A project on call options on dividend paying stocks

The value of financial derivatives depends on whether the underlying asset pays a dividend. Assuming that the underlying asset is a stock, we can model the payment of a dividend D at time $t_D > 0$ in the binomial model in the following way. Consider a uniform partition $t_0 = 0 < t_1 < \cdots < t_N = T$ of the interval $[0, T]$ such that $t_D = t_m$. We set $S(0) = S_0 > 0$,

$$S(t_i) = \begin{cases} S(t_{i-1})e^u, & \text{with probability } p_u = p, \\ S(t_{i-1})e^d, & \text{with probability } p_d = 1 - p, \end{cases}$$

for $i = 1, \ldots, m-1$ and for $i = m+1, \ldots, N$, while at time $t = t_m$ we set

$$S(t_m) = \begin{cases} S(t_{m-1})e^u - D, & \text{with probability } p_u = p, \\ S(t_{m-1})e^d - D, & \text{with probability } p_d = 1 - p \end{cases}.$$

For modeling purposes it is convenient to assume that $D = aS(t_{m-1})e^u$, if the price goes up at time t_m, and $D = aS(t_{m-1})e^d$, if the price goes down at time t_m, where $a \in (0, 1)$. This ensures that the stock price does not become negative after the dividend is paid.

The main goal of this project is to study how the value of European and American call options is affected by the dividend payment using the binomial model and Matlab. The specific goals of the project are the following.

- Fix the value of $t_D \in (0, T)$ and study numerically how the initial value (at $t = 0$) C of the European call and the initial value $\widehat{C}$ of the American call depend on $a \in (0, 1)$. Draw the curves $a \to C$ and $a \to \widehat{C}$ in the same plot and discuss the results.
- Fix the value of a and study numerically how $C, \widehat{C}$ depend on $t_D \in (0, T)$. Draw the curves $t_D \to C$ and $t_D \to \widehat{C}$ in the same plot and discuss the results.
- Show numerically that the only time at which it might be optimal to exercise the American call is "just before the dividend is paid" and that there exists $S_a > 0$ such that it is optimal to exercise the American call "just before the dividend is paid" if and only if the stock price at this time is larger than or equal to S_a. Plot the curve $a \to S_a$ and discuss the result.

The results of the project have to be outlined in a report consisting of four sections:

- The first section is the Introduction (1-2 pages). Here is where you state the problem, describe the theoretical framework and discuss the financial applications. You can find plenty of information online, but *remember to cite your sources!*
- The second section is where you report and discuss your results (3-4 pages).
- The third section is the Conclusion. Summarize your most interesting results in this section and comment on possible flaws in your analysis (1 page).
- An appendix where you attach and explain the Matlab codes. The explanation of the codes could appear as comments within the codes themselves.

Remarks:

- The projects can be worked out in groups of max 4 students
- The report has to be submitted in PDF form by e-mail to calogero@chalmers.se. Use OPTIONS 2021 as subject and include all members of the group as recipients.
- Together with the report you have to submit a statement, signed by all members of the group, certifying that this report is your own work and that all members of the group have equally contributed to the assignment. All reports will be subjected to a plagiarism check. Information on how to avoid plagiarism and on Chalmers policy on plagiarism can be found on this link:

  `https://student.portal.chalmers.se/en/chalmersstudies/policy-documents/Pages/Academic-integrity-honesty.aspx`

- The deadline for submission is **January 3rd, 2021 at 23.59**. The grade on the project (max 2 points) will be communicated on January 7th.
- The grade of the project is mostly based on the "aesthetical" quality of the report, e.g., on having nice and informative plots (remember to specify labels for the axes). An incorrect interpretation of the results will not affect the grade, so be bold in your arguments!