Repetitionsmöte

1) Tentan 24:e Augusti 2018

N°. 4 Beräkna Taylor polynomet för $f(x) = e^{x^2}$ av ordning 4 kring $\boxed{a=0}$ med felterm på O form:

x^2 exact

x^2

$\boxed{t=1}$ 7

i) $g(t) = e^{t}$, $t = x^2$ $\quad (t-a)^n$

e^t

$(t-0)^3$ $\quad t = x^2$

$$g(t) = g(0) + g'(0).t + g''(0)\frac{t^2}{2} + O(t^3)$$

$$\boxed{g(t) = 1 + 1t + \frac{1}{2}t^2 + \underbrace{O(t^3)}_{\text{felterm på O form}}}$$

Now we substitut $t = x^2$,

$\boxed{x=1}$

$$g(x^2) = 1 + 1(x^2) + \frac{1}{2}(x^2)^2 + O((x^2)^3)$$

$$\boxed{e^{x^2} = g(x^2) = 1 + x^2 + \frac{1}{2}x^4 + \underbrace{O(x^6)}_{\text{felterm på O form}} \quad x \text{ kring } 0}$$

Another approach

$$f(x) = e^{x^2} \longrightarrow f(0) = e^0 = 1$$

$$f'(x) = e^{x^2}.2x \longrightarrow f'(0) = e^0.2(0) = 1.2.(0) = 0$$

$$f''(x) = (e^{x^2} 2x)(2x) + e^{x^2}.2$$

$$f''(0) = 1.4.0^2 + 2.(e^0) = 2 \neq 0.$$

$$f'''(x) =$$

2) Tentan 21:e december 2016

N°. 4. Betrakta funktionen $f(x) = \ln(\cos(x))$

Ange dess Taylor polynom av grad 4 runt $\boxed{a=0.}$

$$P_4(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \frac{f^{IV}(a)}{4!}(x-a)^4$$

$+O(x^5)$

$a=0$

$$\boxed{f(0)=\ln(\cos(0))=\ln(1)=0}$$

$$\boxed{f'(x)=\frac{d}{dx}\ln(\cos(x))=\frac{1}{\cos x}\cdot(-\sin x)=-\tan x\Big|_{x=0}=0}$$

$$f''(x)=\frac{d^2}{dx^2}\ln(\cos(x))=\frac{d}{dx}\left(-\tan x\right)=-\frac{1}{\cos^2 x}\Big|_{x=0}=-\frac{1}{2!}\cdot\quad(x-0)^2$$

$$\boxed{f'''(x)=\frac{d}{dx}\left(-\frac{1}{\cos^2 x}\right)=-\frac{2}{\cos^3 x}\cdot\sin x\Big|_{x=0}=0}$$

(check the sign)

$$f^{IV}(x)=\frac{d}{dx}\left(-\frac{2}{\cos^3 x}\cdot\sin x\right)=-2\frac{d}{dx}\left(\frac{\sin x}{\cos^3 x}\right)\underset{\circledast}{=}-2(1)=\frac{-2}{4!}\quad(x-0)^4$$

$$\left(\frac{f}{g}\right)'=\frac{g(x)\cdot f'(x)-f(x)\cdot g'(x)}{(g(x))^2}=$$

$$\frac{\cos^3 x\cdot\cos x+\sin x\,(3\cos^2 x)\cdot(+\sin x)}{\cos^3 x\cdot\cos^3 x}=$$

$$\frac{\cos^4 x+3\sin^2 x\cos^2 x}{\cos^3 x\cdot\cos^3 x}=\frac{\cancel{\cos^2 x}\left(\cos^2 x+3\sin^2 x\right)}{\cancel{\cos^2 x}\cdot\cos^4 x}=$$

$$=\frac{\cos^2 x+3\sin^2 x}{\cos^4 x}\Big|_{x=0}=\frac{1+0}{1}=1$$

So now we can obtain $P_4(x)$ - Taylor polynomial

$$P_4(x)=f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\frac{f^{IV}(a)}{4!}(x-a)^4$$

