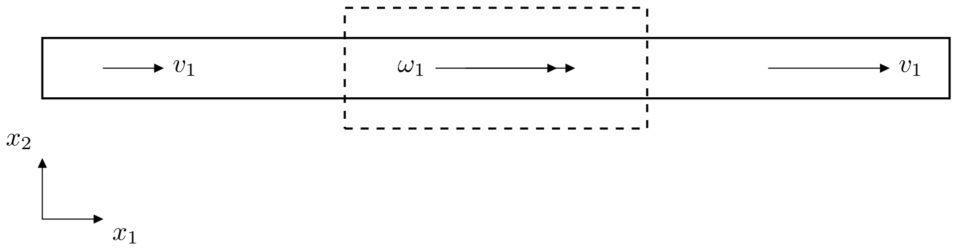

Figure 4.2: Vortex stretching. Dashed lines denote fluid element before stretching. $\dfrac{\partial v_1}{\partial x_1} > 0$.

We recognize the usual unsteady term, the convective term and the diffusive term. Furthermore, we have got rid of the pressure gradient term. That makes sense, because as mentioned in connection to Fig. 4.1, the pressure cannot affect the rotation (i.e. the vorticity) of a fluid particle since the pressure acts through its center. Equation 4.21 has a new term on the right-hand side which represents amplification and bending or tilting of the vorticity lines. If we write it term-by-term it reads

$$\omega_k \frac{\partial v_p}{\partial x_k} = \begin{cases} \omega_1 \dfrac{\partial v_1}{\partial x_1} & +\omega_2 \dfrac{\partial v_1}{\partial x_2} + \omega_3 \dfrac{\partial v_1}{\partial x_3}, \quad p = 1 \\ \omega_1 \dfrac{\partial v_2}{\partial x_1} & +\omega_2 \dfrac{\partial v_2}{\partial x_2} + \omega_3 \dfrac{\partial v_2}{\partial x_3}, \quad p = 2 \\ \omega_1 \dfrac{\partial v_3}{\partial x_1} & +\omega_2 \dfrac{\partial v_3}{\partial x_2} + \omega_3 \dfrac{\partial v_3}{\partial x_3}, \quad p = 3 \end{cases} \tag{4.22}$$

Vortex stretching

The diagonal terms in this matrix represent *vortex stretching*. Imagine a slender, cylindrical fluid particle with vorticity ω_i and introduce a cylindrical coordinate system with the x_1-axis as the cylinder axis and r_2 as the radial coordinate (see Fig. 4.2) so that $\omega_i = (\omega_1, 0, 0)$. We assume that a positive $\partial v_1/\partial x_1$ is acting on the fluid cylinder; it will act as a source in Eq. 4.21 increasing ω_1 and it will stretch the cylinder. The volume of the fluid element must stay constant during the stretching (the incompressible continuity equation), which means that the radius, r, of the cylinder will decrease. For high *Reynolds numbers*, the viscous term is neglible. Hence, the viscous forces on the surface is small. This means than the angular momentum, $r^2\omega_1$, is constant during the elongation (stretching) of the cylinder which gives an increased ω_1. We see that vortex stretching will either make a fluid element longer and thinner with larger ω_1 (as in the example above) or shorter and thicker (when $\partial v_1/\partial x_1 < 0$). The illustratation given here is mainly relevant when a fluid particle actually rotates (as it does in turbulent flow, see Section 5).

Re number= ratio of convective to viscous term

Vortex tilting

The off-diagonal terms in Eq. 4.22 represent *vortex tilting*. Again, take a slender fluid particle, but this time with its axis aligned with the x_2 axis, see Fig. 4.3. Assume is has a vorticity, ω_2, and that the velocity surrounding velocity field is $v_1 = v_1(x_2)$. The velocity gradient $\partial v_1/\partial x_2$ will tilt the fluid particle so that it rotates in clock-wise direction. The second term $\omega_2 \partial v_1/\partial x_2$ in line one in Eq. 4.22 gives a contribution to ω_1. This means that vorticity in the x_2 direction, through the source term $\omega_2 \partial v_1/\partial x_2$, creates vorticity in the x_1 direction..

Vortex stretching and tilting are physical phenomena which act in three dimensions: fluid which initially is two dimensional becomes quickly three dimensional through these phenomena. Vorticity is useful when explaining why turbulence must be three-