

# Wave Energy: A Brief Introduction

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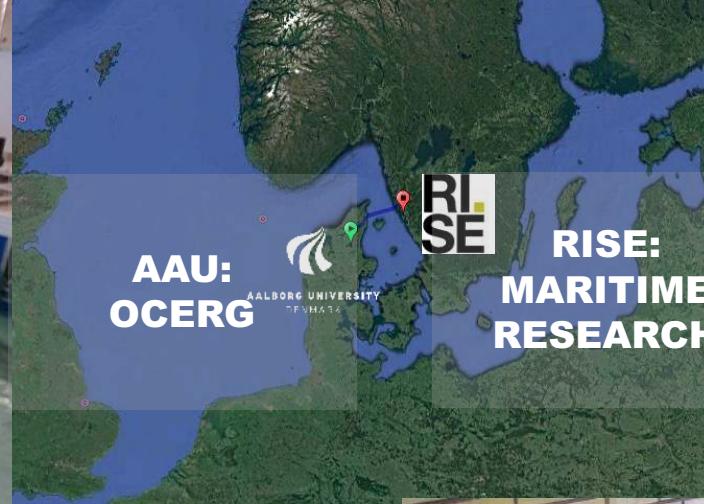
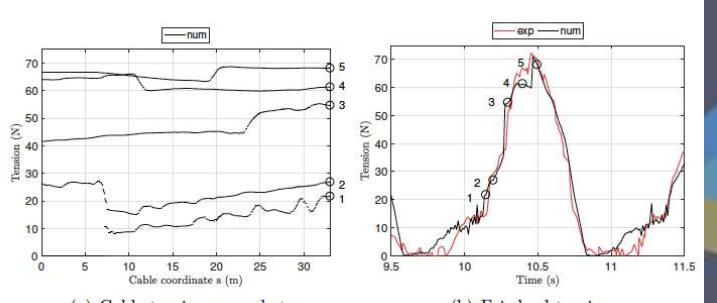
From: [waves4power.com](http://waves4power.com)

GOTHENBURG, 1 OCTOBER 2020

## WAVE BASIN

- Wave generator: 13 x 1.5 m (width and height)
- 30 individually controlled wave paddles
- Accurate generation of 3D waves up to 45 cm (at 3 s period) wave height
- Max significant wave height in the range of 0.25-0.30 m
- Current in the basin (up to 0.15 m/s at 0.5 m water depth)
- Access possible through MarineNet2

## WEPTOS WEC



## W4P FULLSCALE



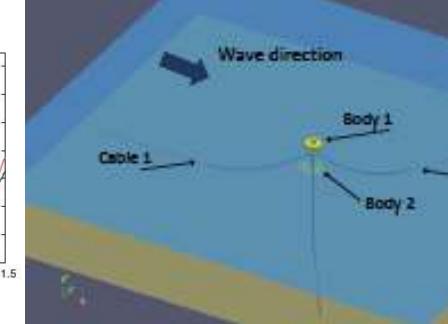
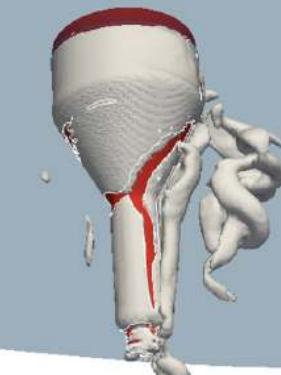
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## TETRASPAR



## BIOFAULING



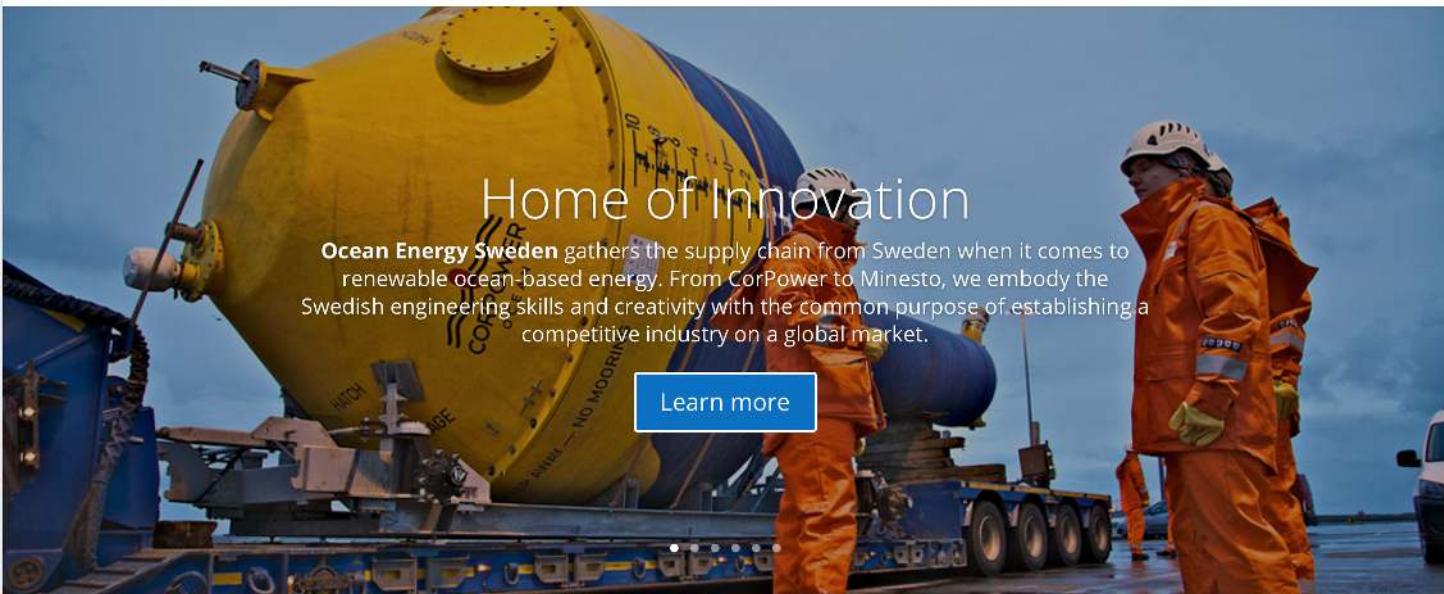


# Ocean Energy Sweden



Ocean Energy Sweden

About Value Chain Events Communication Contact



## Home of Innovation

**Ocean Energy Sweden** gathers the supply chain from Sweden when it comes to renewable ocean-based energy. From CorPower to Minesto, we embody the Swedish engineering skills and creativity with the common purpose of establishing a competitive industry on a global market.

[Learn more](#)

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### OCEAN HARVESTING

Ocean Harvesting Technologies AB develops the novel InfinityWEC, using a flywheel based kinetic energy recovery system to provide advanced force control and constant power output.

[Learn more](#)



### WAVES4POWER

Waves4Power is a Swedish company that develops and sells wave energy systems. Waves4Power's concept is to focus on profitable electricity production with a system that is survivable and easily serviceable.

[Learn more](#)



### MINESTO

Develops a new concept for tidal power plants called Deep Green. The power plant is applicable in areas where no other known technology can operate cost effectively due to its unique ability to operate in low velocities.

[Learn more](#)



### SEABASED

Seabased wave energy plants are complete systems, which has been verified through the grid connection in 2015 of the Sotenäs Wave Power Plant and subsequent generation of power to the Nordic Electricity Grid.

[Learn more](#)



### CORPOWER OCEAN

CorPower Ocean AB has developed a compact high-efficiency Wave Energy Converter. Thanks to patented control technology, the buoys operate in resonance over a wide range of waves frequencies.

[Learn more](#)



### jabeEnergy

Develops jabePower Rig which is based on a hydrokinetic Run on River rig-technic (RoR). It is a vertical and mobile turbine, requires no fixed construction or dam facilities, focused on OFF-GRID or totally "unplugged" areas.

[Learn more](#)



### NoviOcean

Is developing the world's largest and strongest wave power unit. The estimated output is up to 10MW per unit, as well as being very light in relation to output. It has an unusual simple and strong PTO, which also facilitates latching.

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UNIVERSITET**

**RISE**

**TEKNISKA HÖGSKOLEN  
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**KTH  
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OCH KONST**

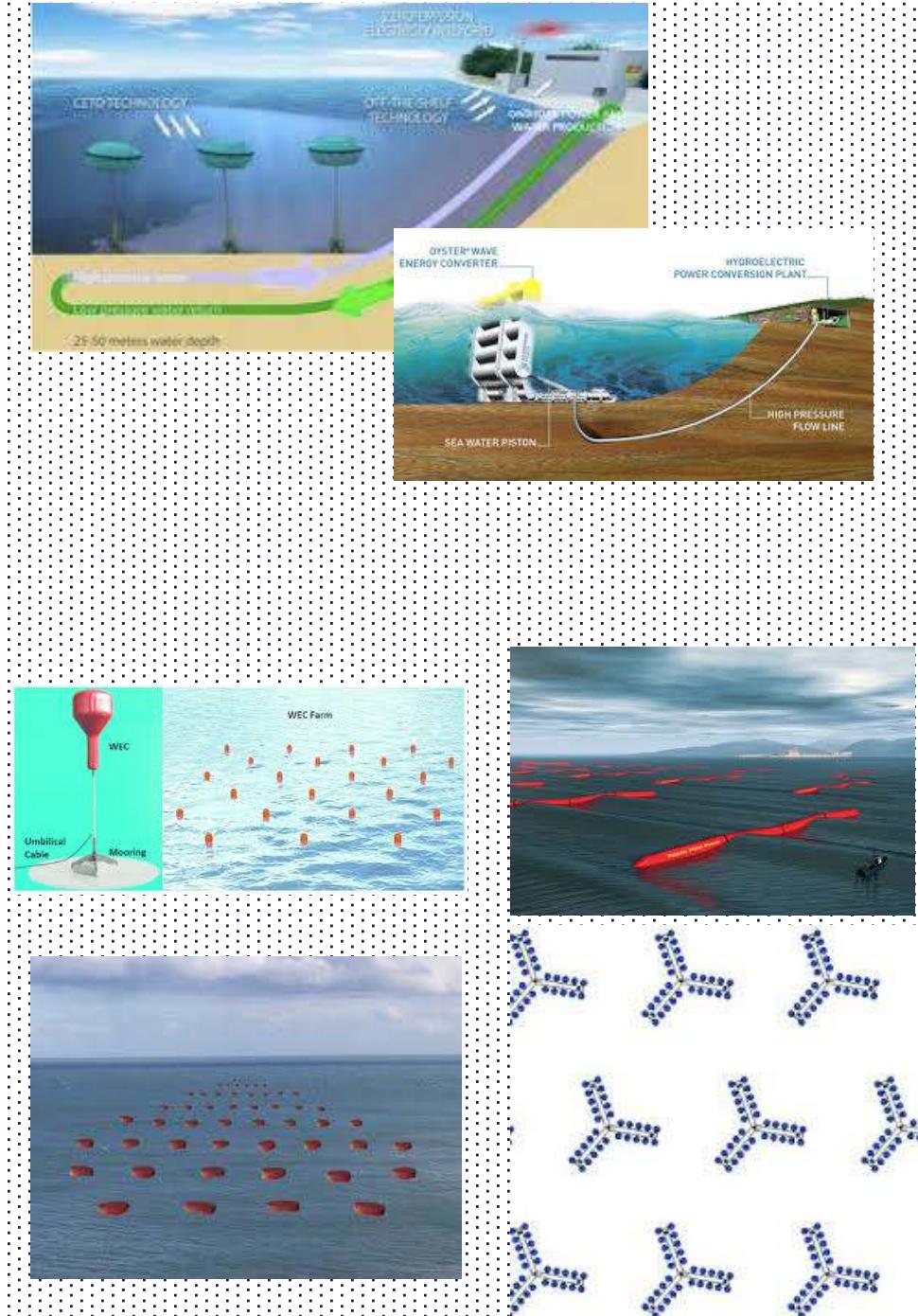
**SSPA**

# OUTLINE

- ⦿ Overview of wave energy – principles
- ⦿ The resource – background on waves
- ⦿ BREAK
- ⦿ Buoy motion
- ⦿ Power and control



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# MARINE ENERGY

Marine energy is diverse (OES definition)

## Tidal/ocean currents



GE Oceande 1.4MW  
From: [www.gerenewableenergy.com](http://www.gerenewableenergy.com)

## Wave energy



Wave Dragon sea trial at Nissum Bredning  
Photo: Wave Dragon AS

## Salinity power



Statkraft Osmotic Power Station (Tofte, Oslo Fjord, Norway)  
(From: OES - An international vision for ocean energy)

## Tidal range



Sihwa Lake Tidal Power Station, Korea  
(From: OES - An international vision for ocean energy)

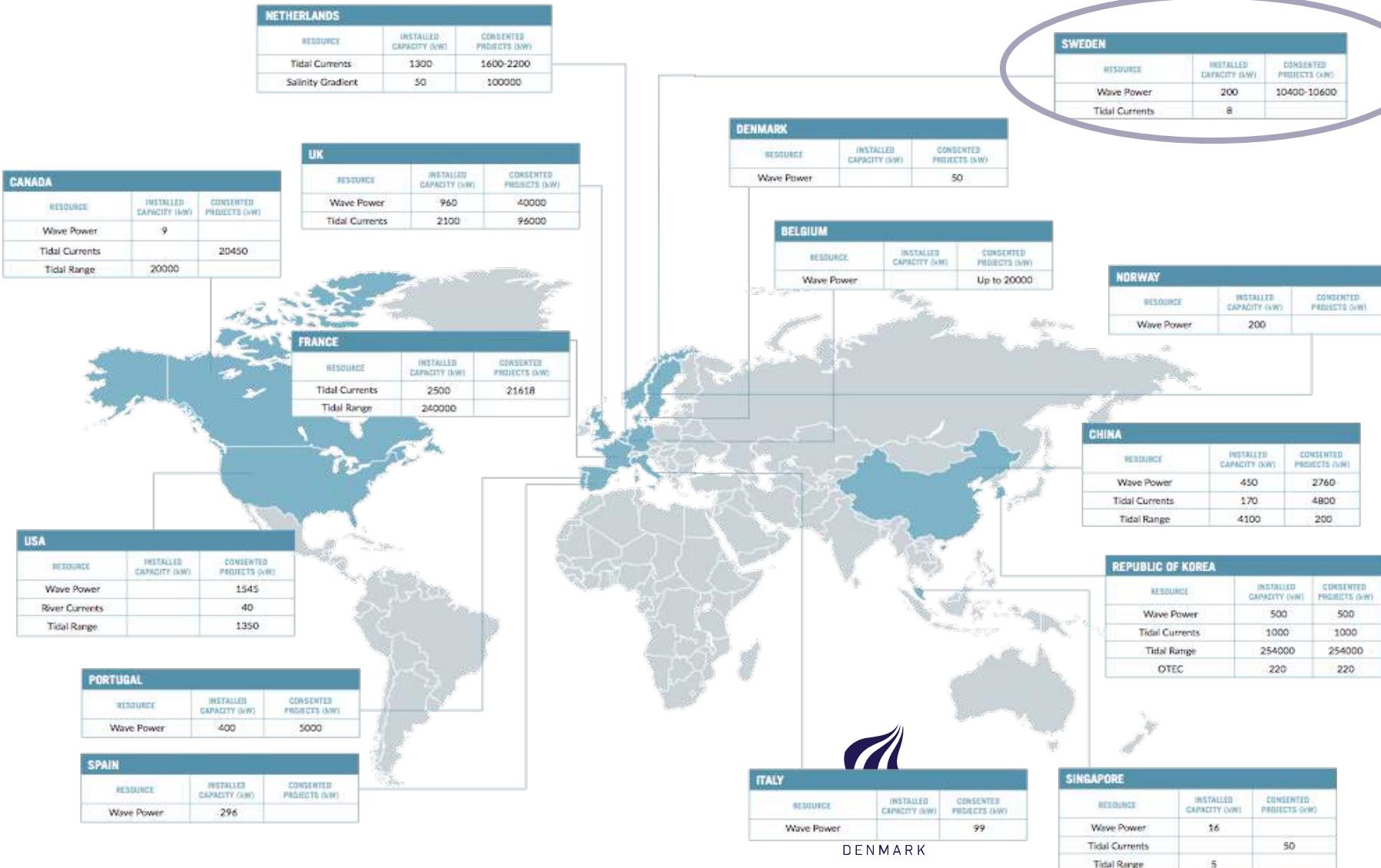
## Ocean thermal energy



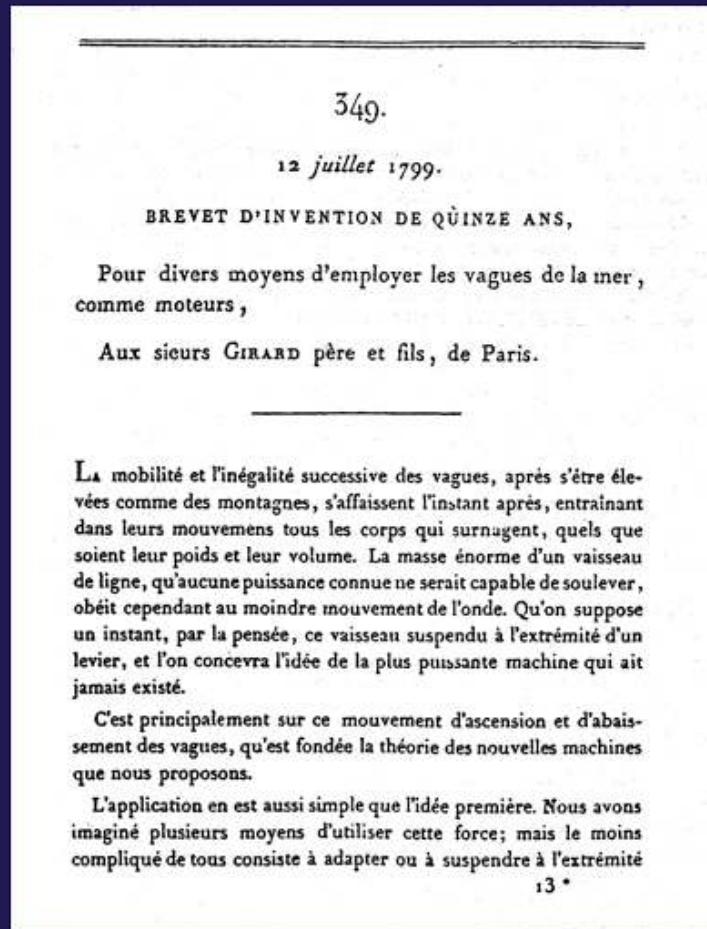
Onshore OTEC in Hawaii  
From: [www.makai.com](http://www.makai.com)

