1 \neq 0 \quad ??$$

$$x^2 - 1 = 0 \implies \boxed{x = \pm 1}$$

$$D_f = \mathbb{R} \setminus \{1, -1\}$$

$$\lim_{x \to 1_+} f(x) = \lim_{x \to 1_+} 1 - \overset{+\infty}{\frac{3}{\underbrace{x^2 - 1}_{1_+ - 1 \;\; 0_+}}} = -\infty$$