MVE550 2020 Lecture 14.1
Review

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Lecture 14

- Lecture 14.1: Quick review of some of the central issues.
- Lecture 14.2: Where to go from here?
- Lecture 14.3: Tips for the exam.

Introduction to Bayesian inference
Compendium Chapter 1

- Basic ideas of Bayesian inference.
- The idea of conjugacy and how to use it in computations.
- Computations using discretizations and numerical integration.

Discrete time discrete state space Markov chains
Dobrow Chapters 2 and 3

- Notation. Basic properties. Basic computation.
- A lot of concepts! (see Lecture 3) Being able to use them.
- Computation with stationary distributions, limiting distributions, absorbing chains, time reversibility, random walks on graphs...

Branching processes
Dobrow Chapter 4

- Basic theory.
- Probability generating functions.
- Computation of the probability of extinction.

Hidden Markov models, more Bayesian inference
Compendium Chapters 2,3,4

- The idea and usage of Hidden Markov Models.
- Inference for parameters of (discrete time discrete state space) Markov chains and HMMs.
- Inference for parameters of branching processes.

MCMC
Dobrow Chapter 5, Compendium Chapter 5

- The Metropolis Hastings algorithm and how to use it. Gibbs sampling.
- MCMC for Bayesian inference.
- Perfect sampling.

Poisson processes
Dobrow Chapter 6

- Definitions.
- Theory and properties.
- Using it as a model for practical computations.

Continuous time discrete state space Markov chains Dobrow Chapter 7

- Theory and basic properties. Limiting behaviour. Absorbing states.
- Using it as a model for practical computations.
- Time reversibility, queueing theory, exponential matrix. . .

Brownian motion
Dobrow Chapter 8

- Basic ideas and properties.
- Doing computations with Brownian motion.
- Variants of Brownian motion. Martingales.

MVE550 2020 Lecture 14.2
Where to go from here

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Stochastic processes

- Infinite collections of random variables. We have only looked at a few examples.
- For example Brownian motion: An entire PhD level course exclusively about Brownian motion is given in reading period 3 by Jeff Steif.
- A number of proofs actually need measure theory, which is not covered in this course.
- For those with an interest, I strongly recommend Chapter 9 of Dobrow, introducing Stochastic Calculus!
- Some related courses of possible interest:
  - MVE170 / MSG800 Basic Stochastic Processes, given by Patrik Albin in reading period 2.
  - MVE140 / MSA150 Foundations of Probability Theory, given by Sergei Zuev in reading period 2.
  - TMS165 / MSA350 Stochastic Analysis, given by Patrik Albin in reading period 1.
  - TMV100 / MMA100 Integration Theory, given by Jeff Steif in reading period 1.
  - MVE330 / MSF200 Stochastic Processes, given by Jakob Bjrnberg in reading period 4 (2022).

Stochastic modelling

- Generally, stochastic modelling means probabilistic modelling of real phenomena, whether the model is a stochastic process or a finite collection of random variables.
- A large number of application areas: Basically any system with uncertainty.
- Stochastic Differential Equations (SDE) and Stochastic Partial Differential Equations (SPDE), see Dobrow Chapter 9 for an introduction. Active area at Mathematical Sciences!

Bayesian inference

- Stochastic models to be used for real applications almost always need fitting (inference) of their parameters using data.
- In Bayesian inference, we specify an entire stochastic model, also for the parameters (using a prior). Then we use for prediction the conditional distribution given the observed data.
- In practice, the main difference in most cases to frequentist inference is that Bayesian inference averages over a posterior for the parameters instead of using a single parameter estimate.
- In this course, we have looked at Bayesian inference for
  - Small toy models
  - Discrete-time discrete state space Markov chains, and Hidden Markov Models (HMM).
  - Branching processes
  - In assignment: Poisson processes and Continuous-time discrete state space Markov chains.
- We have also looked at how to use Markov chains (MCMC) for Bayesian inference.

Bayesian inference

- The intentions in this course:
  - To give a small introduction to Bayesian thinking.
  - Exemplify some simple inference tools for our stochastic processes.
- Bayesian inference is part of many later statistics courses, but I would like to advertise my own course, MVE187 / MSA101 Computational methods for Bayesian inference, in reading period 1. Some subjects covered:
  - More on conjugacy and simple computations.
  - Much more on MCMC.
  - Hamiltonian MCMC.
  - Information theory and the EM algorithm.
  - Graphical models.
  - Variational Bayes.

Computer simulation

- A goal of the course has been to show the close connection between developed theory and computer simulation from models with randomness.
- Not a course in programming, but hopefully you have gained some insight into the R language.
- R is very widely used in many statistics fields.

MVE550 2020 Lecture 14.3
Tips for the exam

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Studying for the exam

- Make sure you have *some* general understanding of *all* parts of the course.
- Make sure you have tried out and played around with the small R codes on Canvas. For me, trying out actual computation of things is very helpful to make them more concrete.
- You don't have to memorise proofs (in particular this year as all aids are allowed) but going through them can be a great way to increase understanding.
- Do old exam questions: NOTE: This years Zoom exam will be similar, except it will not contain questions that can very easily be looked up.
- Don't be afraid to ask if you get stuck (but dont ask too early)! Ask me or Anton or other students.

Practical matters before exam

- To take the exam you need to be signed up. Last date for signing up is Thursday 17 December!
- The exam will be a Zoom monitored exam. You should have received information about how such exams are done (and probably you have some experience). In summary:
  - Questions are found as an assignment in a *special* Canvas room, created just for the exam. This is also where you submit your answers.
  - Connect to Zoom early for ID control. Then: 4 hours of work followed by 30 minutes of scanning your answers.
  - Exam guards will check your ID, monitor that you follow rules, etc.
  - If you have questions for me, you can chat with the exam guards, and they will put you in a breakout room with me.
  - If there are technical breakdowns that cannot be solved with the exam guards, don't hesitate to phone me (number 031-772-3579, also listed on the exam paper with the questions) or mail me.

Working with the exam questions

- All aids are allowed, except communicating with somebody else.
- Reference your source! Not necessary if your source is part of the course study material listed on Canvas. If you use any other material, it must be referenced.
- Compared to pre-pandemic exams, this one will *not* contain questions that you can easily look up. Otherwise, it will be quite similar.
- Regarding computation: Previously in this course, students have been allowed to use Chalmers-approved calculators. NOW: I *allow* you to do computations with R, or any other computational tool, but I will not *require* you to do such computations. But you should have access to at least a calculator.
- I may however ask you to write down how something may be computed with R. (If you cannot remember the exact syntax just write clearly what you intend to compute).

General exam tips

- Write clearly, and *precisely*, with full sentences. If I'm not sure what you mean, I will *not* necessarily make the interpretation that is most in your interest.
- As long as you are clear and cover everything you want to say, writing short is better than writing long.
- Make sure you answer all parts of a question, and answer exactly what is asked for!
- Make sure you attempt to answer all questions: I cannot give you any points for a question if you do not answer anything.
- Distribute your time wisely.

Assignments

- The second assignment has now been graded by Anton. The third has deadline Thursday 17 December, and will be graded by me within two *work-weeks*, so probably in January.
- If what you have handed in by the deadlines is not enough to pass you on the assignments, you will have opportunities to improve your answers *before* the exam. For all our sakes, *get this done now, don't wait!*.
- Note that passing the assignments and passing the exam are separate items in Ladok: They can in principle be done in different years.
- Please ask me any remaining questions by mail, or at the course meetings!