REFLECTIONS ON THE FATE OF SPACETIME

String theory carries the seeds of a basic change in our ideas about spacetime and in other fundamental notions of physics.

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Our basic ideas about physics went through several upheavals early this century. Quantum mechanics taught us that the classical notions of the position and velocity of a particle were only approximations of the truth. With general relativity, spacetime became a dynamical variable, curving in response to mass and energy. Contemporary developments in theoretical physics suggest that another revolution may be in progress, through which a new source of "fuzziness" may enter physics, and spacetime itself may be reinterpreted as an approximate, derived concept. (See figure 1.) In this article I survey some of these developments.

Let us begin our excursion by reviewing a few facts about ordinary quantum field theory. Much of what we know about field theory comes from perturbation theory; perturbation theory can be described by means of Feynman diagrams, or graphs, which are used to calculate scattering amplitudes. Textbooks give efficient algorithms for evaluating the amplitude derived from a diagram. But let us think about a Feynman diagram intuitively, as Feynman did, as representing a history of a spacetime process in which particles interact by the branching and rejoining of their world-lines. For instance, figure 2 shows two incident particles, coming in at a and b, and two outgoing particles, at c and d. These particles branch and rejoin at spacetime events labeled x, y, z and w in the figure.

According to Feynman, to calculate a scattering amplitude, one sums over all possible arrangements of particles branching and rejoining. Moreover, for a particle traveling between two spacetime events x and y, one must in quantum mechanics allow for all possible classical trajectories, as in figure 3. To evaluate the propagator of a particle from x to y, one integrates over all possible paths between x and y, using a weight factor derived from the classical action for the path.

So when one sees a Feynman diagram such as that of figure 2, one should contemplate a sum over all physical processes that the diagram could describe. One must integrate over all spacetime events at which interactions—branching and rejoining of particles—could have occurred, and integrate over the trajectories followed by the particles between the various vertices. And, of course, to actually predict the outcome of an experiment, one must (as in figure 4) sum over all possible Feynman diagrams—that is, all possible sequences of interactions by which a given initial state can evolve into a given final state.

This beautiful recipe—formulated in the early days of quantum field theory—brought marvelous success and efficient, precise computations. Yet this recipe also exhibits certain of the present-day troubles in physics. One important property of a Feynman graph is that the graph itself, regarded as a one-dimensional manifold, is singular; that is, at the branching and joining points, the graph does not look like a true one-dimensional manifold. Everyone can agree, in figure 2 for instance, that x, y, z and w were the spacetime events at which interactions occurred. Two central difficulties spring directly from this:

Infinities. Quantum field theory is plagued with infinities, starting with the infinite electrostatic self-energy of the electron. The infinities come from the singularities of the Feynman diagrams. For instance, in figure 2, the potential infinities come from the part of the integration region where the spacetime events x, y, z and w all nearly coincide. Sometimes the infinities can be "renormalized" away; that is the case for electrodynamics and for the weak and strong interactions in the Standard Model of elementary-particle physics. But for gravity, renormalization theory fails, because of the nature of the inherent nonlinearities in general relativity. So we come to a key puzzle: The existence of gravity clashes with our description of the rest of physics by quantum fields.

Too Many Theories. There are many quantum field theories, depending on many free parameters, because one can introduce fairly arbitrary rules governing the branching and joining of particles. For instance, one could permit higher-order branchings of particles, as in figure 5. With every elementary branching process, one can (with certain restrictions) associate a "coupling constant," an extra factor included in the evaluation of a Feynman diagram. In practice, the Standard Model describes the equations that underlie almost all the phenomena we know, in a framework that is compelling and highly predictive—but that also has (depending on precisely how one counts) roughly seventeen free parameters whose values are not understood theoretically. The seventeen parameters enter as special factors associated with the singularities of the Feynman diagrams. There must be some way to reduce this ambiguity!

