Questions for breakout rooms

(a) Complete the proof by induction in the computation of $\mathrm{Var}\left(Z_n\right)$ below: Remember from lecture 6.1:

- Using the notation $\mu = \mathrm{E}\left(X_i\right)$ and $\sigma^2 = \mathrm{Var}\left(X_i\right)$, we get

$$\begin{aligned} \mathrm{Var}\left(Z_n\right) &= \mathrm{Var}\left(\mathrm{E}\left(Z_n \mid Z_{n-1}\right)\right) + \mathrm{E}\left(\mathrm{Var}\left(Z_n \mid Z_{n-1}\right)\right) \\ &= \mathrm{Var}\left(\mu Z_{n-1}\right) + \mathrm{E}\left(\sigma^2 Z_{n-1}\right) \\ &= \mu^2 \mathrm{Var}\left(Z_{n-1}\right) + \sigma^2 \mu^{n-1} \end{aligned}$$

- From this we prove by induction

$$\mathrm{Var}\left(Z_n\right) = \sigma^2 \mu^{n-1} \sum_{k=0}^{n-1} \mu^k = \begin{cases} n\sigma^2 & \text{if } \mu = 1 \\ \sigma^2 \mu^{n-1}(\mu^n - 1)/(\mu - 1) & \text{if } \mu \neq 1 \end{cases}$$

(b) If the offspring distribution is a Poisson distribution with parameter $\lambda = 2$, what is the expectation and variance of Z_3, the size of the third generation? (You may look up the properties of the Poisson).