$$f(x)=P_4(x)+O((x-a)^5)\longleftarrow$$

$$E_5(x)=\underbrace{\frac{f^{(V)}(s)}{5!}}(x-a)^5,\quad s\in(x,a)$$

$$P_4(x)=-\frac{1}{2}x^2-2\frac{1}{4!}x^4=-\frac{1}{2}x^2-\frac{1}{12}x^4$$

4.3.2

$$f(x)=P_4(x)+\underbrace{O(x^5)}$$

$$E_5(x)=\frac{f^{IV}(s)}{5!}(x-a)^5\qquad s\in(x,a)$$

$$f = L(x) + E_2(x) \Rightarrow f - L(x) = E_2 < 0 \Rightarrow$$

$$f < L(x)$$

$$f - L(x) = E_2 > 0$$

$$f > L_2$$

$$E_3(x) = \frac{f'''(\xi)}{3!}\underbrace{(x-a)^3}$$

$$E_5(x) = \frac{f^{V}(\xi)}{5!}(x-a)^5$$

$$f(x) = \underbrace{P_2(x)} + E_3(x)$$

$$f(x) - P_2(x) = E_3(x) < 0 \Rightarrow f(x) < P_2(x) \Rightarrow (P_2 - \epsilon, P_2)$$

$$f(x) - P_2(x) = E_3(x) > 0 \Rightarrow f(x) > P_2(x) \Rightarrow (P_2, P_2 + \epsilon)$$

$$f(x) - P_4(x) = E_5(x)$$

$$f = \frac{1}{\underbrace{x-5}}$$

$$\boxed{D_f = \mathbb{R}}$$

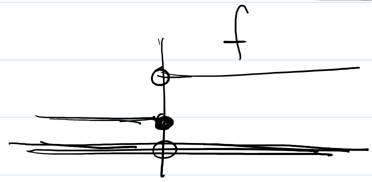
$$D_f = \mathbb{R} \setminus \{5\} = (-\infty, 5) \cup (5, +\infty)$$

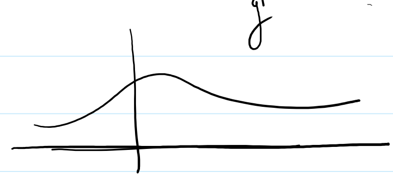
$$D_f = (-\infty, 2) \cup (7, +\infty)$$

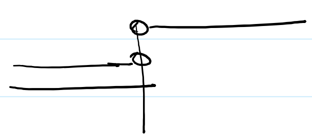
$]a, b[$

$[a, b]$

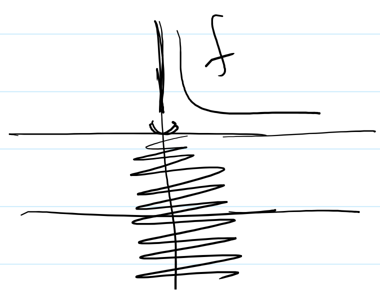
V_f



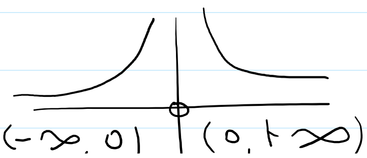




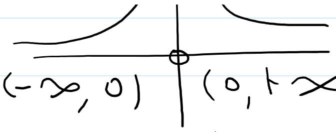
$\mathbb{R}_+ \setminus \{0\}$



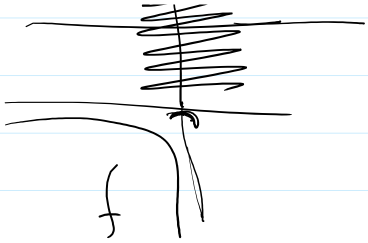
$V_f =$



$V_f =$



$\cup$ D_f

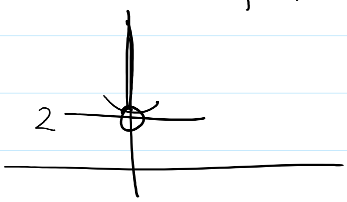


$\underbrace{\mathbb{R}_+ \setminus \{0\}}$ $= \underbrace{(0, \infty)}$

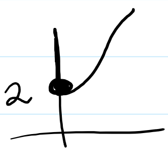
$\underbrace{\mathbb{R} \setminus \{2, 3, 5\}}$ $(-\infty, 2) \cup (2,3) \cup (3,5) \cup (5, +\infty)$

2 3 5

$f''(x)$ $\underline{f' \geq 0}$ $\underline{f' > 0}$ $f' \geq 0$ $\underline{f' = 0}$



$V_f = \underline{\underline{(2}}, +\infty)$



$V_f = [2, +\infty)$