String theory

We have one real candidate for changing the rules; this is string theory. In string theory the one-dimensional trajectory of a particle in spacetime is replaced by a two-dimensional orbit of a string. (See figure 6.) Such strings can be of any size, but under ordinary circum-

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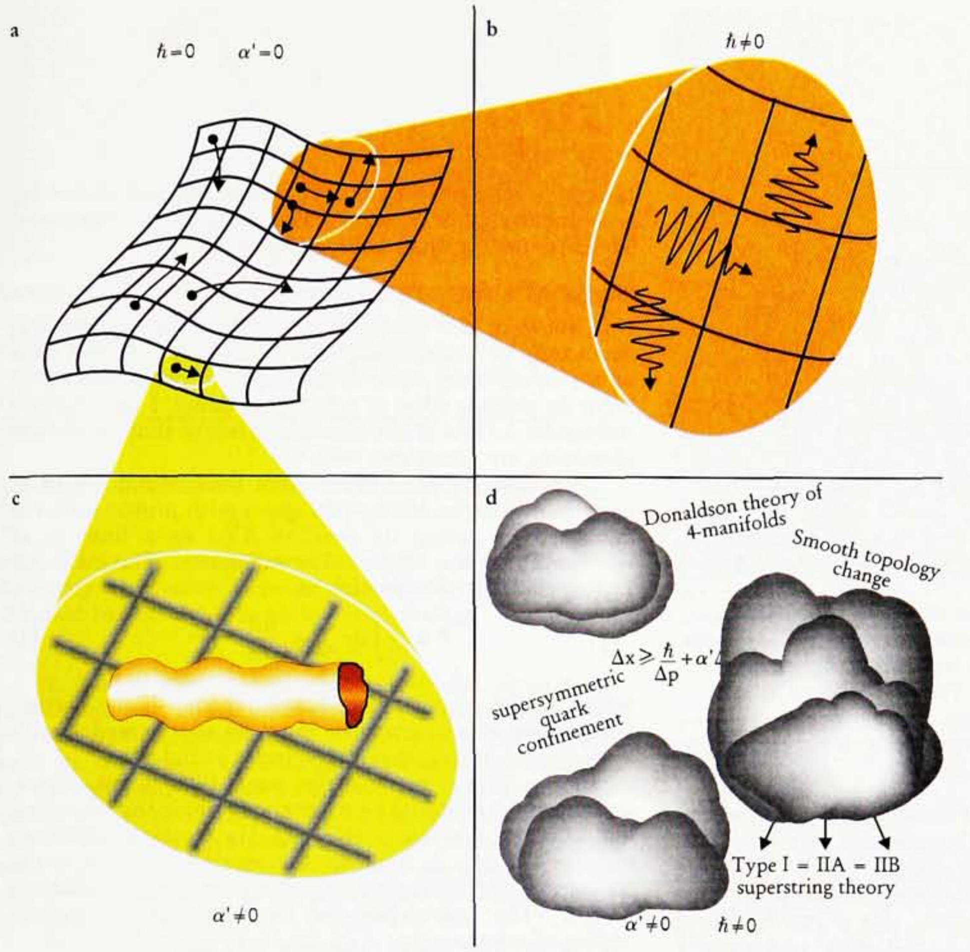


FOUR VIEWS OF REALITY. **a:** In classical physics, particles have definite locations and follow exact trajectories in a precise, curved spacetime. **b:** Closer examination reveals the effects of quantum mechanics, $\hbar \neq 0$. Wavepackets propagate through spacetime, their positions and velocities uncertain according to Heisenberg. **c:** In string theory, point particles are replaced by tiny loops having a "string tension" $\alpha' \neq 0$. Even ignoring quantum mechanics ($\hbar = 0$), the concept of spacetime becomes "fuzzy" at scales comparable to $\sqrt{\alpha'}$. **d:** The full theory, employing both a string tension and quantum effects, is only beginning to take shape. Remarkable results are being uncovered that may overturn our conventional notions of spacetime. FIGURE 1

stances they are quite tiny, around 10^{-32} cm in diameter, a value deduced by comparing the predictions of the theory for Newton's constant and the fine structure constant to the experimental values. This is so small (about sixteen orders of magnitude less than the distances directly probed by high-energy experiments) that for many purposes the replacement of particles by strings is not very important; for other purposes, though, it changes everything. The situation is somewhat analogous to the introduction of Planck's constant $\hbar$ in passing from classical to quantum physics: For many purposes, $\hbar$ is so tiny as to be unimportant, but for many other purposes it is crucial. Likewise, in string theory one introduces a new fundamental constant $\alpha' \approx (10^{-32}\ \mathrm{cm})^2$ controlling the tension of the string. Many things then change.

