Options and Mathematics: Lecture 18

December 2, 2020

Exercises

Execise 5.22

Consider a N-period binomial market with $r \neq 0$ and let $S(t)$ denote the price of the stock at time $t \in \{0, \ldots, N\}$. The Asian call, resp. put, with maturity $T = N$ and strike K is the non-standard European style derivative with pay-off

$$Y_{\text{call}} = \left[\left(\frac{1}{N+1}\sum_{t=0}^{N} S(t)\right) - K\right]_+, \quad \text{resp.} \quad Y_{\text{put}} = \left[K - \left(\frac{1}{N+1}\sum_{t=0}^{N} S(t)\right)\right]_+.$$

Denote by $AC(0)$ and $AP(0)$ the binomial price at time $t = 0$ of the Asian call and put, respectively. Use the risk-neutral pricing formula to prove the following put-call parity identity:

$$AC(0) - AP(0) = e^{-rN}\left[\frac{S(0)}{N+1}\frac{e^{r(N+1)} - 1}{e^r - 1} - K\right].$$

Execise 5.23

Use the risk-neutral pricing formula to show the discounted binomial price $\{\Pi^*_Y(t)\}_{t=0,\dots,N}$ of European derivatives is a martingale in the risk-neutral probability measure and to give an alternative proof of the recurrence formula for the binomial price of European derivatives.

Next consider the augmented binomial market consisting of the stock, the risk-free asset and a European derivative on the stock priced by the risk-neutral pricing formula. Use the martingale property of $\{S^*(t)\}_{t=0,\dots,N}$ and $\{\Pi^*_Y(t)\}_{t=0,\dots,N}$ to show that this market does not admit self-financing arbitrages.

Exercise 5.32

Consider a 2-period binomial model with the parameters $u = \log(5/4)$, $d = \log(1/2)$, $r = 0$, $S(0) = 64/25$, $p \in (0, 1)$.

Compute the price at time $t \in \{0, 1, 2\}$ of the American put on the stock with maturity $T = 2$ and strike price $K_2 = \frac{11}{5}$ and identify the possible optimal exercise times prior to maturity.

Next consider the compound option which gives to its owner the right to buy the American put at time $t = 1$ for the price $K_1 = \frac{8}{25}$. Compute the price of the compound option at time $t = 0$ and the hedging portfolio for the compound option (assume $B(0) = 1$).

Derive the strategy that maximises the expected return for the owner of the compound option, where as usual we assume that the investor can only exercise, and not sell, derivatives.

Exercise 5.33

Consider a portfolio that is long 1 share of the American put option in Section 4.2. Assume $p = 1/2$ and compute the expected value and the variance of the rate of return of the portfolio.

ANSWER: $\mathbb{E}[R] = 1/8$.

Exercise 5.34

Repeat the previous exercise for the compound option in Exercise 3.3.

ANSWER: $\mathbb{E}[R] = 11/16$.