TMV151

Frågestund läsvecka 2

Ö1.25a

Avgör om $\int_{-\infty}^{0} e^{x} \cdot \frac{1}{x}\, dx$ är konvergent.

$$\int_{-\infty}^{0} e^{x} \cdot \frac{1}{x}\, dx = \left\{ \begin{array}{l} y = -x \\ dy = -dx \end{array} \right\} = -\int_{\infty}^{0} \frac{e^{-y}}{-y}\, dy$$

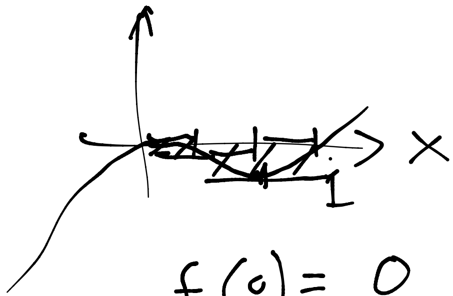
$$= -\int_{0}^{\infty} \frac{e^{-y}}{y}\, dy = -I$$

$$I = \int_{0}^{1} \frac{e^{-y}}{y}\, dy + \underbrace{\int_{1}^{\infty} \frac{e^{-y}}{y}\, dy}_{\geq 0} \geq$$

$$\geq \int_{0}^{1} \frac{e^{-y}}{y}\, dy \geq \int_{0}^{1} \frac{e^{-1}}{y}\, dy = e^{-1} \underbrace{\int_{0}^{1} \frac{1}{y}\, dy}_{= \infty}$$

Alltså $I = \infty$ $\quad \int_{-\infty}^{0} \frac{e^{x}}{x}\, dx = -\infty$ divergent.

P1.3 Bestäm övre och undre Riemann-summor till $f(x) = x^3 - x^2$ på $[0,1]$ med $P = \{0, \frac{1}{3}, \frac{2}{3}, 1\}$



$$f'(x) = 3x^2 - 2x = 0$$

$$x = 0, \quad x = \frac{2}{3}$$

$$f(0) = 0 \qquad f\left(\frac{1}{3}\right) = \frac{1}{27} - \frac{1}{9} = -\frac{2}{27}$$

$$f\left(\frac{2}{3}\right) = \frac{8}{27} - \frac{4}{9} = -\frac{4}{27}$$

$$f(1) = 0$$

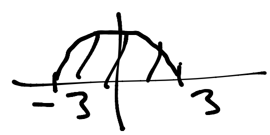
$$I_{min}(f,P) = f\left(\frac{1}{3}\right)\cdot\frac{1}{3} + f\left(\frac{2}{3}\right)\frac{1}{3} + f\left(\frac{2}{3}\right)\frac{1}{3}$$

$$= -\frac{10}{81}$$

$$I_{max}(f,P) = f(0)\frac{1}{3} + f\left(\frac{1}{3}\right)\frac{1}{3} + f(1)\frac{1}{3}$$

$$= -\frac{2}{81}$$

Ö2.3a



$$\int_{-3}^{3} \sqrt{9 - x^2}\, dx =$$

$$= \left\{ \begin{array}{ll} x = 3\cos\theta &, \; 3 = 3\cos\theta \Rightarrow \theta = 0 \\ dx = -3\sin\theta\, d\theta &, \; -3 = 3\cos\theta \Rightarrow \theta = \pi \end{array} \right\}$$

$$= -\int_{\pi}^{0} \sqrt{9 - 9\cos^2\theta} \cdot 3\sin\theta\, d\theta =$$

$$= 9\int_0^{\pi} \sin\theta \cdot \sqrt{1-\cos^2\theta}\, d\theta =$$

$$= 9\int_0^{\pi} \sin^2\theta\, d\theta = \left\{ \begin{array}{l} \cos 2\theta = \cos^2\theta - \sin^2\theta \\ = 1 - 2\sin^2\theta \end{array} \right\}$$

$$= 9\int_0^{\pi} \frac{1-\cos 2\theta}{2}\, d\theta = \frac{9}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{9\pi}{2}$$

Ö2.5b

$$I = \int_{-\pi/4}^{\pi/4} e^x \cdot \cos x\, dx = \left[e^x \sin x\right]_{-\pi/4}^{\pi/4}$$

$$-\int_{-\pi/4}^{\pi/4} e^x \sin x\, dx = e^{\pi/4} \cdot \frac{1}{\sqrt{2}} + e^{-\pi/4} \frac{1}{\sqrt{2}}$$

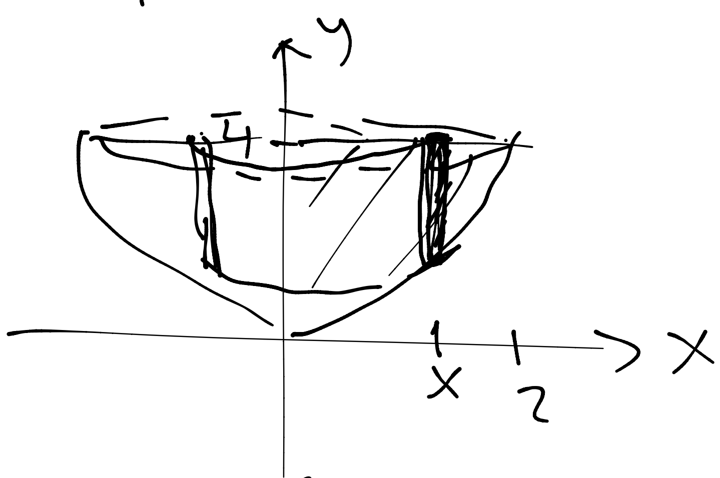
$$-\left[e^x(-\cos x)\right]_{-\pi/4}^{\pi/4} + \int_{-\pi/4}^{\pi/4} e^x(-\cos x)\, dx$$

$$= \frac{1}{\sqrt{2}}\left(e^{\pi/4} + \cancel{e^{-\pi/4}}\right) + \frac{1}{\sqrt{2}}\left(e^{\pi/4} - \cancel{e^{-\pi/4}}\right)$$

$$- I \Rightarrow 2I = \sqrt{2}\, e^{\pi/4} \Rightarrow$$

$$\Rightarrow I = \frac{e^{\pi/4}}{\sqrt{2}}$$

Ö 2,16c $\quad 0 \le x \le 2, \quad x^2 \le y \le 4$



$$V = \int_0^2 2\pi x(4-x^2)\,dx = 2\pi \int_0^2 4x - x^3\,dx$$

$$= 2\pi \left[2x^2 - \frac{x^4}{4} \right]_0^2 = 16\pi - 8\pi = 8\pi$$