## WORLDWIDE OCEAN POWER INSTALLED CAPACITY

**EU goals of 2050 : 100 GW marine energy in EU**  
**Compare to 2015 wind: 161 GW in EU**  
**Current status (2015) : 0.5 GW marine energy worldwide**  
**out of which 12 MW is not tidal range...**



# WAVE ENERGY IS AN OLD IDEA...



# SALTHERS DUCK

## Wave power

S. H. Salter

Bionics Research Laboratory, University of Edinburgh, Edinburgh EH1 2QL, UK

*Solar energy is one form of income on which we can afford to live. Here is another proposal: the use of power from the waves at sea.*

The amount of power in a wave train can be estimated by calculating the change of potential energy as the water in a wave above sea level falls into the trough in front of the wave (Fig. 1). If the sea water has a density  $\rho$ , and the width of the wave front is  $W$ , then the mass of water in the half sinusoid above sea

©1974 Nature Publishing Group

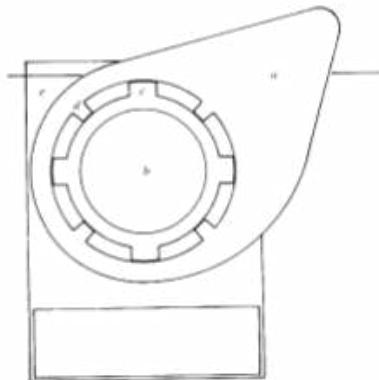
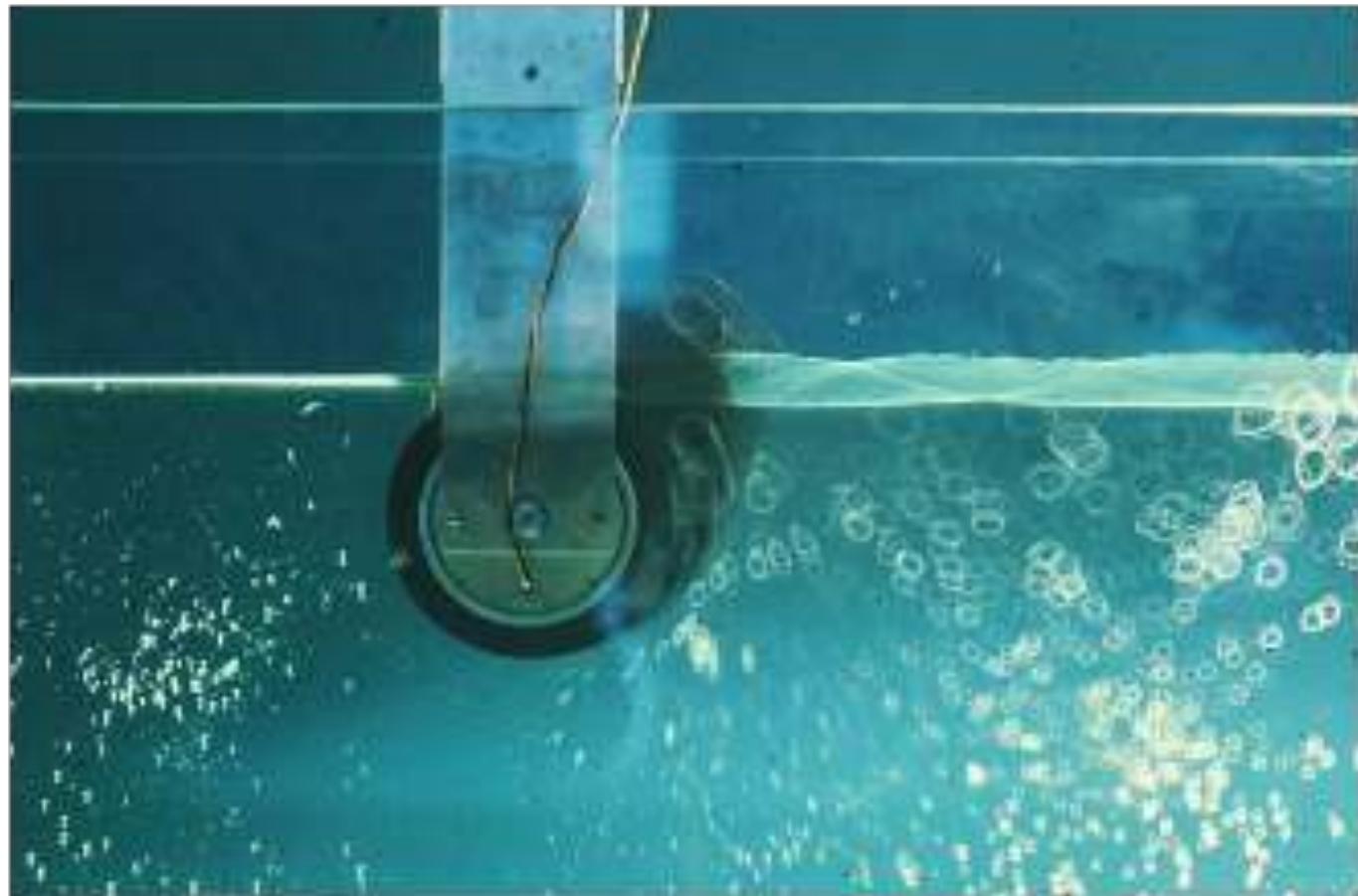


Fig. 5. A vertical section through vane and spline pump. *a*, vane; *b*, a hollow cylindrical member; *c*, paraxial ridges; *d*, inward facing ridges on the vane; *e*, vertical fin between this vane and the next. For the North Atlantic the diameter of the cylindrical portion will be between 10 and 20 m.



# WHY NOT COMMERCIAL?

## COSTLY

LCOE is high but a few tech companies are on the SET plan track

Energy source	LCoE [€/kWh]	Reference
Coal (Germany)	0.06 – 0.10	[4]
Onshore wind (Germany)	0.04 – 0.08	[4]
Offshore wind (Germany)	0.08 – 0.14	[4]
Solar photovoltaic (Germany)	0.04 – 0.12	[4]
Wave estimate (UK)	0.17	[5]

- REPETED FAILURES

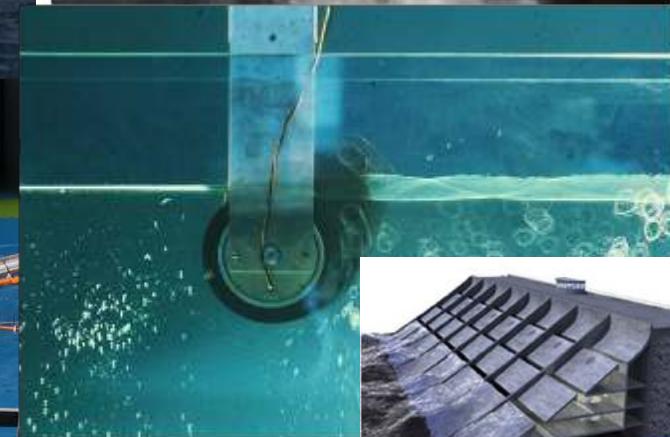
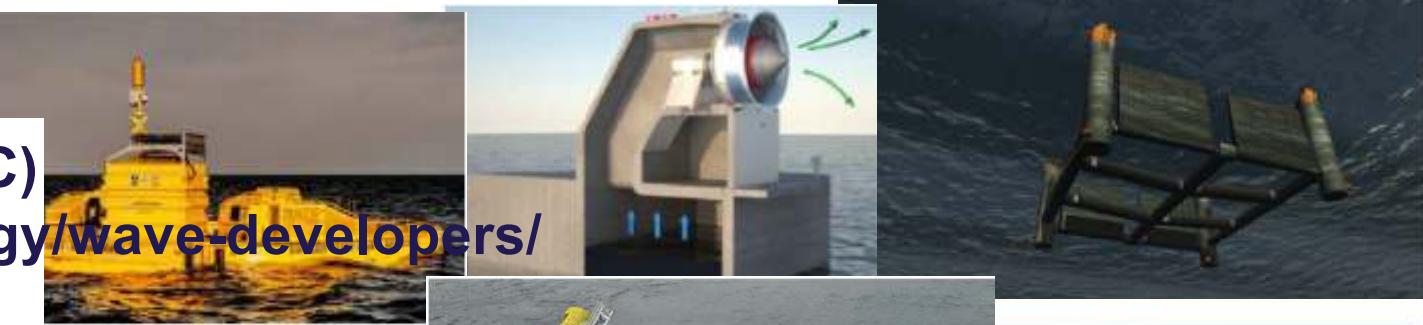
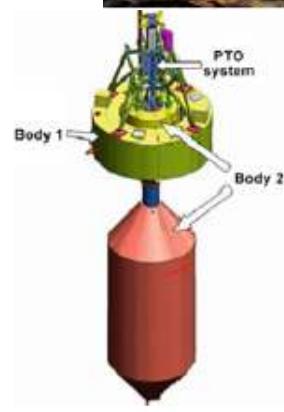
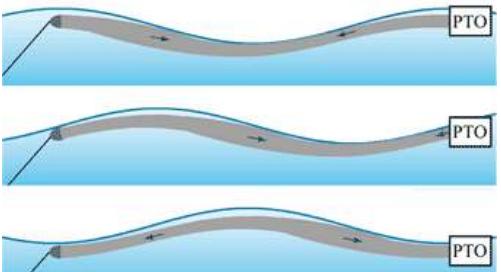


## DISPERSED DESIGNS

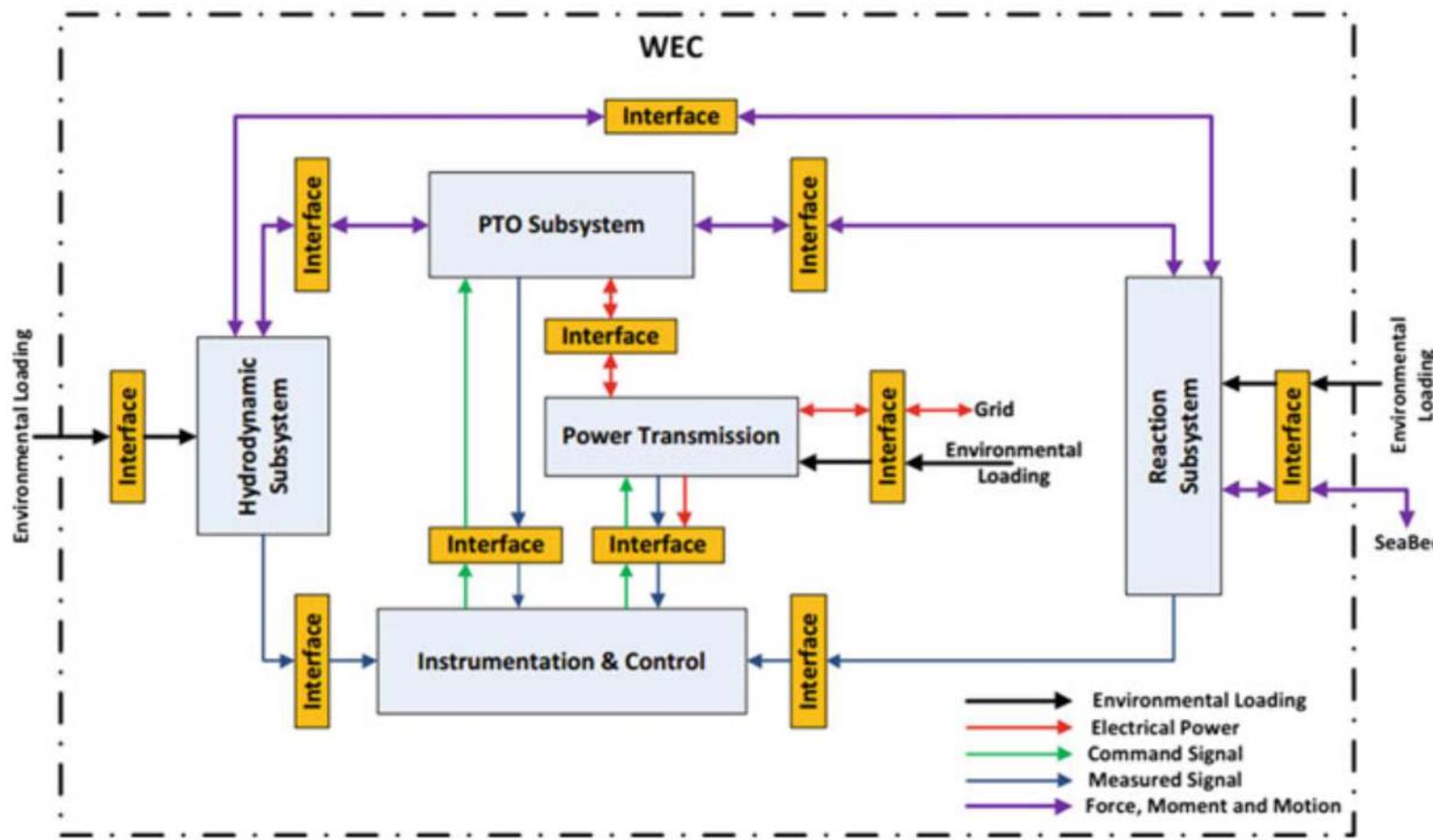
Over 200 concepts world wide (EMEC)

<http://www.emec.org.uk/marine-energy/wave-developers/>

1000s of patents filed



# WEC SUBSYSTEMS

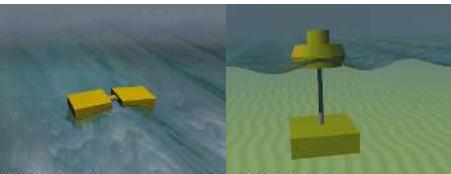


# WEC CATEGORIES



## Oscillating water columns

- Fixed structure
  - Isolated: *Pico*
  - In breakwater: *Mutriku*
- Floating: *Oceanlinx, Leancon*



