Laplacetransformer

2. f, h, j 4. d, e (Lecture notes)

Formler: $\mathcal{L}(f) = F$

$\mathcal{L}(tf) = -F'(s)$

$\mathcal{L}(e^{ct}f) = F(s-c)$

$\mathcal{L}(\cos(t)) = \dfrac{s}{s^2+1}$

$\mathcal{L}(\sin(bt)) = \dfrac{b}{s^2+b^2}$

$\mathcal{L}(\tfrac{1}{t}f) = \int_s^\infty F(w)\,dw$

$\mathcal{L}(1) = \int_0^\infty e^{-st}dt = \left[-\frac{1}{s}e^{-st}\right]_0^\infty = \frac{1}{s}$

2.f) $\mathcal{L}(te^{-t}\cos(t)) = -\left(\mathcal{L}(e^{-t}\cos(t))\right)' = -\left(\dfrac{s+1}{(s+1)^2+1}\right)' =$

$$= -\frac{1\cdot(\overbrace{s^2+2s+2}^{(s+1)^2+1}) - (\overbrace{2s+2}^{2(s+1)})(s+1)}{(s^2+2s+2)^2} = \frac{s^2+2s}{(s^2+2s+2)^2}$$

h) $\mathcal{L}(t\sin\tfrac{t}{2}) = -\left(\mathcal{L}(\sin\tfrac{t}{2})\right)' = -\left(\dfrac{\frac{1}{2}}{s^2+\frac{1}{4}}\right)' = -\tfrac{1}{2}\left((s^2+\tfrac{1}{4})^{-1}\right)' =$

$$= \frac{1}{2}\frac{2s}{(s^2+\frac{1}{4})^2} = \frac{16s}{(4s^2+1)^2}$$

j) $\mathcal{L}(\tfrac{1}{t}(1-\cos(t))) = \int_s^\infty \mathcal{L}(1-\cos(t))(w)\,dw =$

$$= \int_s^\infty \frac{1}{w} - \frac{w}{w^2+1}\,dw = \left[\log(w) - \underbrace{\tfrac{1}{2}\log(w^2+1)}_{\log(\sqrt{w^2+1})}\right]_s^\infty = \left[\log\left(\frac{w}{\sqrt{w^2+1}}\right)\right]_s^\infty =$$

$$= 0 - \log\left(\frac{s}{\sqrt{s^2+1}}\right) = \log\left(\frac{\sqrt{s^2+1}}{s}\right)$$

$$= \log\left(\sqrt{\frac{s^2+1}{s^2}}\right) = \frac{1}{2}\log\left(1+\frac{1}{s^2}\right)$$

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$$\mathcal{L}(e^{ct}) = \frac{1}{s-c}$$

$$\mathcal{L}\left(\int_0^t f(\tau)\,d\tau\right) = \frac{1}{s}F(s)$$

4.d) $F(s) = \dfrac{s+1}{s^3+s^2-6s} = \dfrac{1}{s}\,\dfrac{s+1}{s^2+s-6}$

$$\frac{s+1}{s^2+s-6} = \frac{s+1}{(s+3)(s-2)} = \frac{A}{s+3} + \frac{B}{s-2} = \frac{s(A+B)+3B-2A}{(s+3)(s-2)} = (*)$$

$$\Rightarrow \begin{cases} A+B=1 \\ 3B-2A=1 \end{cases} \Rightarrow \begin{cases} A=\frac{2}{5} \\ B=\frac{3}{5} \end{cases}$$

$$\Rightarrow (*) = \frac{\frac{2}{5}}{s+3} + \frac{\frac{3}{5}}{s-2} \Rightarrow \frac{2}{5}e^{-3t} + \frac{3}{5}e^{2t}$$

$$f(t) = \int_0^t \frac{2}{5}e^{-3\tau} + \frac{3}{5}e^{2\tau}\,d\tau = \left[-\frac{2}{15}e^{-3\tau} + \frac{3}{10}e^{2\tau}\right]_0^t = \frac{3}{10}(e^{2t}-1) - \frac{2}{15}(e^{-3t}-1)$$

e) $F(s) = \dfrac{3s}{s^2+2s-8} = \dfrac{3s}{(s-2)(s+4)} = \dfrac{A}{s-2} + \dfrac{B}{s+4} = \dfrac{s(A+B)-2B+4A}{(s-2)(s+4)}$

$$\begin{cases} A+B=3 \\ -2B+4A=0 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=2 \end{cases} \Rightarrow \frac{1}{s-2} + \frac{2}{s+4}$$

$$f(t) = e^{2t} + 2e^{-4t}$$