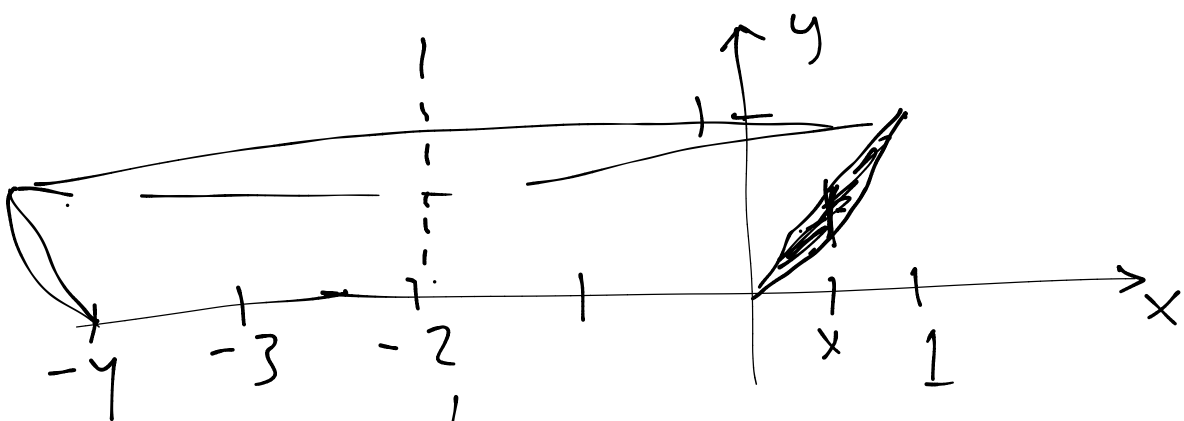
TMV151

Frågestund läsvecka 3

2.17c $0 < x < 1$, $x^2 < y < x$

Bestäm volym när området roterar runt $x = -2$.



$$V = 2\pi \int_0^1 (x+2)(x - x^2)\,dx =$$

$$= 2\pi \int_0^1 x^2 + 2x - x^3 - 2x^2\,dx$$

$$= 2\pi \left[\frac{x^3}{3} + x^2 - \frac{x^4}{4} - \frac{2x^3}{3} \right]_0^1$$

$$= 2\pi \left(\frac{1}{3} + 1 - \frac{1}{4} - \frac{2}{3} \right) = \frac{5\pi}{6}$$

3.6b) $u'(x) = 5u^{4/5}(x)$, $u(0)=0$

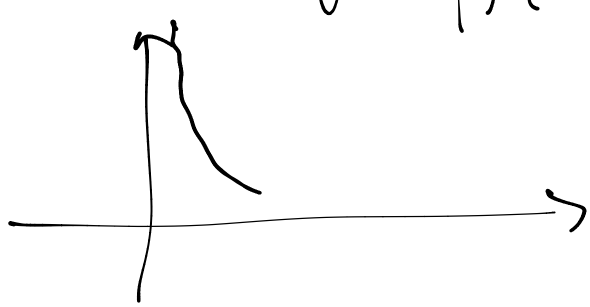
Är lösningen unik? Annars hitta två lösningar.

$f(u) = 5u^{4/5}$

$f'(u) = 4u^{-1/5}$ ej begr i 0

alltså ej Lipschitz kont.

Picard garanterar ej unik lösning. $|f(u)-f(v)| \leq L_f|u-v|$



En lösning ges av $\underline{\underline{u=0}}$.

$u' = 5u^{4/5}$ $u(0)=0$

$\frac{du}{dx} = 5u^{4/5}(x)$

$$\underbrace{\int \frac{1}{5u^{4/5}}\,du}_{u^{1/5}} = \int 1\,dx = x + C$$

$u^{1/5} = x + C$, $u(0) = 0 \Rightarrow C = 0$

$u^{1/5} = x \Rightarrow u(x) = \underline{\underline{x^5}}$

$$\frac{du}{dx} = \underline{f(u)} \cdot \underline{h(x)} \Rightarrow \int \frac{du}{f(u)} = \int h(x)\,dx$$

(3,7b) Lös $u'(x) = -x u(x)$, $u(0) = 1$ med fixpunktsiteration

3,15c) $u''(x) + 2x\,(u'(x))^2 = 0$

$u(0) = 0$, $u'(0) = \frac{1}{4}$

Reduktion till första ordningen

$v = u'$ $\quad v'(x) + 2x\,v^2 = 0$

$v(0) = \frac{1}{4}$

$v'(x) = -\underline{2x} \cdot \underline{v^2(x)} \Rightarrow$

$$\frac{dv}{dx} = -2x v^2 \Rightarrow \int \frac{1}{v^2}\,dv = \int -2x\,dx$$

$$-v^{-1} = -x^2 + C$$

$$-v^{-1}(0) = C = -4 \Rightarrow \frac{-1}{v} = -x^2 - 4$$

$$v(x) = \frac{1}{x^2+4} \Rightarrow u'(x) = v(x)$$

$$u(x) = \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$$

$$u(0) = 0 + C = 0 \Rightarrow$$
$$u(x) = \frac{1}{2}\arctan\left(\frac{x}{2}\right)$$