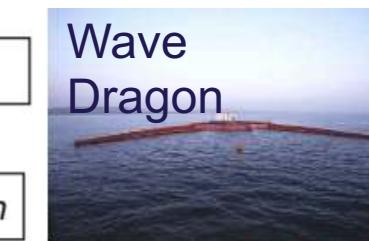
## Wave activated bodies

- Floating
  - Essentially translation: *Wavebob, OPTs PowerBuoy, SeaBased, Fred Olsen's Lifesaver*
  - Essentially rotation: *Pelamis, Crestwing, Dexa, Wavestar, FPP, Weptos*
- Submerged
  - Essentially translation (heave): *Carnegies CETO*
  - Rotation (bottom-hinged): *WaveRoller, Oyster, RME,*



## Overtopping

- Fixed structure (without concentration): *SSG*
- Floating structure (with concentration): *Wave Dragon*



# Waves4Power

## Göteborg 2010

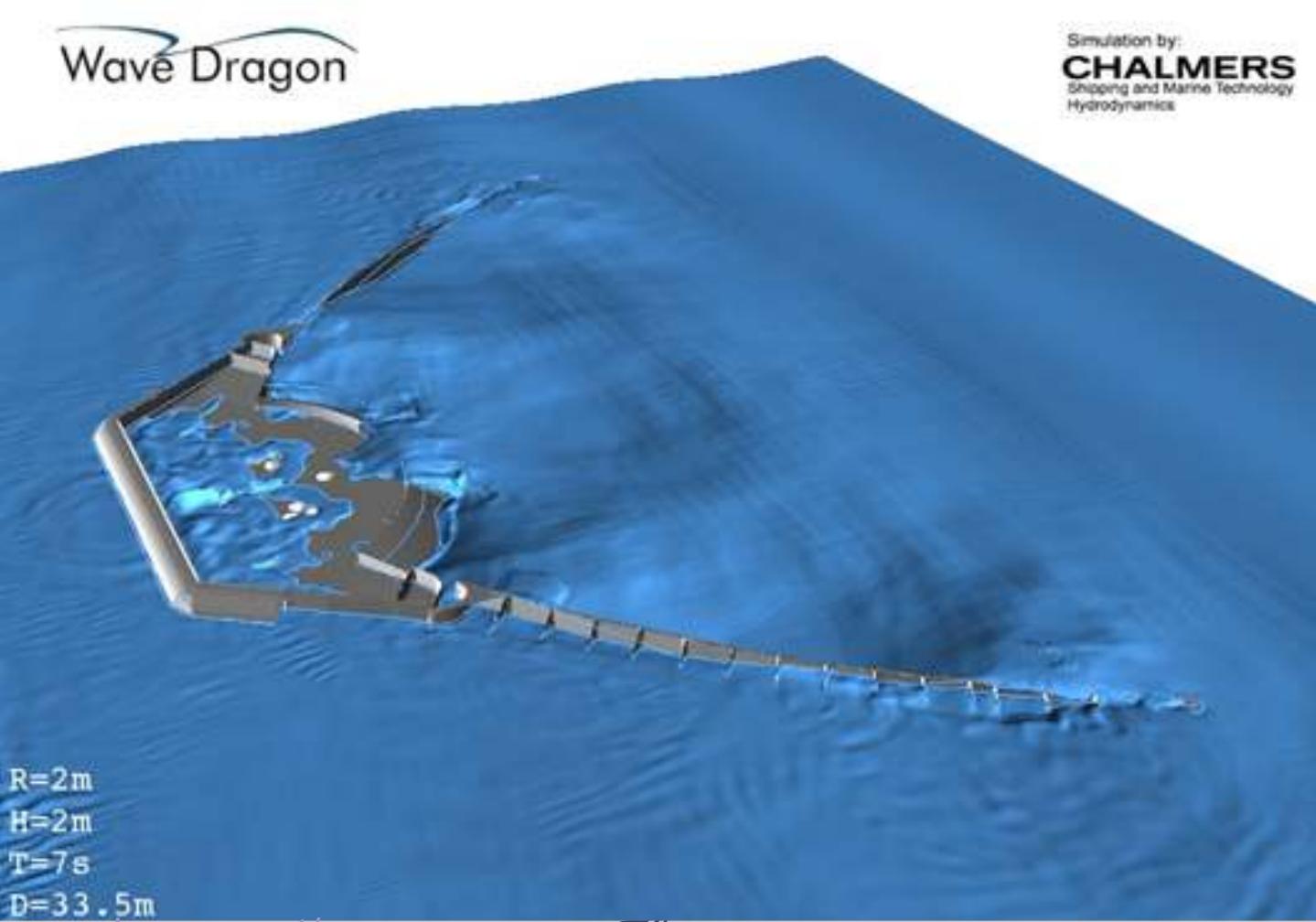


RUNDE  
2017

# Wave Dragon



# WEC Modelling Using OpenFOAM



<http://www.wavedragon.net/>



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# Oyster 1

<http://www.aquamarinepower.com/>

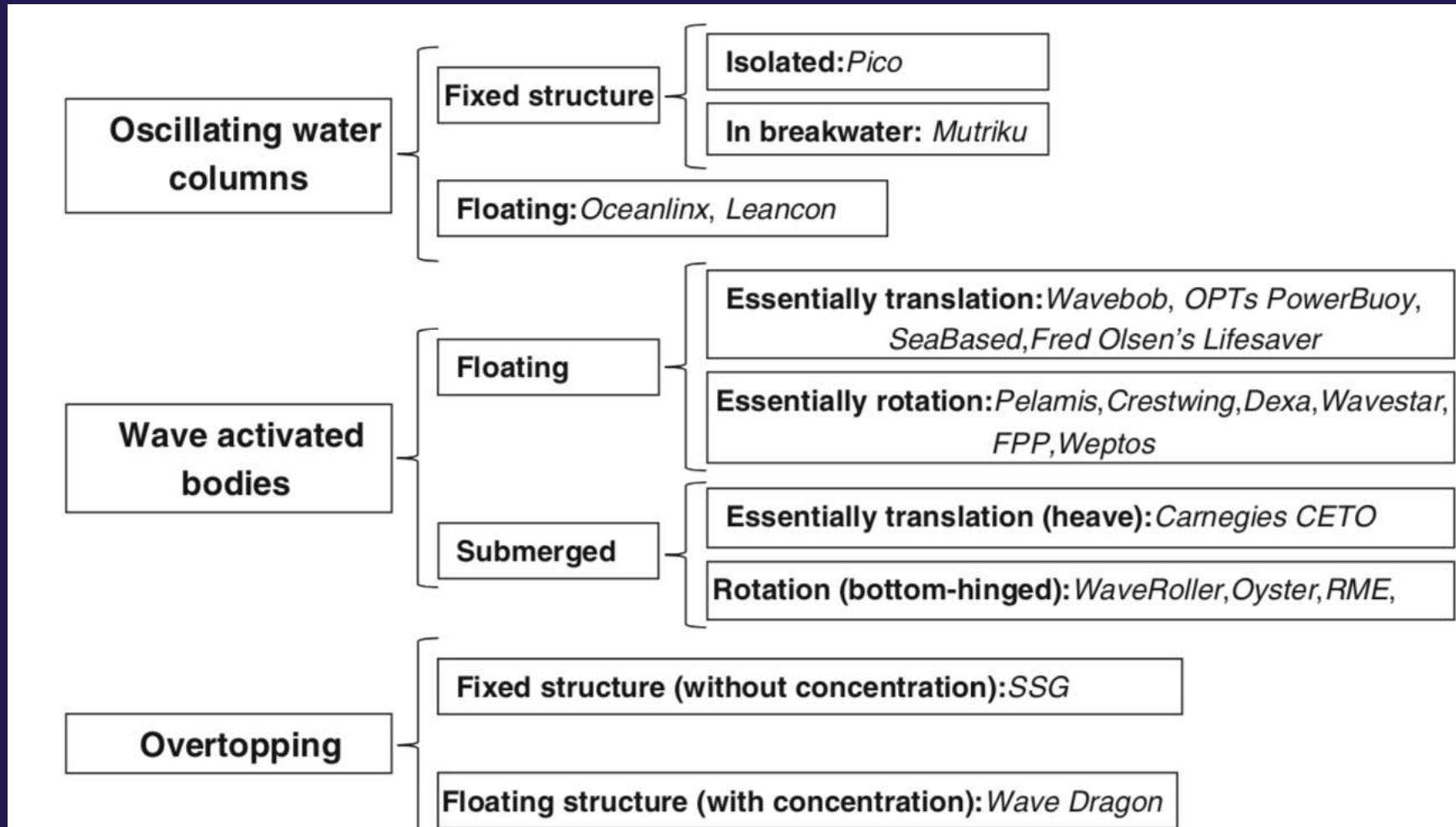


# Oyster 800, June 2012 at EMEC



# THINK-SHARE-PAIR 3MIN

- ⦿ WHICH TECHNOLOGY DO YOU THINK HAS BEEN MOST SUCCESSFUL?
- ⦿ WHY DO YOU THINK SO?

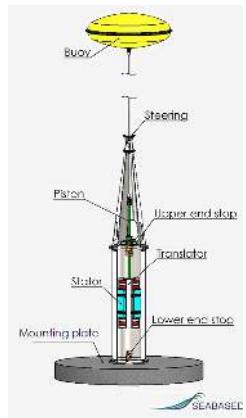


# TECHNOLOGY CONVERGENCE

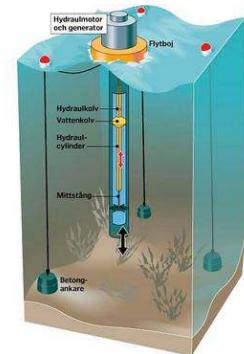
**SWEDEN - exciting times!**  
Technology convergence towards point absorbers

19

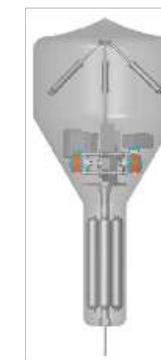
**Seabased**



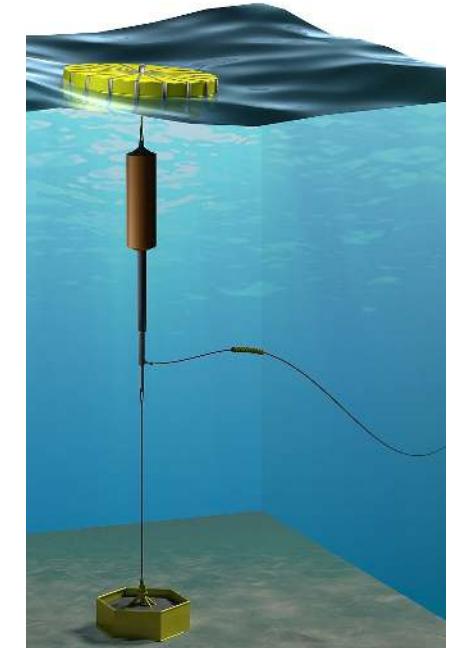
**Waves4Power**



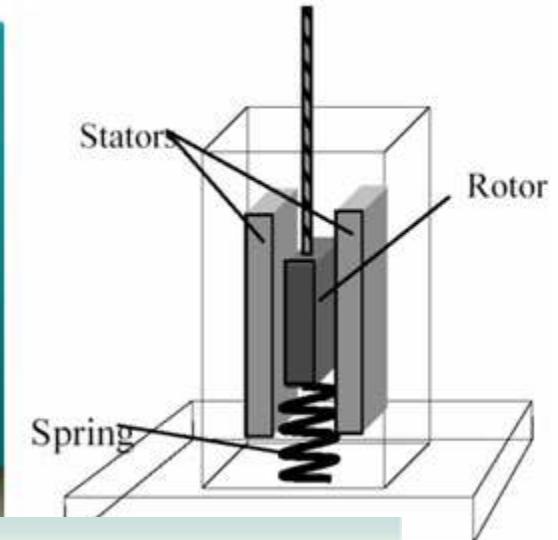
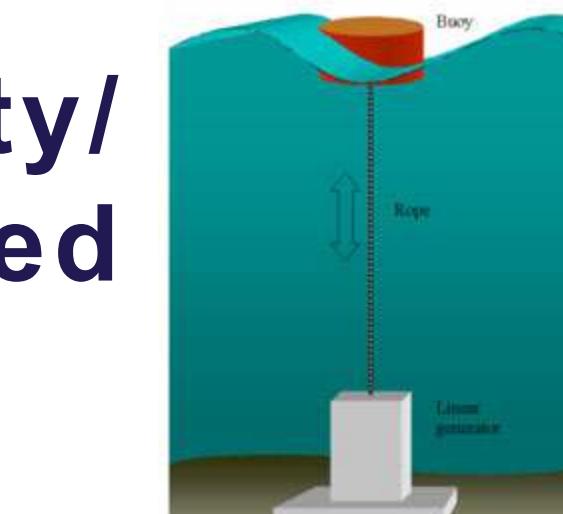
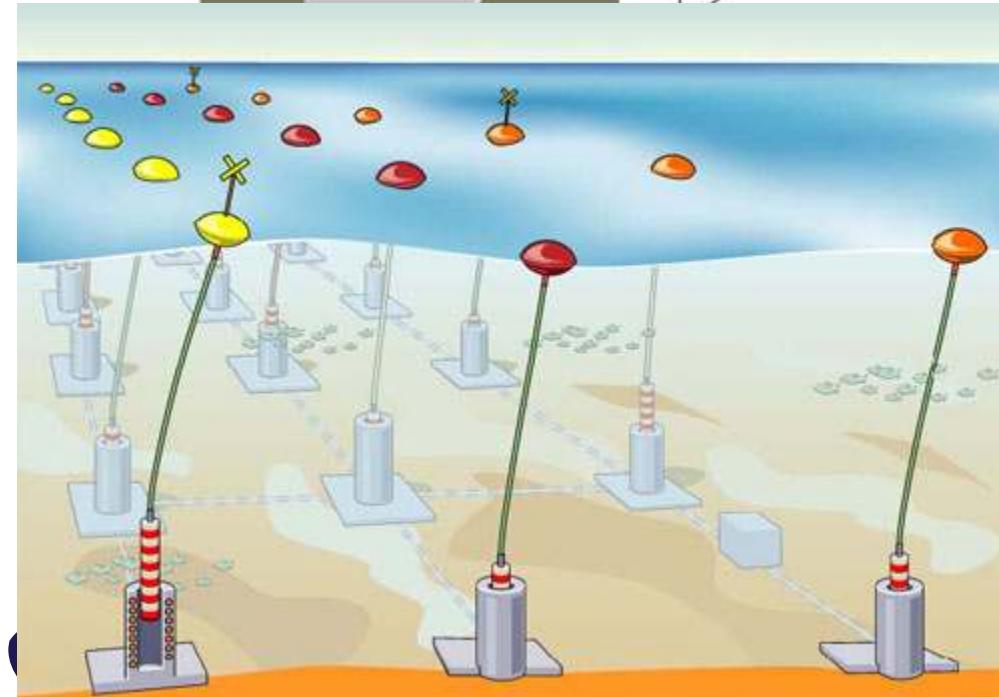
**CorPower**



**Ocean Harvesting**



# Uppsala University/ Seabased



Seabased / Fortum  
Permission to build  
24 June 2010 (M 3086-09).  
EU Permit Nov 2011  
Contract 2011-12-09

Data:

Max 420 units  
First phase 42 units  
Installed power 10 MW  
Area 0.5 km<sup>2</sup>  
Generator height 10 m  
Base area 35 m<sup>2</sup>  
Design life time 20 yrs  
Prod. 25 GWh/yr (28 %)