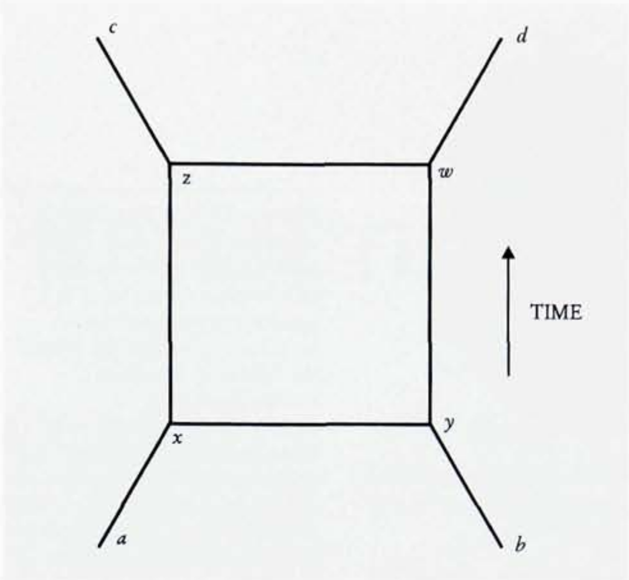
One consequence of replacing world-lines of particles by world-tubes of strings is that Feynman diagrams get smoothed out. World-lines join abruptly at interaction events, as in figure 7a, but world-tubes join smoothly, as in figure 7b. There is no longer an invariant notion of when and where interactions occur, so from the description above of the origin of the problems of field theory, we might optimistically hope to have finiteness, and only a few theories.

These hopes are realized. In fact, once one replaces world-lines with world-tubes, it is all but impossible to construct any consistent theories at all. That such theories do exist was established through a long and complex process stretching over roughly fifteen years, from the late 1960s to the early 1980s.[1] Moreover, there are only a few such theories; in fact, the very latest discoveries strongly suggest that they are all equivalent to each other so that apparently there is really only one such theory.

Moreover, these theories have (or this one theory has) the remarkable property of *predicting gravity*—that is, of requiring the existence of a massless spin-2 particle whose couplings at long distances are those of general relativity. (There are also calculable, generally covariant corrections that are unfortunately unmeasureably small under ordinary conditions.) This result is in striking contrast to the situation in conventional quantum field theory, where gravity is impossible because of the singularities of the Feynman graphs.

String theory (especially the heterotic string) also generates Yang–Mills gauge fields and gauge invariance in close parallel with gravity. Further, if one assumes that the weak interactions violate parity, one is practically forced to consider models with the right gauge groups and fermion quantum numbers for the conventional description of particle physics. Thus, the innocent-sounding operation of replacing world-lines with world-tubes forces upon us not only gravity but extra degrees of freedom appropriate for unifying gravity with the rest of physics. Since 1984, when generalized methods of "anomaly cancellation" were discovered and the heterotic string was introduced, one has known how to derive from string theory uncannily simple and qualitatively correct models of the strong, weak, electromagnetic and gravitational interactions.

Apart from gravity and gauge invariance, the most

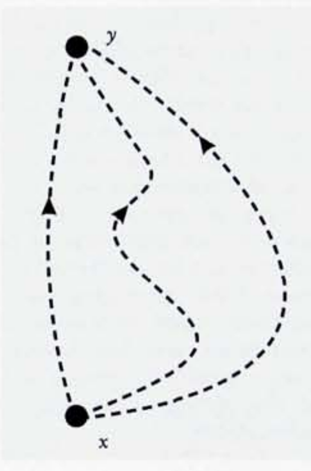


A FEYNMAN DIAGRAM with two incident particles at spacetime events a and b, and two outgoing particles at c and d. Time flows vertically. The particles interact by branching and rejoining at the spacetime events x, y, z and w. Those vertices lead to fundamental problems in field theory. FIGURE 2

important general prediction of string theory is supersymmetry, a symmetry between bosons and fermions that string theory requires (at some energy scale). Searching for supersymmetry is one of the main goals of the next generation of particle accelerators. Its discovery would be quite a statement about nature and would undoubtedly provide a lot of clues about how theorists should proceed.

If this is the good news, what is the bad news? Perhaps what is most glaringly unsatisfactory is this: Crudely speaking there is wave–particle duality in physics, but in reality everything comes from the description by waves, which are then quantized to give particles. Thus a massless classical particle follows a lightlike geodesic (a sort of shortest path in curved spacetime), while the wave description of such particles involves the Einstein, Maxwell or Yang–Mills equations, which are certainly much closer to the fundamental concepts of physics. Unfortunately, in string theory so far, one has generalized only the less fundamental point of view. As a result, we understand in a practical sense how to do many computations in string theory, but we do not yet understand the new underlying principles analogous to gauge invariance. The situation is illustrated in figure 8.

