6.13) Beräkna ekv. för tangent och normal till kurvan

$$y = f(x) = \frac{x^3+1}{x^2+1} \quad \text{för } x=-1$$

$$T: \; y - f(a) = f'(a)(x-a)$$

$$N: \; y - f(a) = -\frac{1}{f'(a)}(x-a)$$

$(i \; x=a)$

Vi behöver $a=-1$, $f(a) = f(-1) = \frac{(-1)^3+1}{(-1)^2+1} = \frac{0}{2} = 0$

$$f'(a) = \frac{3}{2}$$

$$f(x) = \frac{g_1(x)}{g_2(x)}$$

$$g_1(x) = x^3+1 \Rightarrow g_1'(x) = 3x^2$$

$$g_2(x) = x^2+1 \Rightarrow g_2'(x) = 2x$$

$$f'(x) = \frac{g_1'(x)\,g_2(x) - g_1(x)\,g_2'(x)}{(g_2(x))^2} = \frac{3x^2(x^2+1) - (x^3+1)2x}{(x^2+1)^2} = \frac{3x^4+3x^2-2x^4-2x}{(x^2+1)^2} = \frac{x^4+3x^2-2x}{(x^2+1)^2}\bigg|_{x=-1} = \frac{\overset{1}{(-1)^4} + \overset{3}{3(-1)^2} + \overset{2}{2}}{\underset{4}{(1+1)^2}} = \frac{6}{4} = \frac{3}{2}$$

Vi stoppar in allt:

$$T: \; y - 0 = \frac{3}{2}(x+1) \Rightarrow y_T = \frac{3}{2}x + \frac{3}{2}$$

$$N: \; y - 0 = -\frac{2}{3}(x+1) \Rightarrow y_N = -\frac{2}{3}x - \frac{2}{3}$$

6.16) f) Derivera $y(r) = (2r-1)\sqrt{r+5}$

$y(x) = e^{\sin(x^2 - \tan(x))} * (x^2+1)^{100} * \sqrt{x}$

$f_1(r) = 2r - 1 \Rightarrow f_1'(r) = 2$

$f_2(r) = \sqrt{r} \Rightarrow f_2'(r) = \frac{1}{2\sqrt{r}}$

$f_3(r) = r+5 \Rightarrow f_3'(r) = 1$

$y(r) = f_1(r) * f_2(f_3(r))$

$r+5$

$\uparrow$

$f_2(f_3(r)) = f_2(r+5) = \sqrt{r+5}$

$f_2(r) = \sqrt{r}$

3 regler: kedjeregeln, produktregeln, kvotregeln

$$y'(r) = f_1'(r) * f_2(f_3(r)) + f_1(r) * \underbrace{(f_2(f_3(r)))'}_{f_2'(f_3(r)) * f_3'(r)} = f_1'(r)\, f_2(f_3(r)) + f_1(r)\, f_2'(f_3(r))\, f_3'(r) =$$

(kedjeregeln)

$$= 2 * \sqrt{r+5} + (2r-1)\frac{1}{2\sqrt{r+5}} * 1$$

6.15 a,b Derivera

a) $g(x) = (5x+3)^{100}$

b) $h(t) = (4-t)^{\frac{3}{2}}$

Kedjeregeln

$$\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$$

$$g(x) = (5x+3)^{100} = g_1(g_2(x)) \quad , \quad g_1(x) = x^{100} \quad , \quad g_2(x) = 5x+3$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$g_1'(x) = 100x^{99} \qquad g_2'(x) = 5$$

$$g'(x) = g_1'(g_2(x))\, g_2'(x) = 100(5x+3)^{99} \cdot 5 = 500(5x+3)^{99}$$

$$h(t) = (4-t)^{\frac{3}{2}} = h_1(h_2(t)) \quad , \quad h_1(t) = t^{\frac{3}{2}} \quad , \quad h_2(t) = 4-t$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$h_1'(t) = \frac{3}{2} t^{\frac{1}{2}} = \frac{3}{2}\sqrt{t} \qquad h_2'(t) = -1$$

$$h'(t) = h_1'(h_2(t)) \cdot h_2'(t) = \frac{3}{2}\sqrt{4-t} \cdot (-1) = -\frac{3}{2}\sqrt{4-t}$$

6.21 f) $y(x) = x(x^4 - x + 1)^{10} = f_1(x)\, f_2(f_3(x))$

produktregeln

derivera med kedjeregeln

$f_1(x) = x$

$f_2(x) = x^{10}$

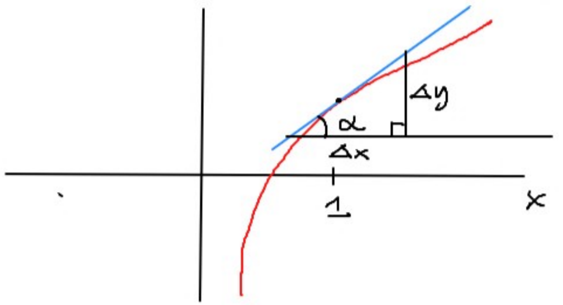
$f_3(x) = x^4 - x + 1$

$$\frac{d}{dx}\left(f(x) g(x) \right) = f'(x) g(x) + f(x) g'(x)$$

G.12 Beräkna lutningsvinkeln för tangenten till $y = \frac{x^2-x-1}{2x-1} = \frac{g_1(x)}{g_2(x)}$

$$\begin{cases} g_1'(x) = 2x-1 \\ g_2'(x) = 2 \end{cases}$$

i $x=1$



$f(x) = kx+m \Rightarrow f'(x) = k = \frac{\Delta y}{\Delta x}$

$$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

$$\tan\alpha = \frac{\Delta y}{\Delta x} = k$$

$$\alpha = \arctan(k)$$

Vill ha $y'(1)$

$$y'(x) = \frac{g_1'(x)g_2(x) - g_1(x)g_2'(x)}{(g_2(x))^2} = \frac{(2x-1)(2x-1)-(x^2-x-1)\cdot 2}{(2x-1)^2} = 1 - 2\left.\frac{(x^2-x-1)}{(2x-1)^2}\right|_{x=1} =$$

$$= 1 - 2\frac{(1-1-1)}{(2-1)^2} = 1 - 2\cdot\frac{(-1)}{1} = 3 \Rightarrow \alpha = \arctan(3)$$

Derivera

6.21) f) $y(x) = x(x^4-x+1)^{10} = f_1(x)\, f_2(f_3(x))$

$f_1(x) = x \quad \Rightarrow \quad f_1'(x) = 1$

$f_2(x) = x^{10} \quad \Rightarrow \quad f_2'(x) = 10x^9$

$f_3(x) = x^4-x+1 \quad \Rightarrow \quad f_3'(x) = 4x^3-1$

$y'(x) = f_1'(x)\, f_2(f_3(x)) + f_1(x)\,(f_2(f_3(x)))' = f_1'(x)\, f_2(f_3(x)) + f_1(x)\, f_2'(f_3(x))\, f_3'(x) = *$

$(f_2(f_3(x)))' = f_2'(f_3(x)) \cdot f_3'(x)$

$* = 1\,(x^4-x+1)^{10} + x \cdot 10\,(x^4-x+1)^9 \cdot (4x^3-1) = (x^4-x+1)^{10}\left(1 + \frac{10(4x^3-1)}{x^4-x+1}\right)$