# The IPS Buoy Elskling at Gamla Gumman, Göteborg 1980 - 1981

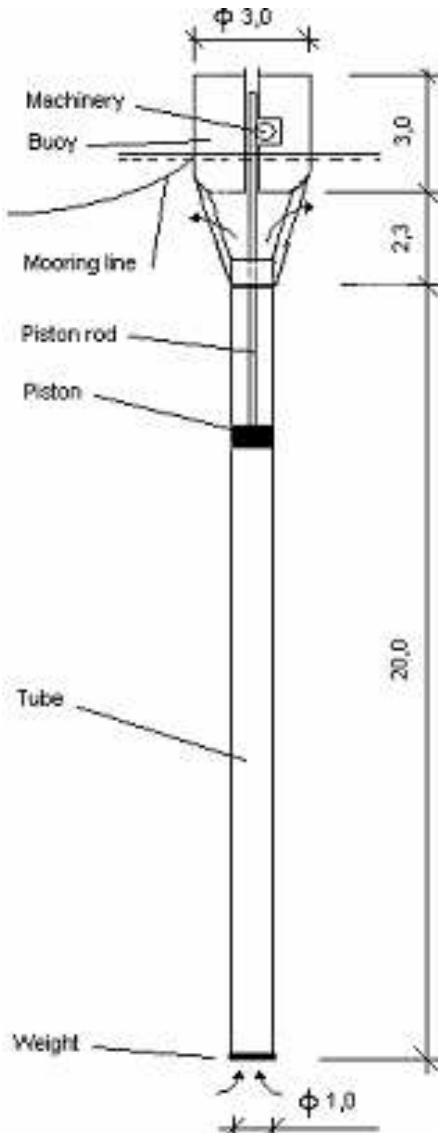


Figure 4.1 The IPS-Buoy Elskling I

# Waves4Power

## Göteborg 2010



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# Power-take-off machinery

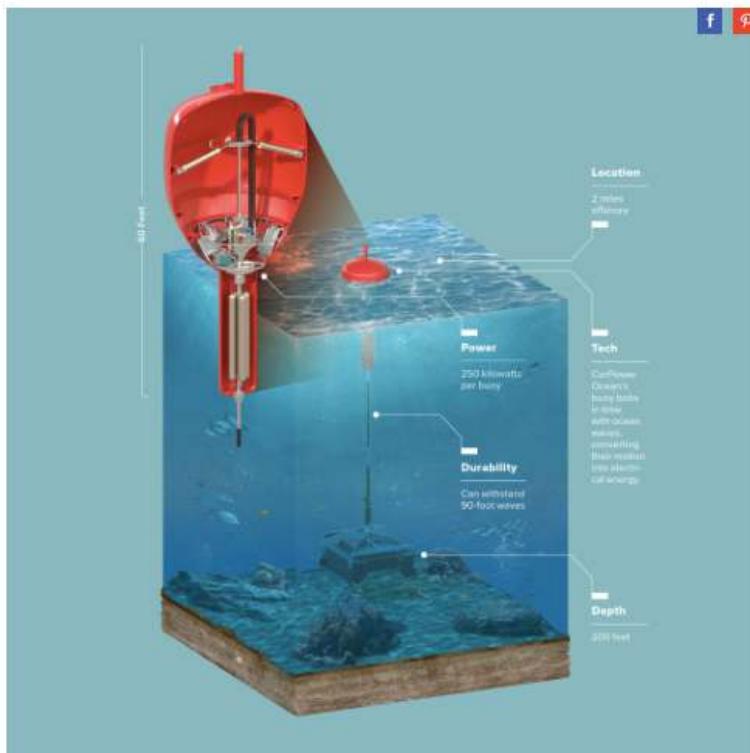
## Hydraulic motor, generator and ackumulator



# CORPOWER: PASSIVE CONTROL

RACHEL NUWER MAGAZINE 04.06.16 4:10 PM

## THE WONDER-BUOY THAT MAY FINALLY MAKE WAVE ENERGY FEASIBLE



5 times higher Annual Energy / mass compared to conventional point absorbers without phase control. [MWh/ton]

3 times higher Annual Energy / PTO Force compared to conventional point absorbers without phase control. [MWh / kN]

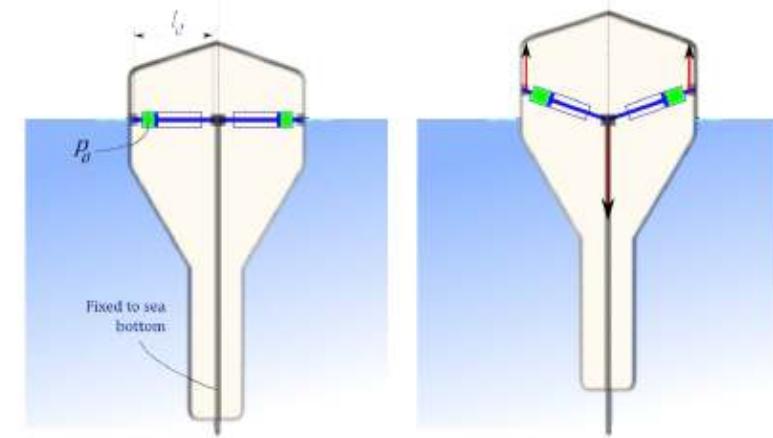
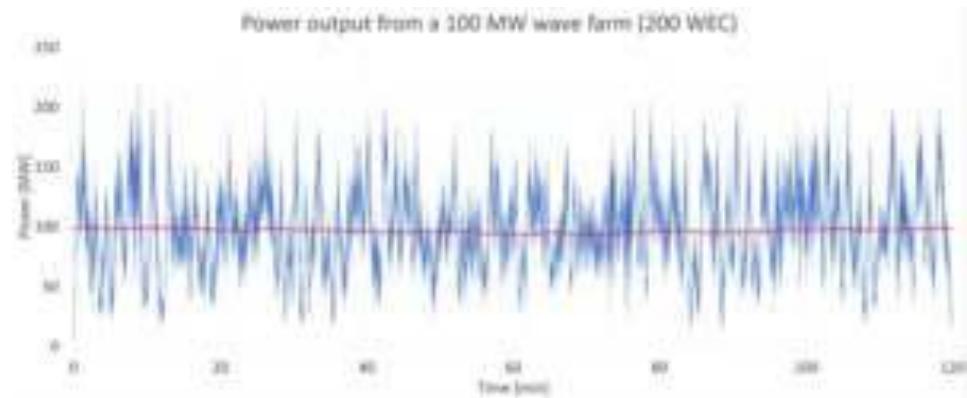
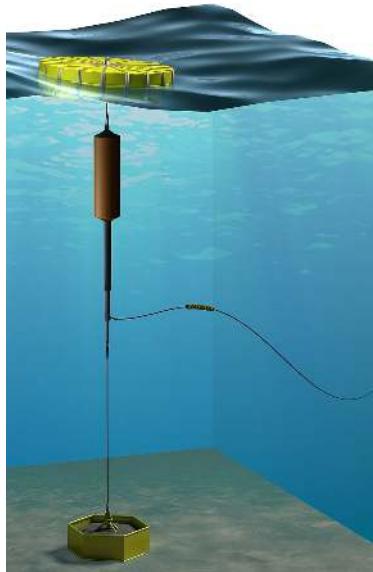


Fig. 5. Illustration of how the buoy displacement induces a vertical spring force with negative stiffness. Buoy in mean vertical position (left) and displaced upwards (right).



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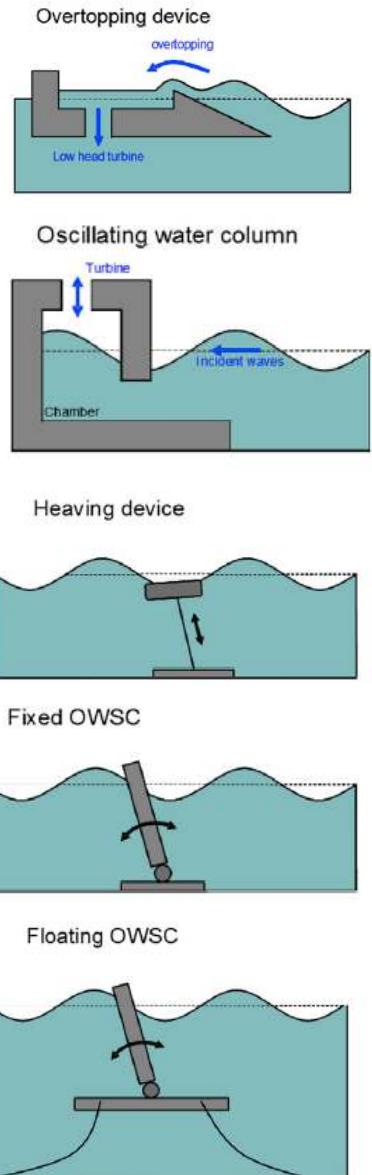
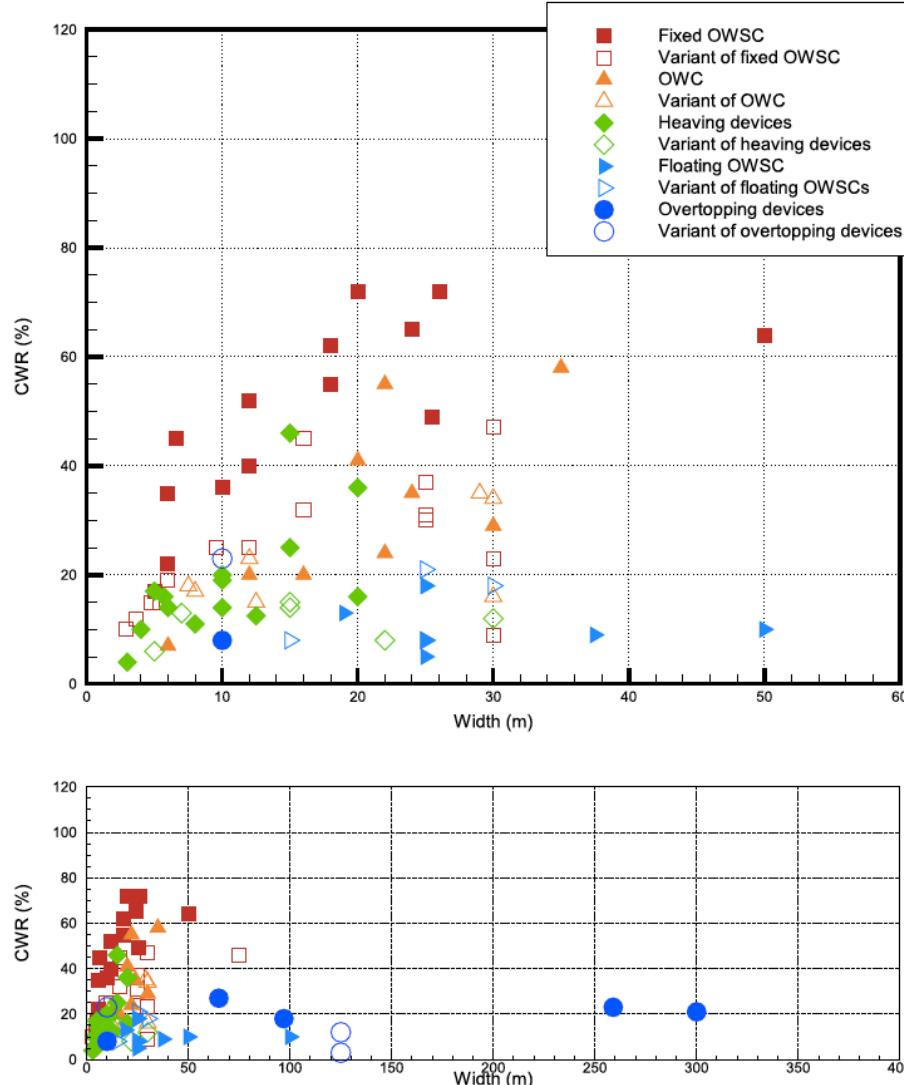
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# Why point absorbers? Have they better efficiency?

A. Babarit / Renewable Energy 80 (2015) 610–628

$CWR = P/JD = \text{capture width ratio} = \frac{\text{absorbed power}}{\text{wave resource}} / \text{characteristic dimension of device}$

- 158 estimates of CWR
- Fixed OWSCs most efficient.
- Heaving devices and OWCs medium
- Floating OWSCs and overtopping devices least efficient devices



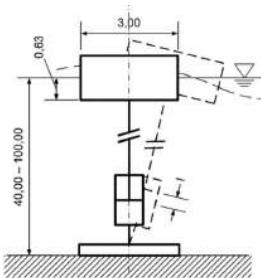


Fig. 1. Components of the Seahorse WEC.

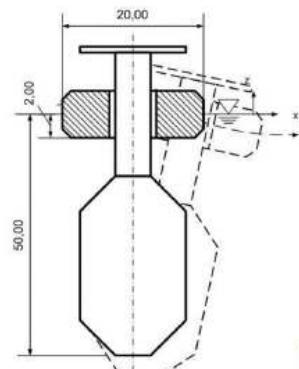
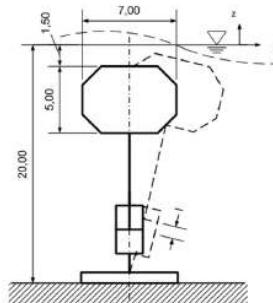


Fig. 3. 1/4 scale model of the Wavebob at sea.

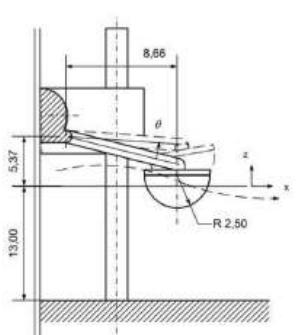


Fig. 4. Picture of the test section of the Wavestar WEC.

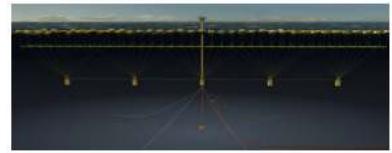
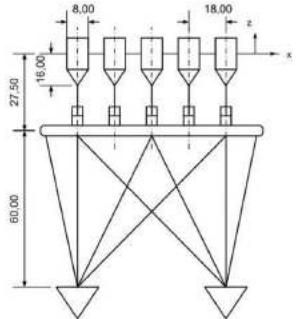


Fig. 5. Artistic view of the Pontoon Power Converter WEC.

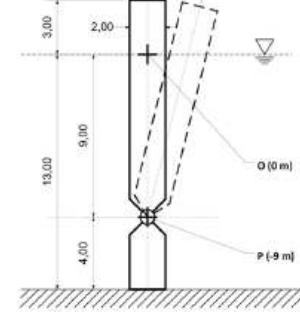


Fig. 6. Photo of the OceanWEC energy converter.