SOME CLASSICAL TRAJECTORIES for a particle propagating from x to y; they all contribute to the Feynman propagator. FIGURE 3



Some of the symptoms

Not knowing the concepts by which string theory will eventually be understood, here I can only describe some of the symptoms, some of the curious phenomena that occur in physics when $\alpha' \neq 0$. In so doing, I hope to give the reader a taste of the conceptual issues that theoretical physicists are grappling with.

But first we need some more background. A point particle moving in Minkowski space with proper time τ is described by giving its position $X^i(\tau)$ as a function of τ—here X^i are the Minkowski coordinates. The action, or Lagrangian, for this particle is

$$I = \frac{1}{2}\int \mathrm{d}\tau \sum_{ij} \eta_{ij} \frac{\mathrm{d}X^i}{\mathrm{d}\tau}\frac{\mathrm{d}X^j}{\mathrm{d}\tau} \qquad (1)$$

where η_{ij} the metric of Minkowski space. If the particle is massless, the Lagrangian must be supplemented with a constraint saying that the velocity is lightlike.

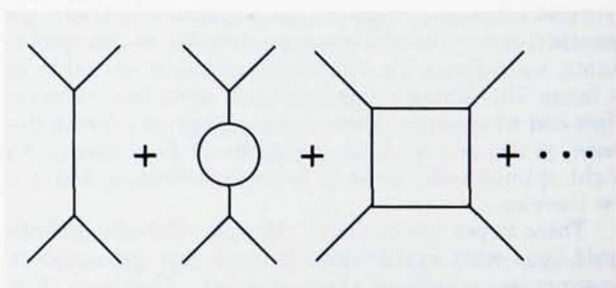
For a string, because the world-tube is two-dimensional, one has not just a proper time τ along the trajectory, but a proper position σ as well. We combine them into coordinates $\sigma^\alpha = (\sigma,\tau)$ along the world-tube. Then the motion of the string is described by giving functions $X^i(\sigma^\alpha)$. The Lagrangian for the string is the obvious analog of equation 1:

$$I = \frac{1}{2\alpha'}\int \mathrm{d}^2\sigma \sum_{ij\alpha} \eta_{ij} \frac{\mathrm{d}X^i}{\mathrm{d}\sigma^\alpha}\frac{\mathrm{d}X^j}{\mathrm{d}\sigma^\alpha} \qquad (2)$$

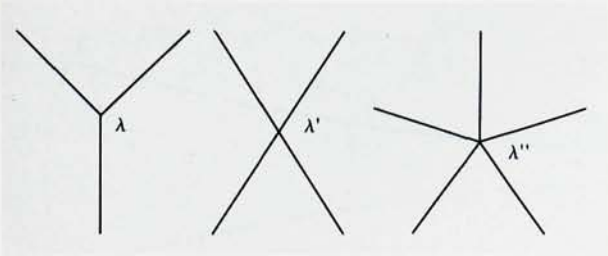
This must again be supplemented with a constraint analogous to saying that a particle velocity is lightlike. Notice that the stringy constant α' appears in equation 2 to make the action dimensionless. If one sets $\hbar = c = 1$, as particle physicists often do, then α' has dimensions of length squared.

Now, regardless of its origins, equation 2 is a Lagrangian quite similar to what one might meet in many problems of two-dimensional statistical mechanics or field theory. For instance, the σ^α might be coordinates along the interface between two media and the X^i might be fields of some kind defined on the interface.

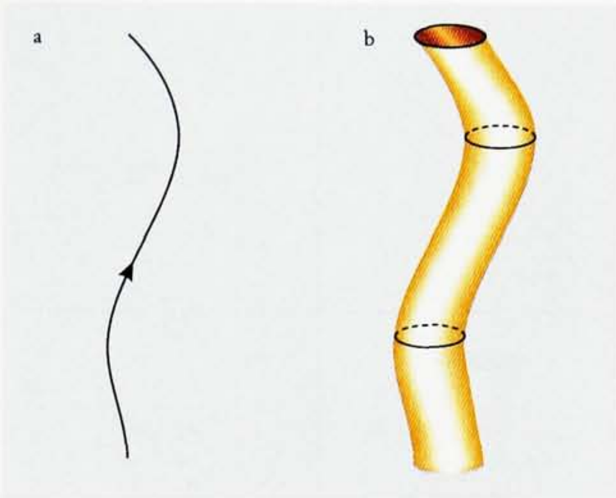
Let us study this problem by standard methods of field theory. First we look at the symmetries. Our prob-



