A project on the American put option

The American put option with strike K and maturity T gives the owner the right to sell the underlying stock for the price K at any time $t \in (0, T]$. The binomial model is the most popular numerical method to compute the fair price of American puts.

The main goal of this project is to perform a parameter sensitivity analysis of the American put and to derive numerically the optimal exercise curve of the option using the binomial model and Matlab. It is assumed that the underlying stock pays no dividend. The specific goals of the project are the following.

- Study numerically how the value of the American put at time $t = 0$ depends on the parameters $N, \alpha, p, \sigma, r, K, T, S_0$ and give an intuitive explanation of the results obtained (see Section 3.3 in the lecture notes for a similar analysis in the case of the European call option).

- Derive numerically the curve $(t_i, S_*(t_i))$ where t_i is an optimal exercise time and $S_*(t_i)$ is the largest stock price for which it is optimal to exercise the American put at time t_i (optimal exercise curve). Create one plot with optimal exercise curves for different values of σ and another plot with optimal exercise curves for different values of r. Describe the qualitative properties of these curves.

- Show numerically that the binomial price at time $t = 0$ of the American put with strike K and maturity T converges, as $T \to +\infty$, to the value $v(S_0)$, where v is the function

$$v(x) = \begin{cases} K - x & 0 \le x \le L \\ (K - L)\left(\frac{x}{L}\right)^{-\frac{2r}{\sigma^2}} & x > L \end{cases}, \quad L = \frac{2r}{2r + \sigma^2} K.$$

  (The American put with infinite maturity is called perpetual American put. Of course, it does not exist in real markets but it is useful for theoretical considerations.)

The results of the project have to be outlined in a report consisting of four sections:

- The first section is the Introduction (1-2 pages). Here is where you state the problem, describe the theoretical framework and discuss the financial applications. You can find plenty of information online, but *remember to cite your sources!*

- The second section is where you report and discuss your results (3-4 pages).

- The third section is the Conclusion. Summarize your most interesting results in this section and comment on possible flaws in your analysis (1 page).
- An appendix where you attach and explain the Matlab codes. The explanation of the codes could appear as comments within the codes themselves.

Remarks:

- The projects can be worked out in groups of max 4 students
- The report has to be submitted in PDF form by e-mail to calogero@chalmers.se. Use OPTIONS 2021 as subject and include all members of the group as recipients.
- Together with the report you have to submit a statement, signed by all members of the group, certifying that this report is your own work and that all members of the group have equally contributed to the assignment. All reports will be subjected to a plagiarism check. Information on how to avoid plagiarism and on Chalmers policy on plagiarism can be found on this link:

  `https://student.portal.chalmers.se/en/chalmersstudies/policy-documents/Pages/Academic-integrity-honesty.aspx`

- The deadline for submission is **January 3rd, 2021 at 23.59**. The grade on the project (max 2 points) will be communicated on January 7th.
- The grade of the project is mostly based on the "aesthetical" quality of the report, e.g., on having nice and informative plots (remember to specify labels for the axes). An incorrect interpretation of the results will not affect the grade, so be bold in your arguments!