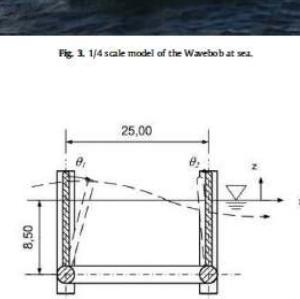
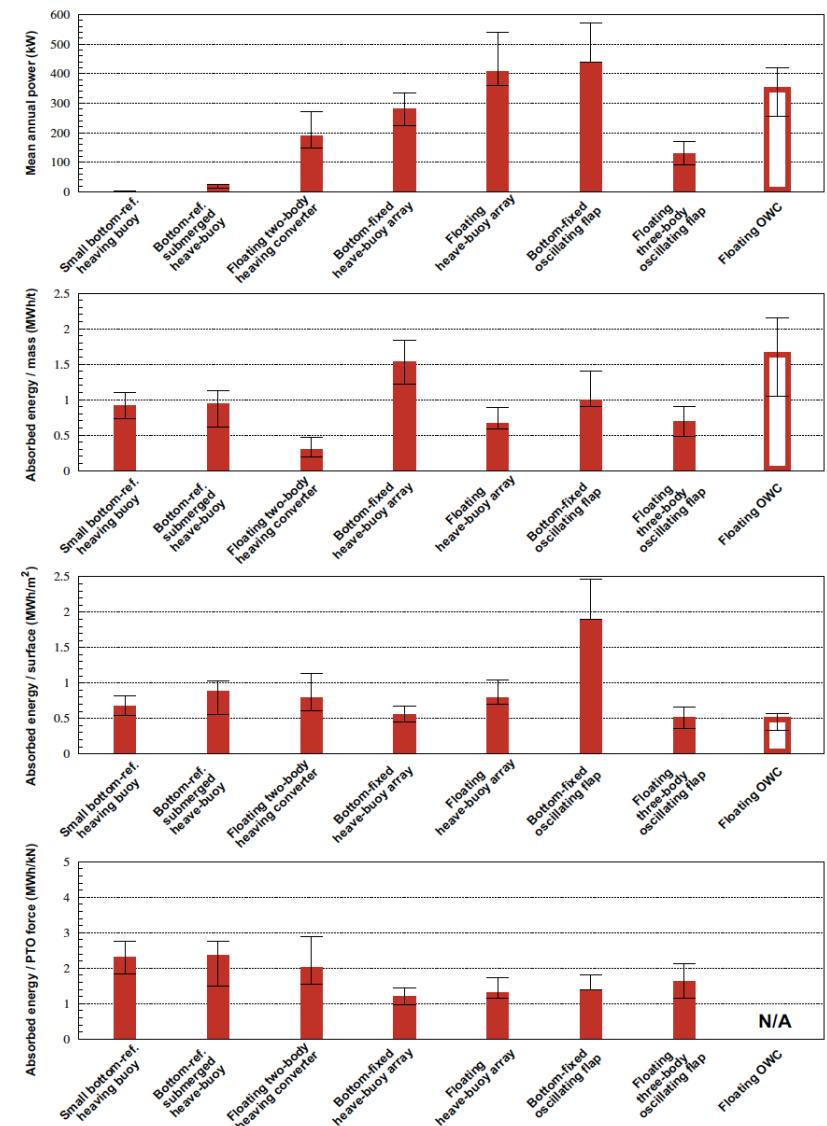
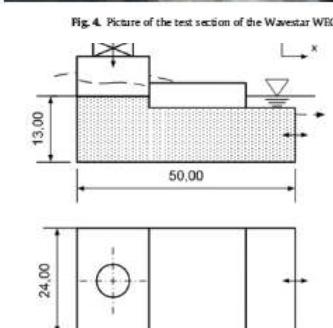
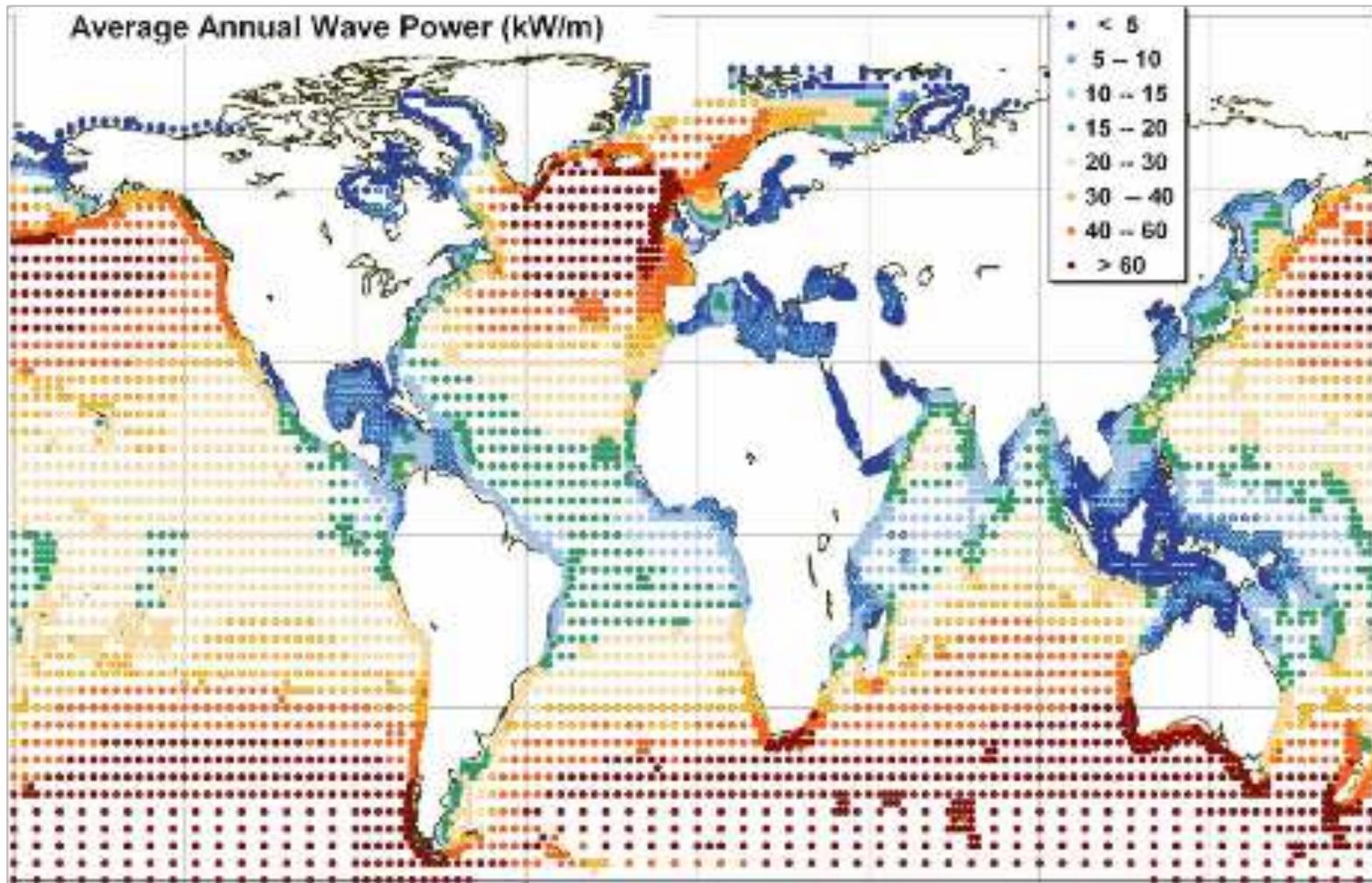
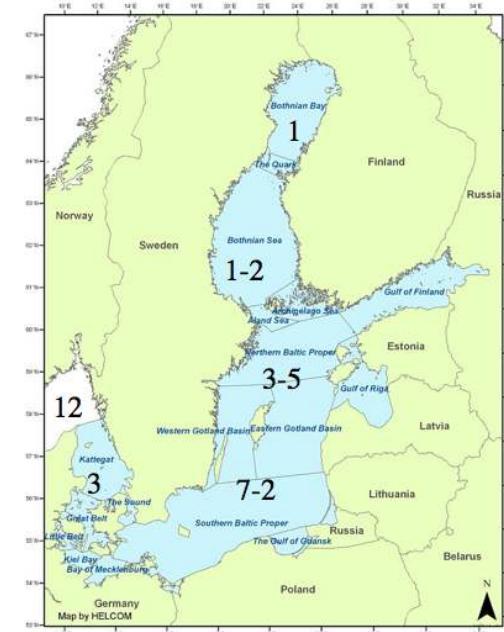
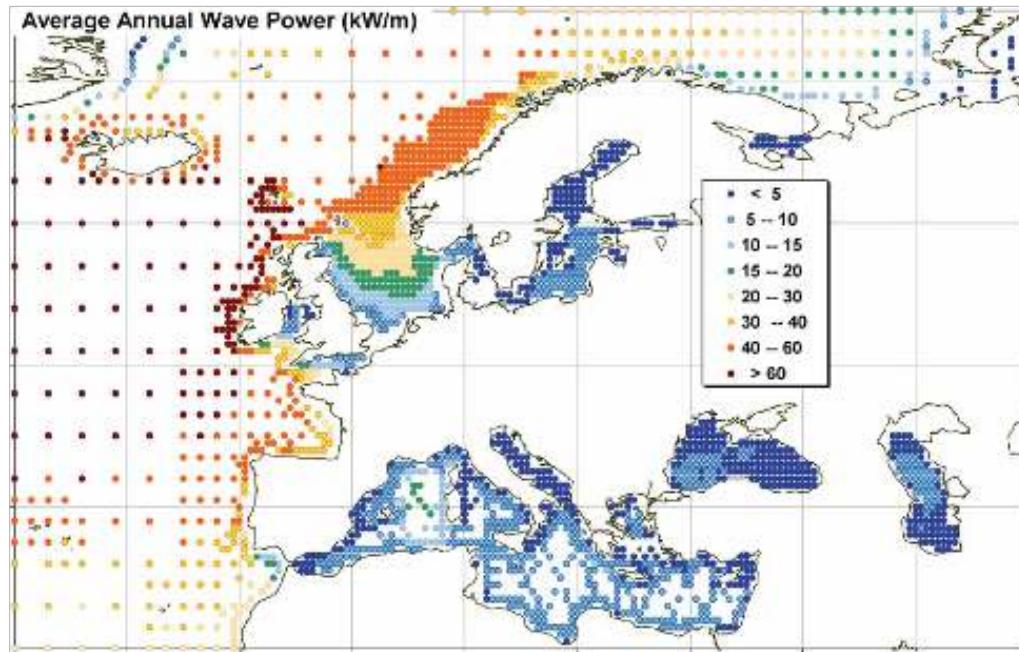


Fig. 7. Artistic view of the Langene WEC.



# RESOURCE

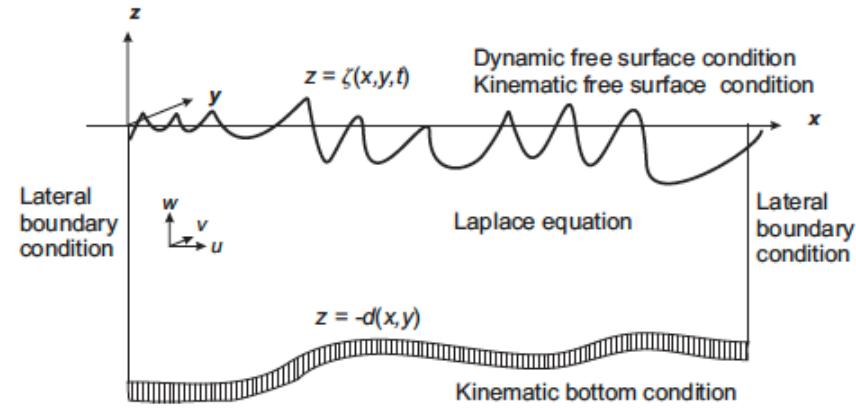




**Table 6.1 | Regional theoretical potential of wave energy (Mørk et al., 2010).**

REGION	Wave Energy TWh/yr (EJ/yr)
Western and Northern Europe	2,800 (10.1)
Mediterranean Sea and Atlantic Archipelagos (Azores, Cape Verde, Canaries)	1,300 (4.7)
North America and Greenland	4,000 (14.4)
Central America	1,500 (5.4)
South America	4,600 (16.6)
Africa	3,500 (12.6)
Asia	6,200 (22.3)
Australia, New Zealand and Pacific Islands	5,600 (20.2)
<b>TOTAL</b>	<b>29,500 (106.2)</b>

# WATER WAVES



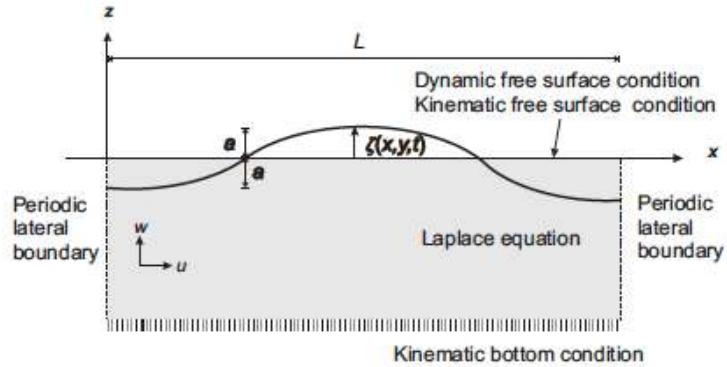
$$\bar{\nabla}^2 \phi + \partial_{zz} \phi = 0 \quad -d < z < \zeta$$

$$g\zeta + \partial_t \phi + \frac{1}{2} \left( \bar{\nabla} \phi \cdot \bar{\nabla} \phi + (\partial_z \phi)^2 \right) = 0 \quad z = \zeta$$

$$\partial_t \zeta + \bar{\nabla} \phi \cdot \bar{\nabla} \zeta - \partial_z \phi = 0 \quad z = \zeta$$

$$\partial_z \phi + \bar{\nabla} \phi \cdot \bar{\nabla} d = 0 \quad z = -d$$

# WATER WAVES: LINEAR



In dimensional variables we write the equations as

$$\partial_{zz}\phi + \partial_{xx}\phi = 0, \quad -d < z < 0,$$

$$\partial_z\phi = 0, \quad z = -d,$$

$$\partial_t\zeta - \partial_z\phi = 0, \quad z = 0,$$

$$g\zeta + \partial_t\phi = 0, \quad z = 0,$$

$$\phi(x,t) = \phi(x+L,t), \phi(x,t) = \phi(x,t+T), \quad \text{at lateral boundaries.}$$

- Linear waves allows for superposition

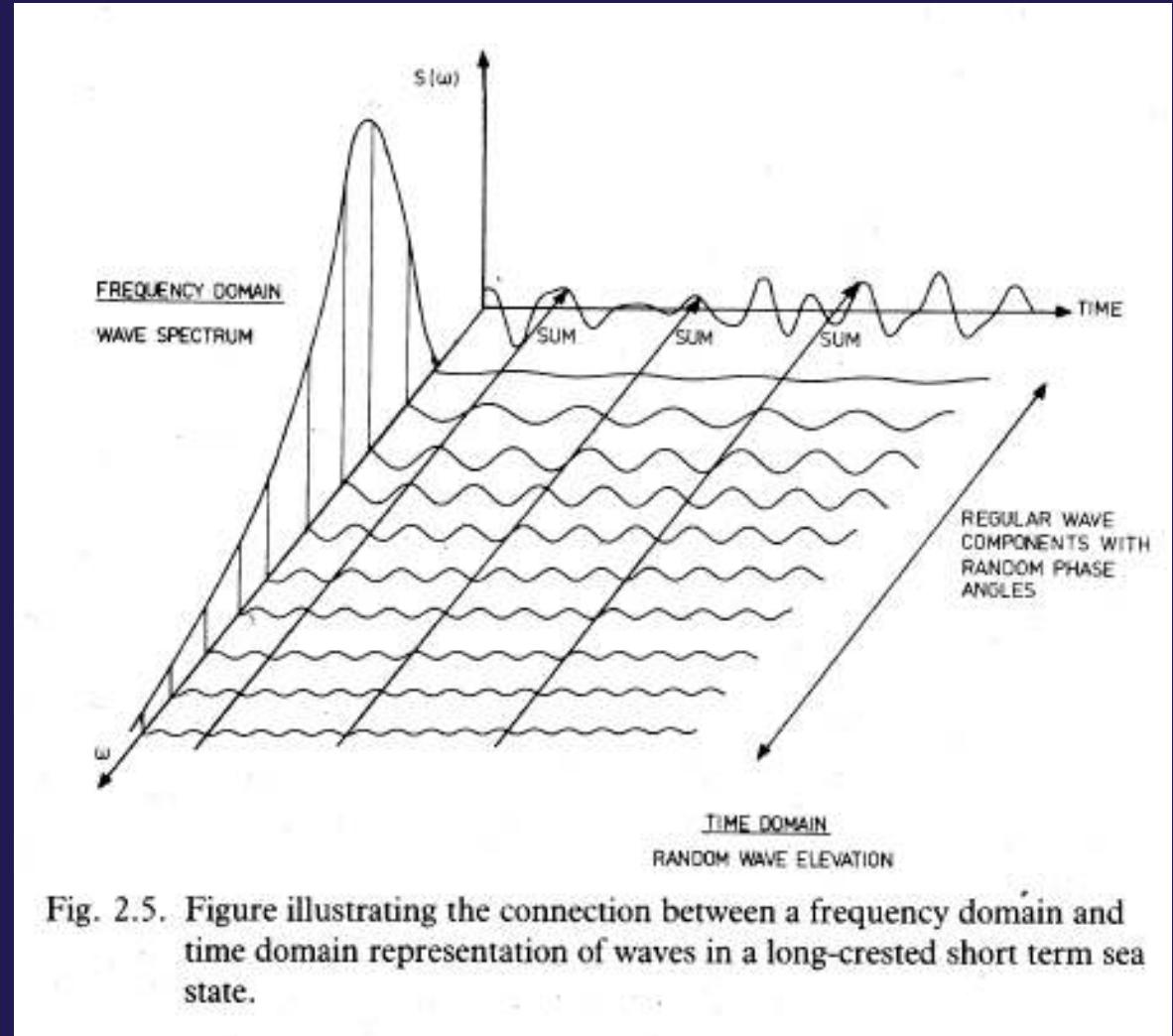
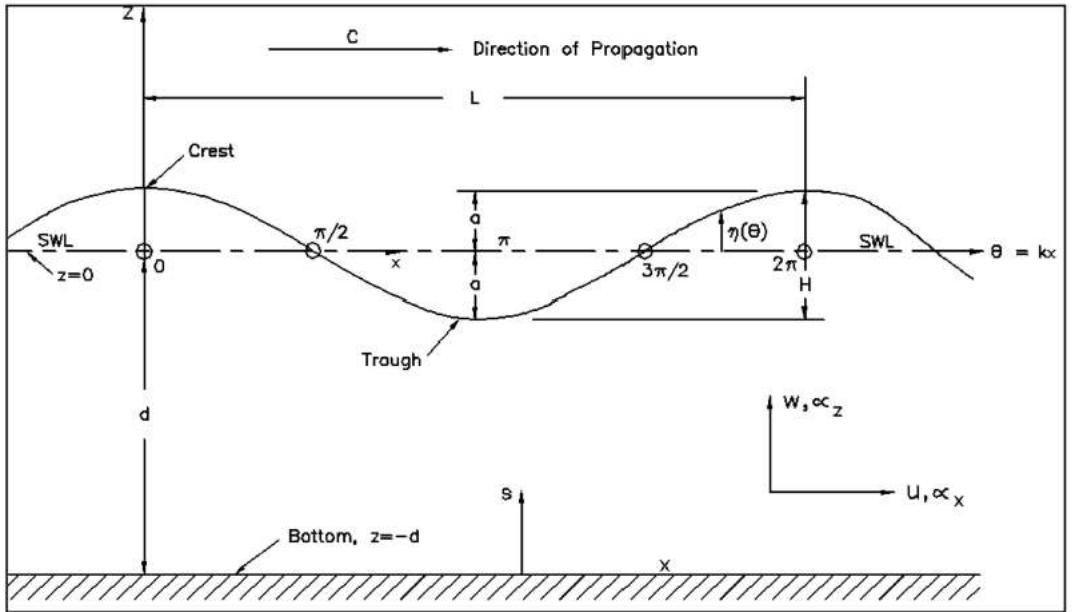