SEVERAL FEYNMAN DIAGRAMS contributing to the same physical process. FIGURE 4



ARBITRARY FACTORS are associated with arbitrary branchings of particles in conventional field theory. In the Standard Model of particle physics, this freedom leads to about 17 parameters whose values are not understood theoretically. FIGURE 5



PARTICLES AND STRINGS. a: A point particle traces out a one-dimensional world-line in spacetime. b: The orbit of a closed string is a two-dimensional tube, or "world-sheet," in spacetime. FIGURE 6

lem had Poincaré invariance—that is, invariance under

$$X^i \to \Lambda^i{}_j X^j + a^i \tag{3}$$

with Λ a Lorentz transformation and a a constant. For simplicity we consider here only the constant translations, obtained by setting Λ to be the unit matrix:

$$X^i \to X^i + a^i \tag{4}$$

In field theory or statistical mechanics, one of the first things that one calculates is the propagator or two-point correlation function $\langle X^i(\sigma)X^j(0)\rangle$. In the present problem, we have a conundrum because it is impossible for the two-point function to be invariant under transformation 4: Under 4, $\langle X^i(\sigma)X^j(0)\rangle$ picks up a nonzero term $a^i a^j$. This term is a c-number, that is, an ordinary number and not an operator, and so is nonzero and cannot be canceled for arbitrary a^i by other contributions, as they are lower order in a^i.

Thus, there are two options. Either the two-point function in question is ill-defined, or Poincaré invariance is spontaneously broken in this theory and would not be observed as a symmetry of physical processes.

In fact, the first option prevails. By the standard recipe, the two-point function of this theory should be

$$\langle X^i(\sigma)X^j(0)\rangle = \eta^{ij}\int \frac{d^2k}{(2\pi)^2}\,\frac{e^{ik\cdot\sigma}}{k^2} \tag{5}$$

The integral is infrared divergent. This divergence means that the "elementary field" X^i is ill-behaved quantum mechanically (but other fields are well-behaved and the theory exists).

This infrared divergence—which is central in string theory—was in fact first studied in the theory of two-dimensional XY ferromagnets. In that context, the infrared divergence means that the system has a low-temperature phase with power law correlations but no long range order. This is an example of a general theme: properties of spacetime in string theory (in this case, unbroken Poincaré invariance) reflect phenomena in two-dimensional statistical mechanics and field theory.

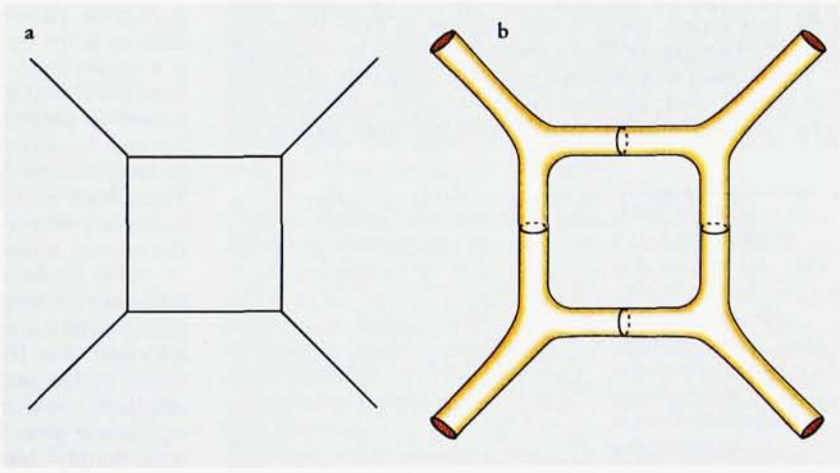
For instance, condensed matter theorists and field theorists are often interested in the anomalous dimensions of operators—how the renormalized operators scale with changes in the length or energy scale. In this case, by studying the anomalous dimension of a certain operator—namely, $(\partial X)^2 e^{ik\cdot X}$—we could go on to explain why string theory predicts the existence of gravity. This tale has been told many times.[2] Here I prefer to convey the radical change that taking $\alpha' \neq 0$ brings in physics.