3,14c) $u'' = 2u' \cdot u$

Let $v = u'$.

$$u''(x) = \frac{d}{dx} v(u(x)) = v'(u) \cdot u'(x)$$
$$= v' \cdot v \quad , \quad v = v(u)$$

$$\cancel{v} v' = 2 \cancel{v} \cdot u \qquad v'(u) = 2u$$

$$\Rightarrow v(u) = u^2 + C \Rightarrow u'(x) = u^2 + C$$

$$\frac{du}{dx} = u^2 + C \Rightarrow \int \frac{1}{u^2+C}\, du = \int dx$$

$$\frac{1}{c}\arctan\left(\frac{u}{c}\right) = x + D$$

$$\arctan\left(\frac{u}{c}\right) = cx + D$$

$$\frac{u}{c} = \tan(cx+D) \Rightarrow$$

$$u(x) = c\tan(cx+D)$$

3,24c) Bestäm partikulärlösning till $u'' - 3u' + 2u = \sin x$

$$u(x) = A\sin x + B\cos x$$

$$u'(x) = A\cos x - B\sin x$$

$$u''(x) = -\underline{A}\sin x - B\cos x$$

$$u'' - 3u' + 2u = \sin x(-A + 3B + 2A)$$

$$+\cos x(-B - 3A + 2B) = \sin x$$

$$3B + A = 1 \Rightarrow 9A + A = 1 \quad A = \frac{1}{10}$$

$$B - 3A = 0 \Rightarrow B = 3A \quad B = \frac{3}{10}$$

$$u_p(x) = \frac{1}{10}\sin x + \frac{3}{10}\cos x$$

$(*)$ $au'' + bu' + cu = f(x)$

En lösning till $(*)$ kallas partikulär lösning u_p

Om $f(x) = 0$ kallas lösningen homogen lösning u_h

Om u_p löser $(*)$ så löser även $u_p + u_h$ $(*)$

Allmänna lösningen

$$u'' - 3u' + 2u = 0$$

Ansätt e^{rx}

$$e^{rx}\underbrace{(r^2 - 3r + 2)}_{\text{karakteristisk ekvation}} = 0$$

$$r^2-3r+2=0 \quad r=\frac{3}{2}\pm\sqrt{\frac{9}{4}-2}$$

$$=\frac{3}{2}\pm\frac{1}{2}$$

$$r_1=2 \ , \ r_2=1$$

$$r^2-3r+2=(r-2)(r-1)$$

$$u_h(x)=Ae^{2x}+Be^{x}$$

$$u(x)=Ae^{2x}+Be^{x}+\frac{1}{10}\sin x+\frac{3}{10}\cos x$$

Antag $u(0)=0$

$$0=u(0)=A+B+\frac{3}{10}$$

$$u'(0)=0$$

$$0=2A+B+\frac{1}{10}$$

$$A+B=-\frac{3}{10} \Rightarrow B=-\frac{3}{10}-A$$

$$2A+B=-\frac{1}{10} \Rightarrow 2A-A-\frac{3}{10}=-\frac{1}{10}$$

$$A=\frac{2}{10} \quad B=-\frac{3}{10}-\frac{2}{10}=-\frac{1}{2}$$

$$u(x)=\frac{2}{10}e^{2x}-\frac{1}{2}e^{x}+\frac{1}{10}\sin x+\frac{3}{10}\cos x$$

Vad händer om man får imaginära rötter

$r^2+1=0 \quad r_1 = i \quad r_2 = -i$

$u'' + u = 0$

$u_h(x) = Ae^{r_1 x} + Be^{r_2 x} = Ae^{ix} + Be^{-ix}$

$\tilde{u}_1(x) = e^{ix} \qquad \tilde{u}_2(x) = e^{-ix}$

$$\frac{\tilde{u}_1 + \tilde{u}_2}{2} = \frac{e^{ix} + e^{-ix}}{2} = \cos x$$

$$\frac{\tilde{u}_1 - \tilde{u}_2}{2i} = \frac{e^{ix} - e^{-ix}}{2i} = \sin x$$

$$\boxed{e^{ix} = \cos x + i \sin x}$$