Fig. 2.5. Figure illustrating the connection between a frequency domain and time domain representation of waves in a long-crested short term sea state.





$$\phi = \frac{ag \cosh(k(d+z))}{\omega \cosh(kd)} \sin(kx - \omega t),$$

$$\zeta = a \cos(kx - \omega t).$$

# LINEAR WAVE

- ⌚ Completely determined by
- ⌚ Amplitude
- ⌚ Period
- ⌚ Still water depth



Relative Depth	Shallow Water $\frac{d}{L} < \frac{1}{20}$ $kd < \frac{\pi}{10}$	Transitional Water $\frac{1}{20} < \frac{d}{L} < \frac{1}{2}$ $\frac{\pi}{10} < kd < \frac{\pi}{2}$	Deep Water $\frac{d}{L} > \frac{1}{2}$ $kd > \frac{\pi}{2}$
1. Wave profile	Same As >	$\eta = \frac{H}{2} \cos \left[ \frac{2\pi x}{L} - \frac{2\pi t}{T} \right] = \frac{H}{2} \cos \theta$	< Same As
2. Wave celerity	$C = \frac{L}{T} = \sqrt{gd}$	$C = \frac{L}{T} = \frac{gT}{2\pi} \tanh \left( \frac{2\pi d}{L} \right)$	$C = C_0 = \frac{L}{T} = \frac{gT}{2\pi}$
3. Wavelength	$L = T\sqrt{gd} = CT$	$L = \frac{gT^2}{2\pi} \tanh \left( \frac{2\pi d}{L} \right)$	$L = L_0 = \frac{gT^2}{2\pi} = C_0 T$
4. Group velocity	$C_g = C = \sqrt{gd}$	$C_g = nC = \frac{1}{2} \left[ 1 + \frac{4\pi d/L}{\sinh(4\pi d/L)} \right] C$	$C_g = \frac{1}{2} C = \frac{gT}{4\pi}$
5. Water particle velocity			
(a) Horizontal	$u = \frac{H}{2} \sqrt{\frac{g}{d}} \cos \theta$	$u = \frac{H}{2} \frac{gT}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos \theta$	$u = \frac{\pi H}{T} e^{\left(\frac{2\pi z}{L}\right)} \cos \theta$
(b) Vertical	$w = \frac{H\pi}{T} \left( 1 + \frac{z}{d} \right) \sin \theta$	$w = \frac{H}{2} \frac{gT}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin \theta$	$w = \frac{\pi H}{T} e^{\left(\frac{2\pi z}{L}\right)} \sin \theta$
6. Water particle accelerations			
(a) Horizontal	$a_x = \frac{H\pi}{T} \sqrt{\frac{g}{d}} \sin \theta$	$a_x = \frac{g\pi H}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin \theta$	$a_x = 2H \left( \frac{\pi}{T} \right)^2 e^{\left(\frac{2\pi z}{L}\right)} \sin \theta$
(b) Vertical	$a_z = -2H \left( \frac{\pi}{T} \right)^2 \left( 1 + \frac{z}{d} \right) \cos \theta$	$a_z = -\frac{g\pi H}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos \theta$	$a_z = -2H \left( \frac{\pi}{T} \right)^2 e^{\left(\frac{2\pi z}{L}\right)} \cos \theta$
7. Water particle displacements			
(a) Horizontal	$\xi = -\frac{HT}{4\pi} \sqrt{\frac{g}{d}} \sin \theta$	$\xi = -\frac{H}{2} \frac{\cosh[2\pi(z+d)/L]}{\sinh(2\pi d/L)} \sin \theta$	$\xi = -\frac{H}{2} e^{\left(\frac{2\pi z}{L}\right)} \sin \theta$
(b) Vertical	$\zeta = \frac{H}{2} \left( 1 + \frac{z}{d} \right) \cos \theta$	$\zeta = \frac{H}{2} \frac{\sinh[2\pi(z+d)/L]}{\sinh(2\pi d/L)} \cos \theta$	$\zeta = \frac{H}{2} e^{\left(\frac{2\pi z}{L}\right)} \cos \theta$
8. Subsurface pressure	$p = \rho g(\eta - z)$	$p = \rho g \eta \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho gz$	$p = \rho g \eta e^{\left(\frac{2\pi z}{L}\right)} - \rho gz$

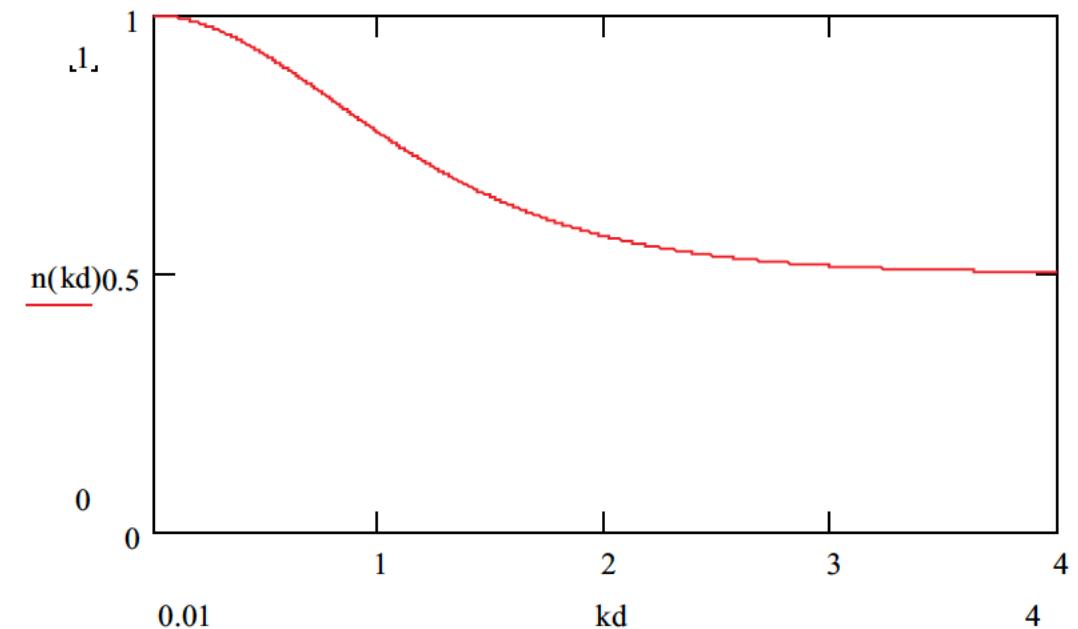
# ENGINEERING WAVE PROPERTIES

# PHASE VS GROUP VELOCITY



The group velocity is sometimes written as  $C_g = nC$  where

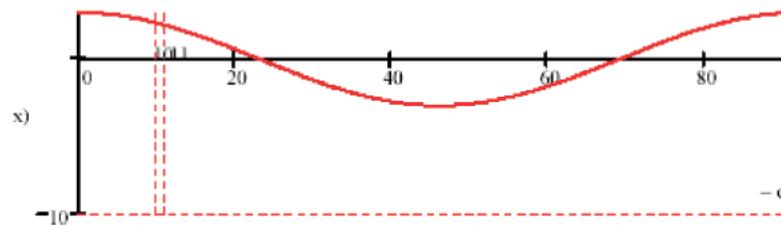
$$n = \frac{1}{2} \left( 1 + \frac{2kd}{\sinh(2kd)} \right)$$



# WAVE ENERGY

the energy in a progressive Airy wave can be estimated by integrating the potential and kinematic energy in the wave over a wavelength.

The potential energy is a result of the displacement of mass from a position of rest, i.e. potential energy at a time, say  $t = 0$ , is the deformation work needed to give the form of the wave:  $\zeta = a \cos(kx)$ .



The mass of the lifted water mass between the two dashed verticals is  $\rho g \zeta dx$  and this mass is lifted the height  $\zeta/2$ .



# WAVE ENERGY

The mean energy over a wavelength thus becomes

$$E_p = \frac{1}{L} \int_0^L \rho g \zeta \frac{\zeta}{2} dx = \frac{\rho g}{2L} \int_0^L \zeta^2 dx = \frac{\rho g a^2}{2L} \int_0^L (\cos(kx))^2 dx$$

We find that the potential energy is proportional to the square of the amplitude

$$E_p = \frac{\rho g a^2}{4}$$

# WAVE ENERGY

The kinetic energy is total kinetic energy contained in all the water from the free surface to the bottom. However, in Airy theory we can only integrate up to the still water level. So finite waves are badly computed.

We consider a small moving parcel with mass  $dm$ . The kinetic energy is

$$dE_k = dm \frac{u^2 + w^2}{2} = \rho dx dz \frac{u^2 + w^2}{2}.$$

Integrate over both the depth and over a wave length:

$$E_k = \frac{1}{L} \int_0^L \int_{-d}^0 \rho \frac{u^2 + w^2}{2} dz dx.$$



# WAVE ENERGY

Substituting the known values for the velocities from Airy theory

$$\begin{aligned} E_k = \frac{\rho}{2L} & \left( \frac{gak}{\omega} \frac{1}{\cosh(kd)} \right)^2 \int_0^L \int_{-d}^0 (\cosh^2(k(d+z)) \cos^2(kx - \omega t) \right. \\ & \left. + \sinh^2(k(d+z)) \sin^2(kx - \omega t)) dz dx \end{aligned}$$

which can be simplified and integrated to give

$$E_k = \frac{\rho g a^2}{4}$$

# WAVE ENERGY

Adding the potential and kinematic energies we obtain the total energy per unit surface area as

$$E = E_p + E_k = \frac{\rho g a^2}{2}$$

Finally, it should be stressed that the energy propagates with the group velocity, so the wave power read:

$$P = EC_g$$

# WAVE POWER RESOURCE

$$J_{reg} = \frac{\rho g^2}{32\pi} H^2 T$$

$$J_{irreg} = \frac{\rho g^2}{64\pi} H_s^2 T$$

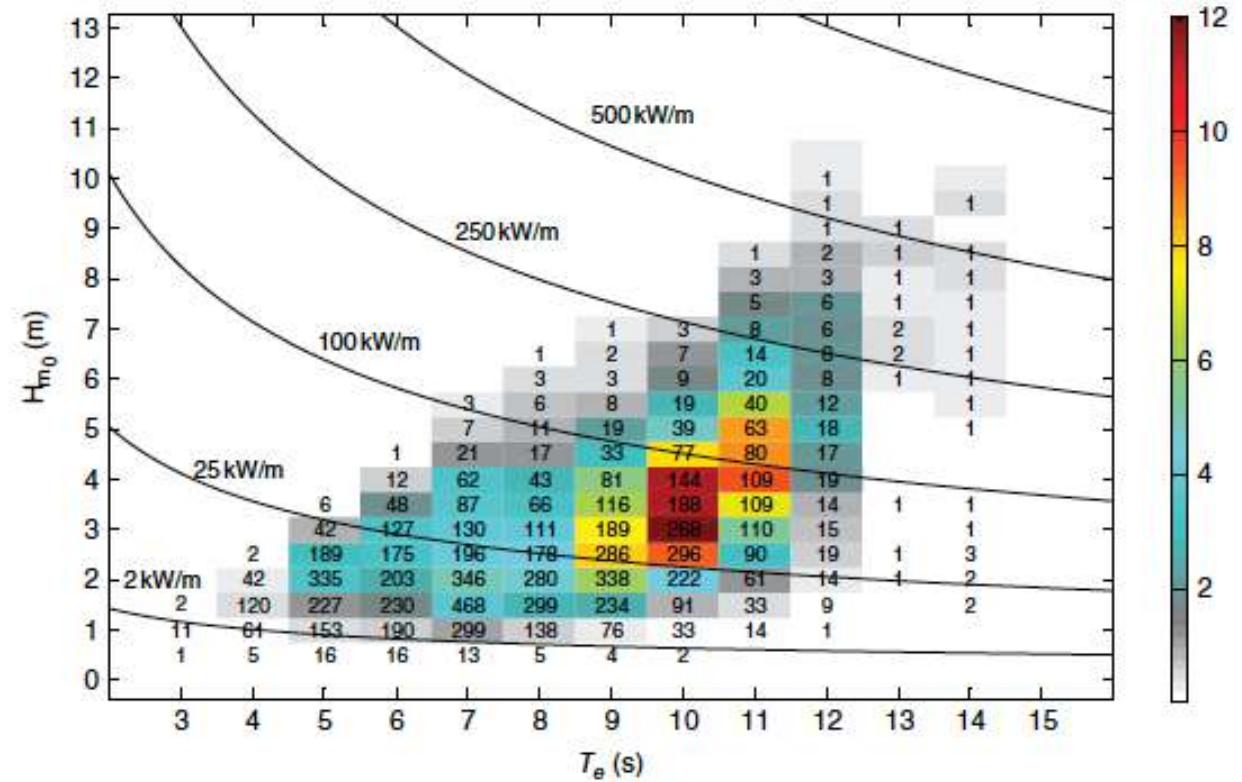


Figure 2.2 Offshore resource characterisation matrix including wave power isolines for the Death Coast (*Costa da Morte*), Galicia, NW Spain. The numbers in the greyed squares represent the occurrence of the respective *energy bins*, expressed in hours in a typical year. The greyscale indicates the contribution of each *energy bin* to the annual resource.

$$T_e = T_{-10} = \frac{m_{-1}}{m_0} \quad m_n = \sum_i f_i^n S_i \Delta f_i \quad S_i = \sum_j S_{ij} \Delta \theta_j$$

# WAVE POWER RESOURCE

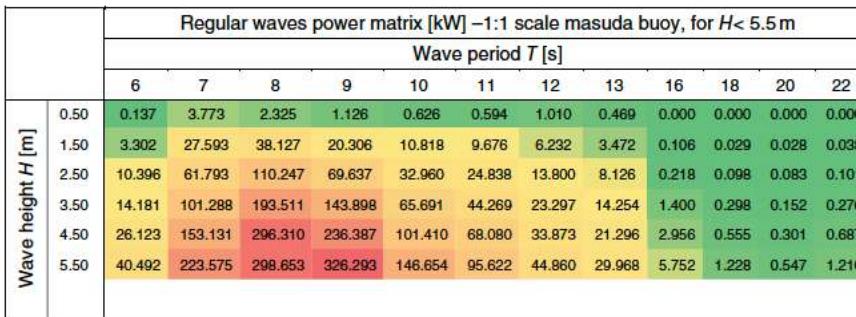


Figure 7.18 Regular waves power matrix of an OWC Masuda buoy model, 1 : 50 scale [38].