In analyzing Poincaré invariance, we took the spacetime metric to be flat—we used the Minkowski metric η_{ij} in equation 2. Nothing prevents us from replacing the flat metric with a general spacetime metric $g_{ij}(X)$, taking the world-tube Lagrangian to be

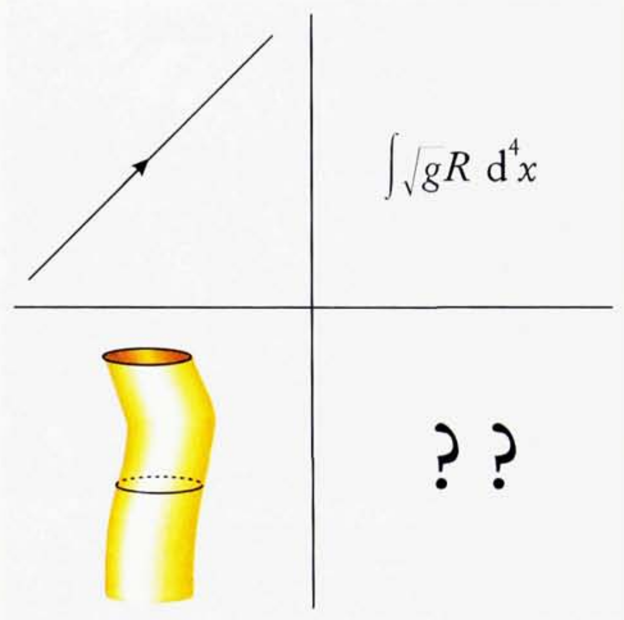
$$I = \frac{1}{2\alpha'}\int d^2\sigma \sum_{ij\alpha} g_{ij}(X)\,\frac{dX^i}{d\sigma^\alpha}\,\frac{dX^j}{d\sigma^\alpha} \tag{6}$$

Simply by writing equation 6, we get, for each classical spacetime metric g, a two-dimensional quantum field theory, or at least the Lagrangian for one.

So spacetime with its metric determines a two-dimensional field theory. And that two-dimensional field theory



STRING THEORY'S SMOOTHING EFFECT is apparent when one compares a Feynman graph (a) with its stringy couterpart (b). The string diagram has no singular interaction points. FIGURE 7



THE "MAGIC SQUARE" OF STRING THEORY. The two rows represent ordinary physics and string theory, respectively, while the two columns represent particles and waves. In the upper left-hand corner, a line drawn at a 45° angle to the horizontal symbolizes a classical massless particle, propagating at the speed of light. In the lower left, we show the stringy analog of a particle's world-line, the world-tube. In the upper right are crown jewels, such as the Einstein–Hilbert action of general relativity. In the lower right should be the synthesis, related to the Einstein–Hilbert action as world-tubes are related to world-lines. FIGURE 8

is *all* one needs to compute stringy Feynman diagrams. The reason that theory suffices is that (as explained above) stringy Feynman diagrams are nonsingular. Thus, in a field theory diagram, as in figure 7a, even when one explains how free particles propagate (what factor is associated with the lines in the Feynman diagram), one must separately explain how particles interact (what vertices are permitted and what factors are associated with them). Because the stringy Feynman diagram of figure 7b is nonsingular, once one understands the propagation of the free string, there is nothing else to say—there are no interaction points whose properties must be described.

Thus, once one replaces ordinary Feynman diagrams with stringy ones, one does not really need spacetime any more; one just needs a two-dimensional field theory describing the propagation of strings. And perhaps more fatefully still, one does not have spacetime any more, except to the extent that one can extract it from a two-dimensional field theory.

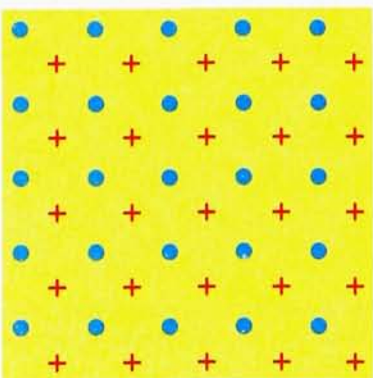
So we arrive at a quite beautiful paradigm. Whereas in ordinary physics one talks about spacetime and classical fields it may contain, in string theory one talks about an auxiliary two-dimensional field theory that encodes the information. The paradigm has a quite beautiful extension: A spacetime that obeys its classical field equations corresponds to a two-dimensional field theory that is conformally invariant (that is, invariant under changes in how one measures distances along the string). If one computes the conditions needed for conformal invariance of the quantum theory derived from the Lagrangian (equation 6), assuming the fields to be slowly varying on the stringy scale, one gets generally covariant equations that are simply the Einstein equations plus corrections of order α'.

