Options and Mathematics: Lecture 5

November 10, 2020

Exercises

Exercise 1.9

Assume that at time t it is known that the underlying stock will pay the dividend $D < S(t_0)$ at time $t_0 \in (t, T)$. Prove the following variant of the put-call parity for $t < t_0$:

$$C(t, S(t), K, T) - P(t, S(t), K, T) = S(t) - KB(t, T) - DB(t, t_0).$$

In the following exercises it is assumed that the arbitrage-free principle holds and that the stock pays no dividend.

Exercise 1.12

Consider the European derivative $\mathcal{U}$ with maturity time T and pay-off Y given by

$$Y = \min[(S(T) - K_1)_+, (K_2 - S(T))_+],$$

where $K_2 > K_1$ and $(x)_+ = max(0, x)$. Draw the graph of the pay-off function of the derivative.

Find a constant portfolio consisting of European calls expiring at time T which **replicates** the value of $\mathcal{U}$, i.e., whose value at any time $t \leq T$ equals the value of $\mathcal{U}$.

Exercise 1.14

Let $\mathcal{U}_1$ be a call stock option with strike K_1 and maturity T and $\mathcal{U}_2$ the physically-settled digital call option on the same stock with strike K_2 and maturity T. Decide whether the following statements are true or false and explain your answer.

(a) If $K_2 \leq K_1$, the value of $\mathcal{U}_2$ is greater or equal than the value of $\mathcal{U}_1$ for all $t < T$;

(b) If $K_2 > K_1$, the value of $\mathcal{U}_1$ is greater or equal than the value of $\mathcal{U}_2$ for all $t < T$.

Exercise 1.16

Consider the European derivative with pay-off Y at maturity T and the derivative with pay-off $Z = \Pi_Y(t_*)$ at maturity $t_* < T$. Show that $\Pi_Z(t) = \Pi_Y(t)$, $t \in [0, t_*]$

Exercise 1.17 (Chooser option)

Let $T_2 > T_1$. A chooser option with maturity T_1 is a contract which gives to the buyer the right to choose at time T_1 whether the derivative transforms (at zero cost) into a call or a put option with strike K and maturity T_2.

Write down the pay-off Y of the chooser option.

Let $r \in \mathbb{R}$ be constant. Show that the value of the chooser option is given by the formula

$$\Pi_Y(t) = C(t, S(t), K, T_2) + P(t, S(t), Ke^{-r(T_2-T_1)}, T_1), \quad t \leq T_1.$$

HINT: You need the result of Exercise 1.16 and the put-call parity.

Exercise 1.28

Find, if possible, constant portfolios consisting of European calls and/or puts that replicate the European derivatives with maturity T and pay-off Y depicted in the following figure.

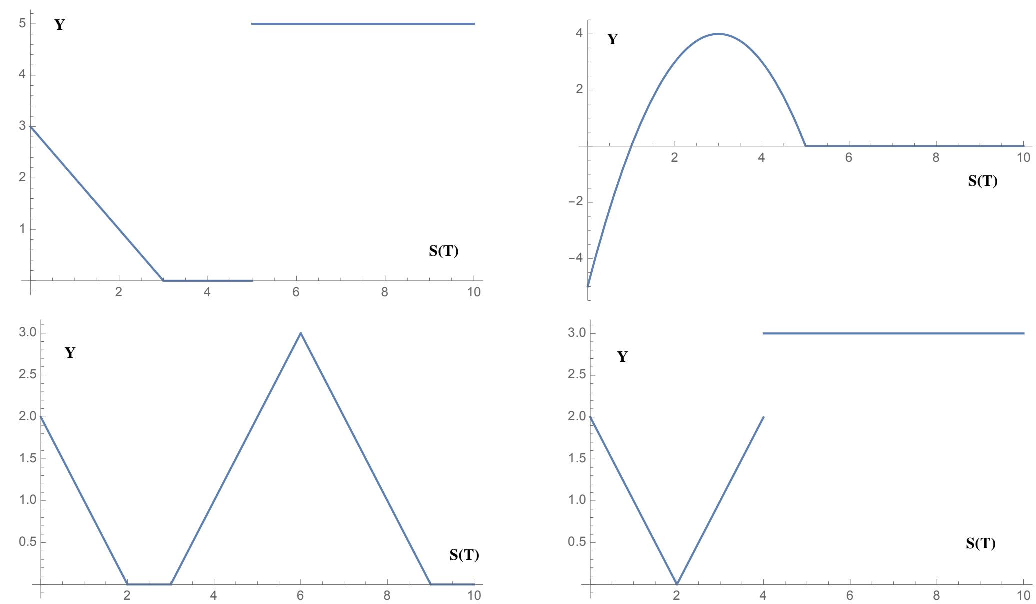

Figure 1: Remark: For $S(T) > 10$ the pay-off is identically zero

Exercise 1.30

Decide whether the following statements are true and motivate your answer:

(a) The value of the European put option is non-decreasing with maturity;

(b) The value of the American put option is non-decreasing with maturity.