X

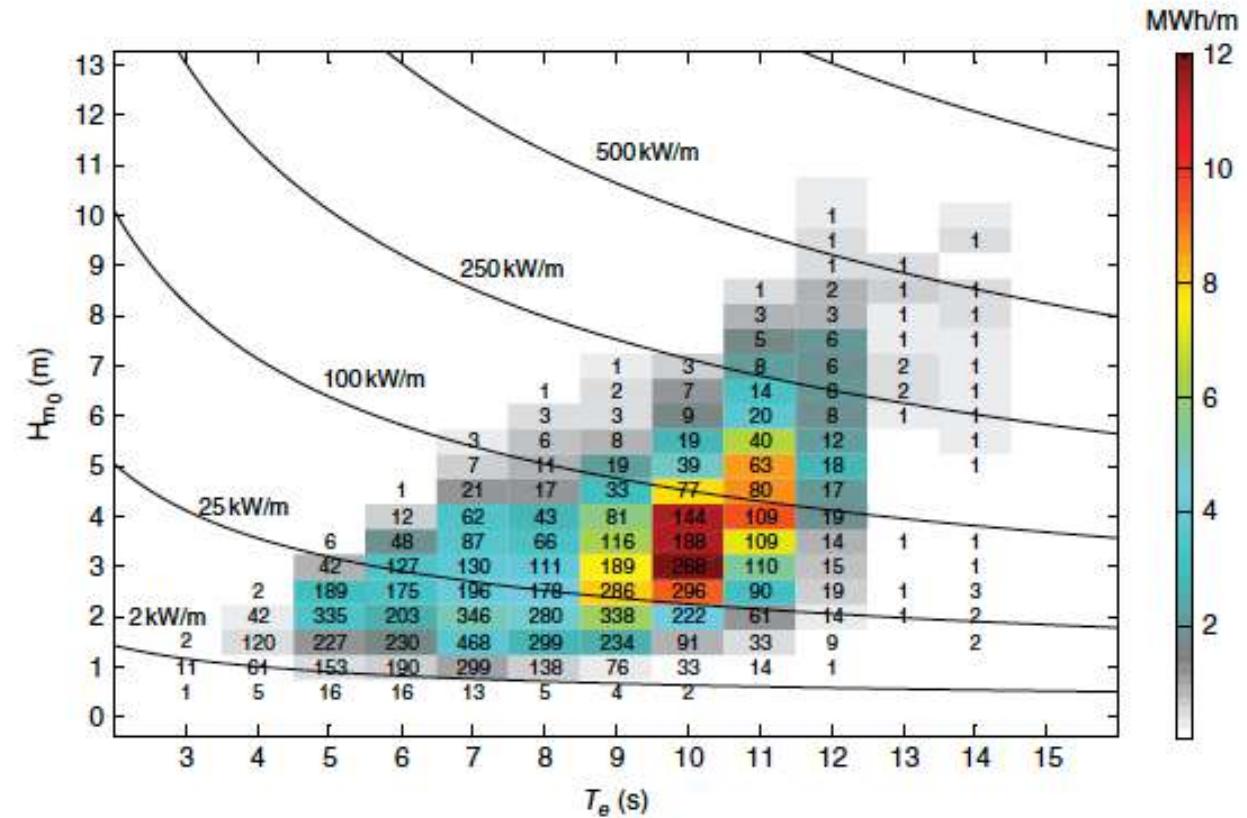


Figure 2.2 Offshore resource characterisation matrix including wave power isolines for the Death Coast (*Costa da Morte*), Galicia, NW Spain. The numbers in the greyed squares represent the occurrence of the respective *energy bins*, expressed in hours in a typical year. The greyscale indicates the contribution of each *energy bin* to the annual resource.



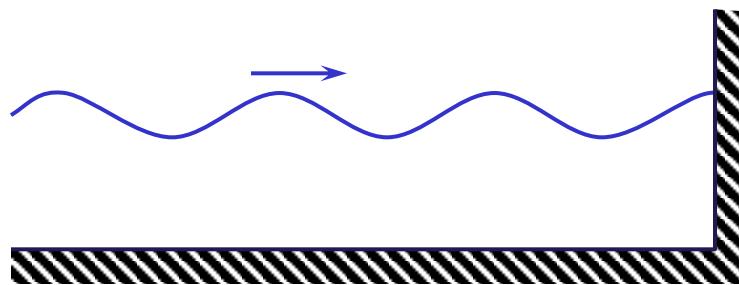
# BREAK

45

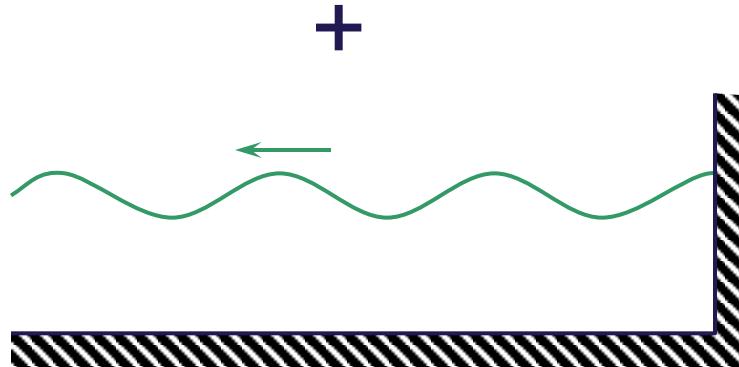


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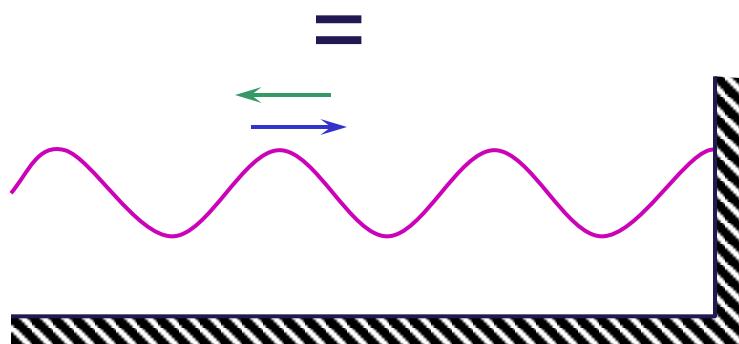
“To absorb a wave is to generate a wave”  
Prof Falnes



Incident wave



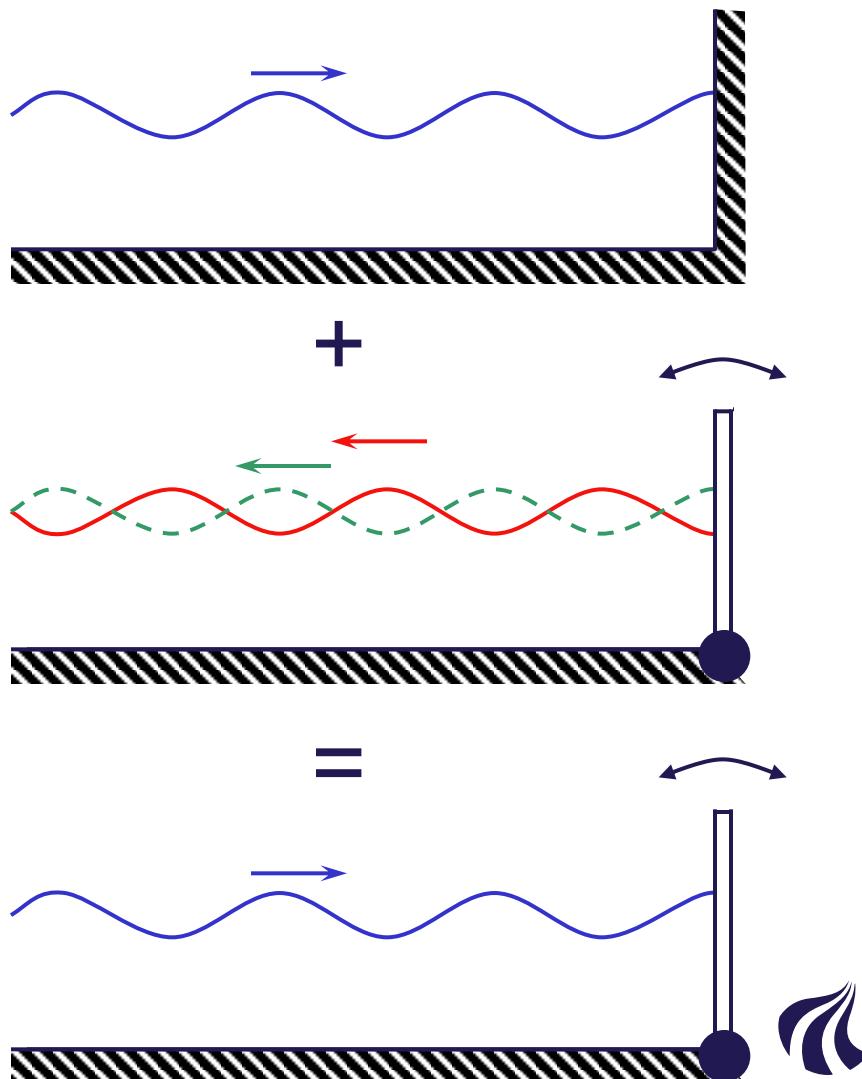
Reflected wave



Inteference pattern: Standing wave



"To absorb a wave is to generate a wave"  
Prof Falnes



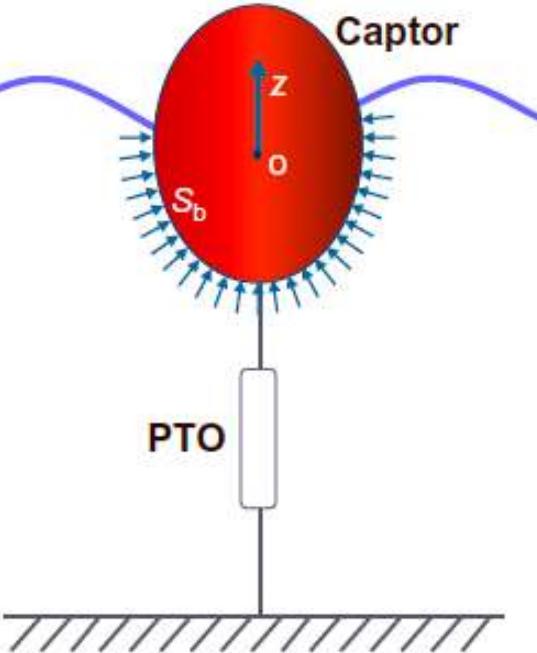
Incident wave

Reflected wave  
Generated wave

Inteference pattern: incident wave  
absorbed as the generated wave  
cancels the reflected wave



# Newton's II Law



$$m\ddot{x} = F_{pe}(t) + F_{re}(t),$$

$m$  is total inertia

$x$  is displacement

$F_{pe}$  force due to external pressure

$F_{re}$  reaction forces



# Newton's II Law

$$m\ddot{x} = F_{pe}(t) + F_{re}(t), \quad \text{Time domain}$$

$$x(t) = \operatorname{Re}(\hat{x}(\omega) \exp^{i\omega t})$$

$$-\omega^2 m \hat{x}(\omega) = \hat{F}_{pe}(\omega) + \hat{F}_{re}(\omega) \quad \text{Frequency domain}$$



# FORCE DUE TO PRESSURE ACTING ON FLOATER

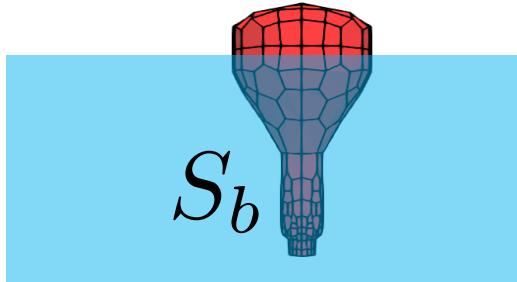
$$-\omega^2 m \hat{x}(\omega) = \hat{F}_{pe}(\omega) + \hat{F}_{re}(\omega)$$

$$\hat{F}_{pe}(\omega) = \hat{F}_{hd}(\omega) + \hat{F}_{hs}(\omega),$$

$\hat{F}_{hd}(\omega)$  is the hydrodynamic force

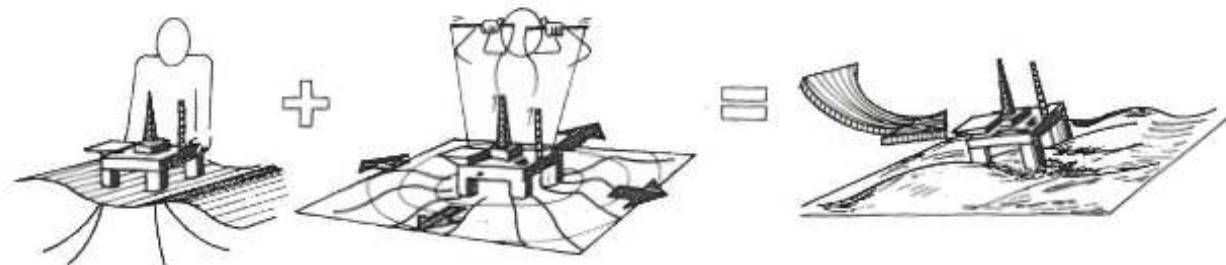
$\hat{F}_{hs}(\omega)$  is the hydrostatic force





$$F_{hd} = \int_{S_b} p_e n dS_b = \rho \int_{S_b} \frac{\partial \phi}{\partial t} n dS_b$$

$$\hat{F}_{hd} = \hat{F}_e + \hat{F}_r$$



Excitation loads

Added mass  
Damping and Restoring  
forces and moments

Faltinsen (1990)

DENMARK

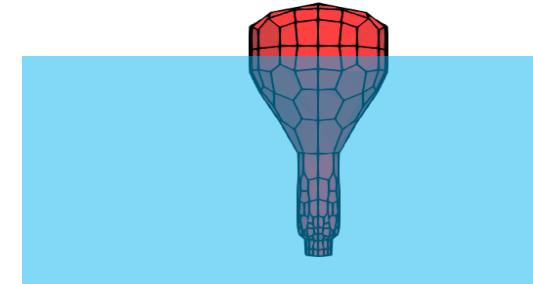
# HYDRODYNAMIC FORCE

- Assumption of linear waves allows superposition of forces

# FROUDE-KRYLOV FORCE

$$\hat{F}_e = \hat{F}_{FK} + \hat{F}_S,$$

$$\hat{F}_{FK} = i\omega\rho \int_{S_b} \hat{\phi}_0 n dS_b$$



**Table 1**

Summary of the main difference between the three Froude-Krylov modelling approaches for axisymmetric buoys.