We are far from coming to grips fully with this paradigm, and one can scarcely now imagine how it will all turn out. But two remarks seem fairly safe. First, all the vicissitudes of two-dimensional field theory and statistical mechanics are reflected in "spacetime," leading to many striking phenomena. Second, once α' is turned on, even in the classical world with $\hbar = 0$, "spacetime" seems destined to turn out to be only an approximate, derived notion, much as classical concepts such as the position and velocity of a particle are understood as approximate concepts in the light of quantum mechanics.

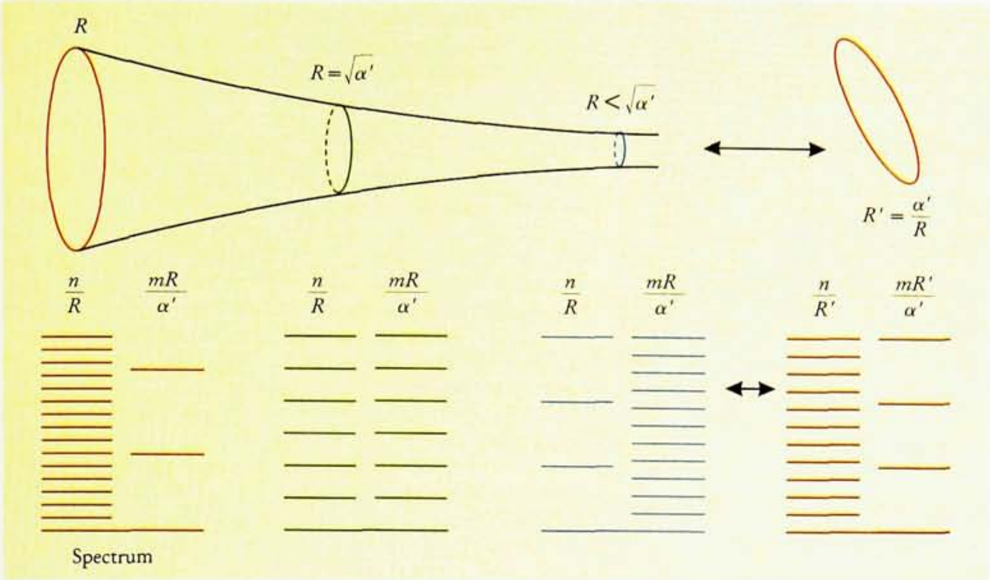
Duality and the minimum length

A famous vicissitude of two-dimensional statistical mechanics is the duality of the Ising model. The Ising model is a simple model of a ferromagnet in two dimensions. As was discovered 60 years ago, the Ising model on a square lattice is equivalent to a "dual" spin system on a "dual lattice," as sketched in figure 9. If the original system is at temperature T, the dual system has temperature $1/T$. Thus, high and low temperatures are exchanged, and if there is precisely one phase transition, it must occur at the critical temperature, $T = 1$.

This duality has an analog if the $\mathbf{Z}_2$ symmetry of the Ising model (spin up and spin down) is replaced by $\mathbf{Z}_n$ (spins pointing in any of n directions equispaced around a circle). For large n, there is an interesting continuum limit, which leads to the following assertion: There is a smallest circle in string theory; a circle of radius R is equivalent to a circle of radius α'/R. By this we mean most simply the following. Imagine that the universe as a whole is not infinite in spatial extent, but that one of the three space dimensions is wrapped in a circle, making it a periodic variable with period $2\pi R$. Then there is a smallest possible value of R. When R is large, things will



A SYSTEM OF ISING SPINS on the lattice indicated by blue dots is equivalent to another spin system on the "dual" lattice indicated by red crosses. In string theory, analogous dualities of an underlying two-dimensional field theory result in dualities of spacetimes. FIGURE 9



SMALL CIRCLES DON'T EXIST. The spectrum of string states on a circle has two components, n/R due to momentum quantization and mR/α' due to wrapping of the string around the circle m times (red, at left). When the circle radius shrinks to about size $\sqrt{\alpha'}$ (green), the "momentum" and "wrapping" states become equivalent. As one tries to compress the circle further (blue), the states become equivalent to those on a larger "dual" circle with "momentum" and "wrapping" states swapped (red, at right). FIGURE 10

look normal, but if one tries to shrink things down until the period is less than $2\pi\sqrt{\alpha'}$, space will re-expand in another "direction" peculiar to string theory, and one will not really succeed in creating a circle with a radius of less than $\sqrt{\alpha'}$.

Technically, this phenomenon arises as follows. A massless particle—or string—on a circle of radius R has quantized momentum $p = n/R$, with integer n, and energy levels

$$E_n = \frac{|n|}{R} \tag{7}$$

A string can also wrap m times around the circle, with energy

$$\tilde{E}_n = \frac{|m|R}{\alpha'} \tag{8}$$

There is a duality symmetry—generalizing the duality of the Ising model—that exchanges the two spectra, exchanging also R with α'/R. (See figure 10.)

As presented here, the argument might seem to apply only to circles wrapped around a periodic dimension of the universe. In fact, similar arguments can be made for any circle in spacetime.

The fact that one cannot compress a circle below a certain length scale might be taken to suggest that the smaller distances just are not there. Let us try to disprove this. A traditional way to go to short distances is to go to large momenta. According to Heisenberg, at a momentum scale p, one can probe a distance $x \approx \hbar/p$. It would appear that by going to large p, one can probe small x and verify that the small distances do exist. However (as described in reference 3), the Heisenberg microscope does not work in string theory if the energy is too large. Instead, the strings expand and—when one accelerates past the string scale—instead of probing short distances one just watches the propagation of large strings. It is roughly as if the uncertainty principle has two terms,

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' \frac{\Delta p}{\hbar} \tag{9}$$

where the first term is the familiar quantum uncertainty and the second term reflects a new uncertainty or fuzziness due to string theory. With the two terms together, there is an absolute minimum uncertainty in length—of order $\sqrt{\alpha'} \approx 10^{-32}$ cm—in any experiment. But a proper theoretical framework for the extra term in the uncertainty relation has not yet emerged.

A somewhat similar conclusion arises if one tries to compute the free energy at high temperature. In field theory, at high temperature T, one gets (in four dimensions) a free energy per unit volume $F \approx T^4/(\hbar c)^3$, as if each box of linear size $\hbar c/T$ contains one quantum of energy T. In string theory, the behavior is similar until one reaches "stringy" temperatures, after which the free energy seems to grow more slowly, roughly as if one cannot divide space into boxes less that 10^{-32} cm on a side, with each such box containing one string.

The duality symmetry described above also has a number of nonlinear analogs, such as "mirror symmetry," which is a relationship between two spacetimes that would be quite distinct in ordinary physics but turn out to be equivalent in string theory. The equivalence is possible because in string theory one does not really have a classical spacetime, but only the corresponding two-dimensional field theory; two apparently different spacetimes X and Y might correspond to equivalent two-dimensional field theories.

A cousin of mirror symmetry is the phenomenon of topology change. Here one considers how space changes as a parameter—which might be the time—is varied. One starts with a spatial manifold X so large that stringy effects are unimportant. As time goes on, X shrinks and stringy effects become large; the classical ideas of spacetime break down. At still later times, the distances are large again and classical ideas are again valid, but one is on an entirely different spatial manifold Y! Quite precise computations of such processes have been developed.

Strings and quantum mechanics

In this article, I have generally suppressed the effects of quantum mechanics, or $\hbar$, and have attempted to explain how physics changes when one turns on α'. My goal has been to explain that the phenomena and the change in viewpoint associated with α'—or string theory—are as striking as those associated with $\hbar$—or quantum mechanics.

Of course, in the real world, $\hbar$ and (if string theory is correct) α' are both nonzero. What happens then? That is perhaps the main focus of current work in the field. We are far from getting to the bottom of things, but lately there have been enough surprising new ideas and discoveries to make up what some have characterized as "the second superstring revolution." (From that point of view, the "first superstring revolution" was the period in the mid-1980s when the scope of superstring theory first came to be widely appreciated.) New dualities—generalizing the duality of Maxwell's equations between electric and magnetic fields—appear when $\hbar$ and α' are considered together. These new symmetries have enabled us to understand that—as I mentioned earlier—there is apparently only one string theory, the previously formulated theories being equivalent. Their richness is illustrated by the fact that (in their field theory limit) they have provided new insights about quark confinement, the geometry of four-dimensional spacetime and many other things. (See PHYSICS TODAY, March 1995, page 17.)

Moreover, these new dualities mix $\hbar$ and α' in a way quite unlike anything previously encountered in physics. The existence of such symmetries that hold only for $\hbar \neq 0$ gives one the feeling that the natural formulation of the theory may eventually prove to be inherently quantum mechanical and thus, in a sense, may entail an explanation of quantum mechanics.

We shall have to leave further discussion of these matters for another occasion. Even so, I hope to have communicated a sense of some of the storm clouds in theoretical physics, and a feeling for the likely fate of the concept of spacetime.

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