	<b>LFK</b> Linear Froude-Krylov (FK)	<b>VFK</b> Algebraic nonlinear FK	<b>RFK</b> Numerical nonlinear FK
Wetted surface $S(t)$	Constant	Instantaneous	Instantaneous
Pitch angle $\delta$	0	0	$\delta$
Free surface $\eta(x, t)$	+	$\eta(t)$	$\eta(x, t)$
Computational time		++	+++

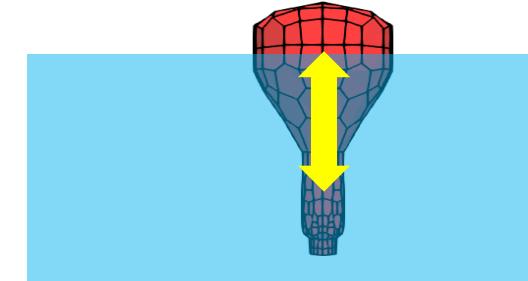
# RADIATION FORCE

$$\hat{F}_r = \omega^2 \rho \hat{x} \int_{S_b} \hat{\phi}_r n dS_b,$$

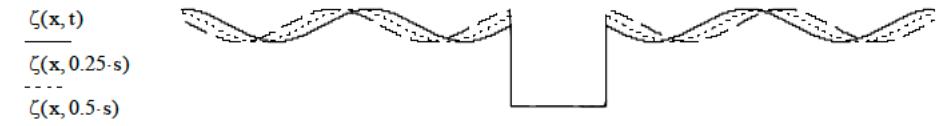
$$\hat{F}_r = -i\omega Z \hat{x}$$

$$Z = -i\omega \rho \int_{S_b} \hat{\phi}_r n dS_b = R + iX = R + i\omega A,$$

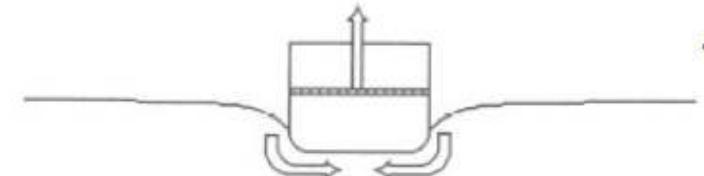
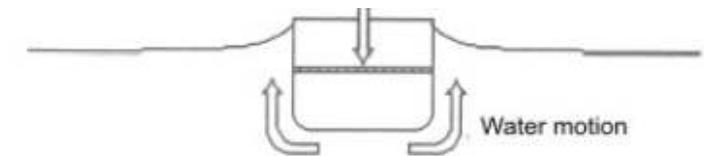
$$\hat{F}_r = -i\omega R \hat{x} + \omega^2 A \hat{x}$$



Radiation damping



Added mass



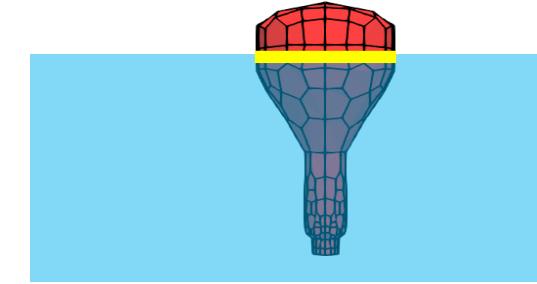
# HYDROSTATIC FORCE

$$\hat{F}_{hs} = -G\hat{x},$$

$$G = \rho g S,$$

$G$  is the hydrostatic stiffness

$S$  is the waterplane area



# REACTION FORCES

$$-\omega^2 m \hat{x}(\omega) = \hat{F}_{pe}(\omega) + \boxed{\hat{F}_{re}(\omega)}$$

$$\hat{F}_{re}(\omega) = \hat{F}_{pto}(\omega) + \hat{F}_m(\omega),$$

$\hat{F}_{pto}(\omega)$  is the reaction force from PTO

$\hat{F}_m(\omega)$  is the reaction force from mooring





# REACTION FORCES

$$\hat{F}_{pto} = -i\omega B_{pto} \hat{x} - K_{pto} \hat{x},$$

$B_{pto}$  is the PTO damping

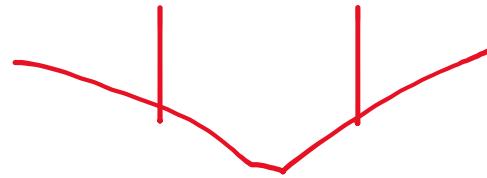
$K_{pto}$  is the PTO spring

$$\hat{F}_m = -K_m \hat{x},$$

$K_m$  mooring spring stiffness

# MOTION

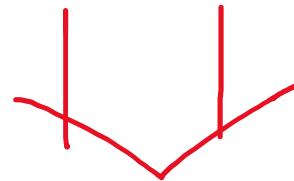
$$-\omega^2 m \hat{x}(\omega) = \hat{F}_{pe}(\omega) + \hat{F}_{re}(\omega)$$



$$\hat{x} = \frac{\hat{F}_e}{[-\omega^2(m + A) + G + K_{pto} + K_m] + [i\omega(R + B_{pto})]}$$

# POWER

$$P = \frac{1}{T} \int_0^T B_{pto} u^2 dt = \frac{1}{2} B_{pto} \omega^2 |\hat{x}|^2$$



$$P = \frac{1}{2} \frac{B_{pto} \omega^2 |\hat{F}_e|^2}{[-\omega^2(m + A) + G + K_{pto} + K_m]^2 - \omega^2(R + B_{pto})^2}$$

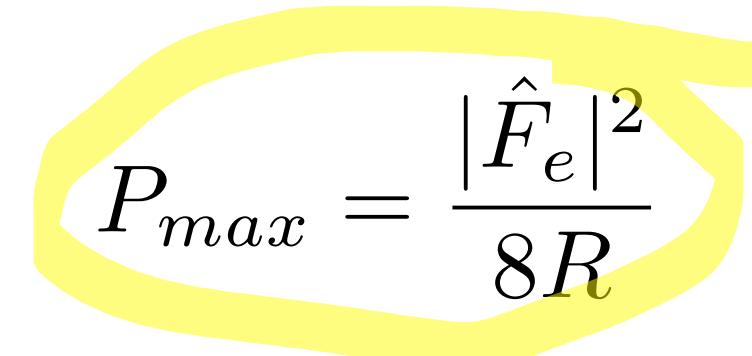
# MAXIMUM POWER

$$P = \frac{1}{2} \frac{B_{pto} \omega^2 |\hat{F}_e|^2}{[-\omega^2(m + A) + G + K_{pto} + K_m]^2 - \omega^2(R + B_{pto})^2}$$

The first condition states that power absorption is maximized at resonance. When this condition is achieved the velocity is in phase with the excitation force

The second condition states that the PTO damping must be equal to the hydrodynamic radiation damping. This highlights that the ability to generate waves is a fundamental aspect in the design of WECs.

- 1  $K_{pto} = \omega^2(m + A) - G - K_m$
- 2  $B_{pto} = R$



$$P_{max} = \frac{|\hat{F}_e|^2}{8R}$$

# THINK- SHARE\_PAIR

- ⌚ ARE WE MIOSSIG SOMETHING HERE?
- ⌚ ASSUMPTIONS?



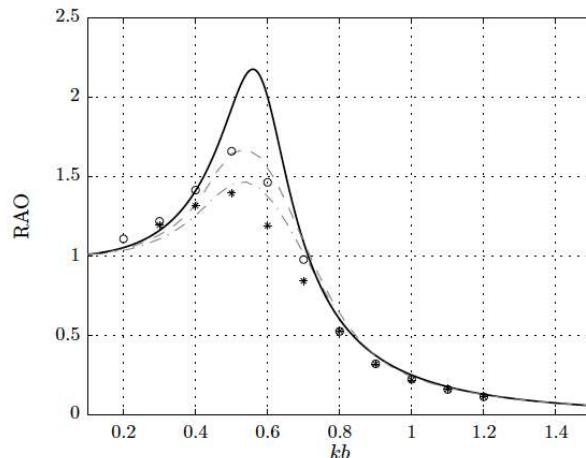


Figure 6.4: Heave Response Amplitude Operator (RAO) showing experimental data due to incident waves of steepness  $A_1 k = \circ 0.05$  and  $\ast 0.10$ , — Linear potential flow prediction (WAMIT), and TD simulation with  $C_D2 = 350 \text{Ns}^2/\text{m}^2$  for —  $A_1 k = 0.05$  and  $- - A_1 k = 0.10$

$$F_V = -C_d \dot{X} |\dot{X}|$$

# MISSING: VISCOUS FORCES

- ➊ Linear theory does not include viscous forces and greatly overpredicts response in the resonance region
- ➋ FIX: use Morison drag. Need to calibrate the Morrison drag – NB: the calibrated drag coefficient will be ‘all-inclusive’ including all nonlinearities (Scaling problem)

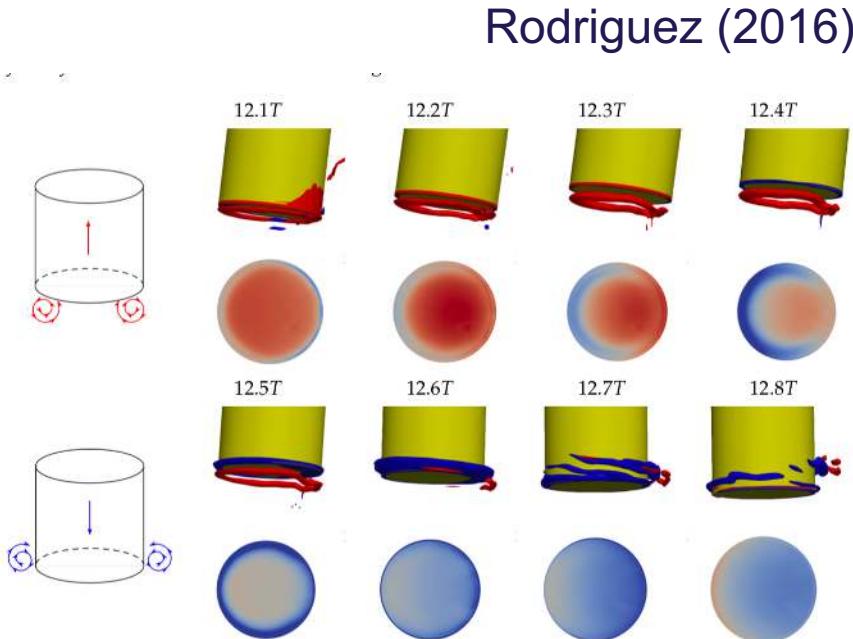


Figure 9. Vortex formation over the wave period  $T$  with corresponding dynamic pressure distribution on the bottom of the buoy for the W1 RANS model scale simulations. Vortices are visualized by the  $Q = 200$  iso-surface and colored by rotation direction (inward rotation is blue and outward is red, see schematics on the left). Top two rows are upward motion, top bottom rows are downward motion. The colour-scale of the pressure distribution goes from blue to red in  $[-0.25, 0.25]$ , in non-dimensionalised values. The wave propagates from left to right in all figures. The  $y$ -direction is into the figure in the top row, and is directed downwards in the bottom row.

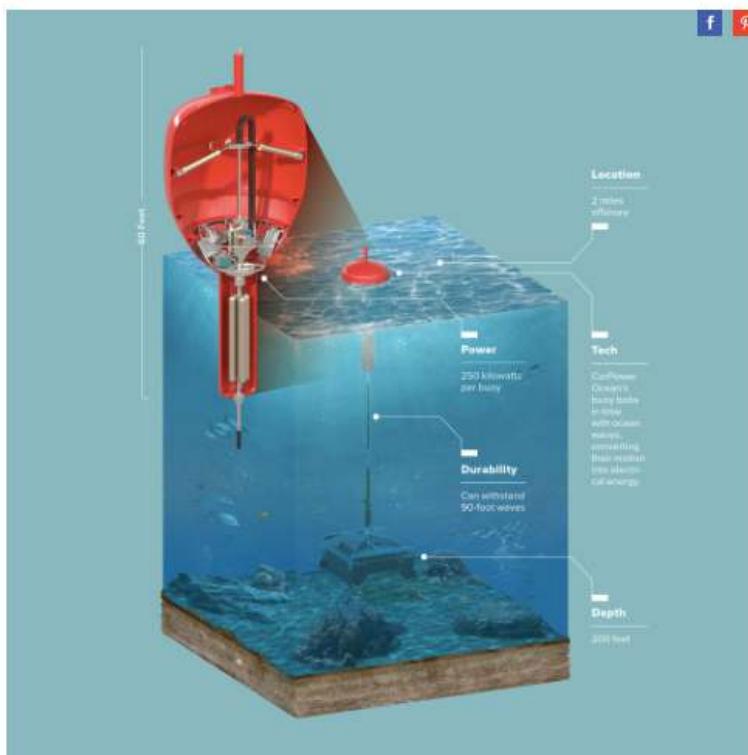
Palm et al (2018)



# CORPOWER: PASSIVE CONTROL

RACHEL NUWER MAGAZINE 04.06.16 4:10 PM

## THE WONDER-BUOY THAT MAY FINALLY MAKE WAVE ENERGY FEASIBLE



5 times higher Annual Energy / mass compared to conventional point absorbers without phase control. [MWh/ton]

3 times higher Annual Energy / PTO Force compared to conventional point absorbers without phase control. [MWh / kN]

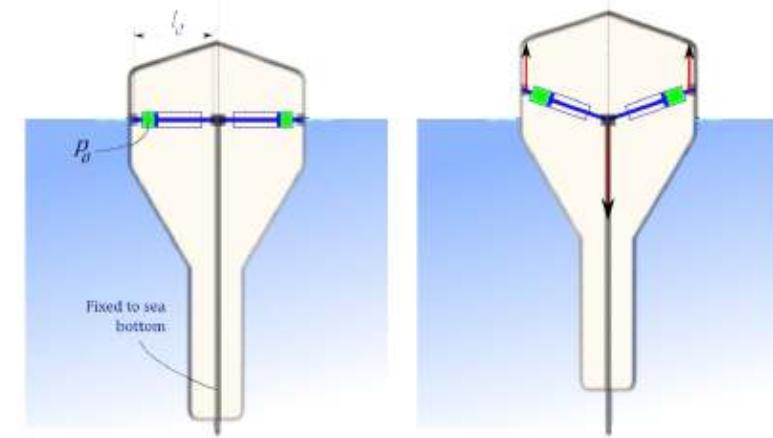


Fig. 5. Illustration of how the buoy displacement induces a vertical spring force with negative stiffness. Buoy in mean vertical position (left) and displaced upwards (right).



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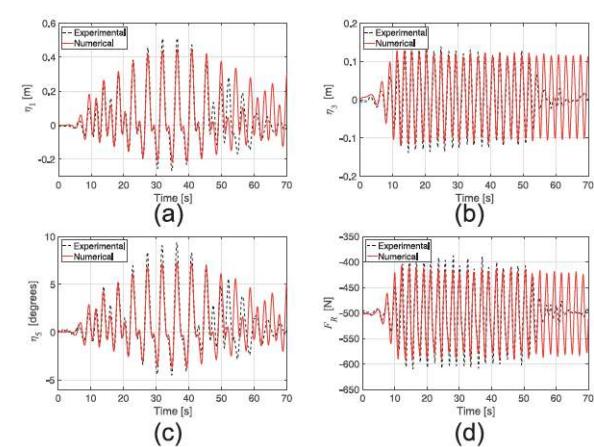
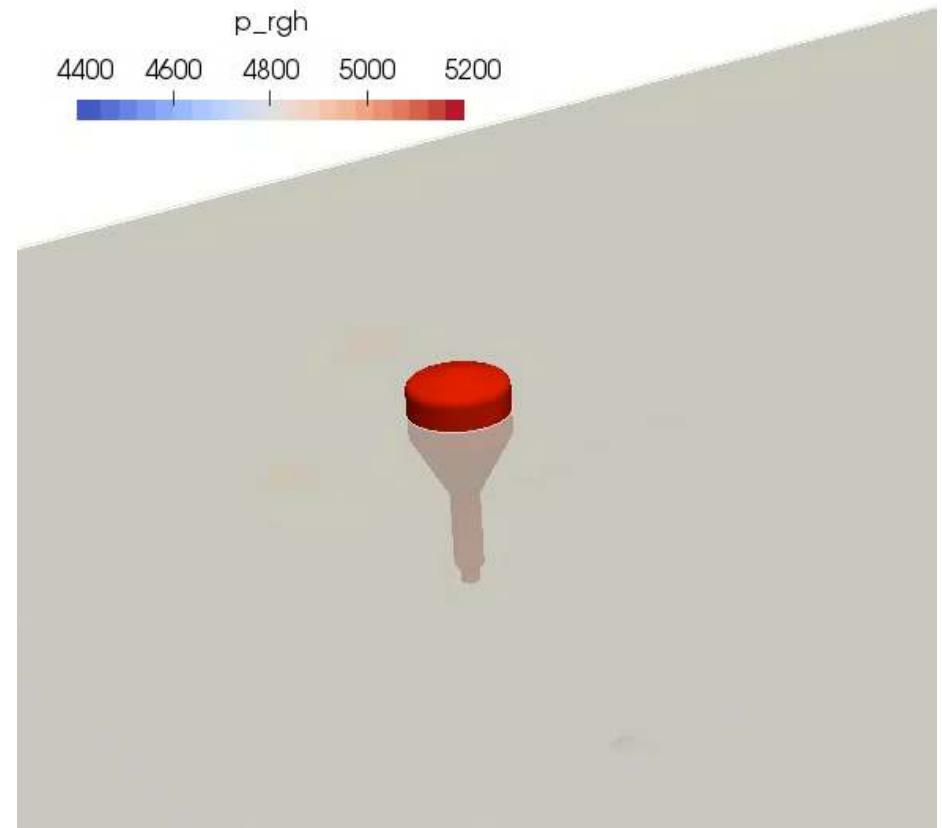
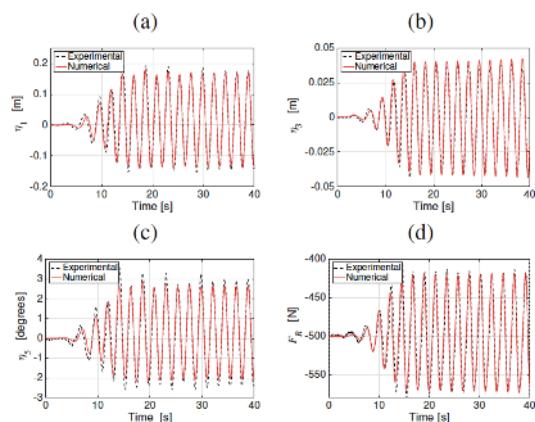
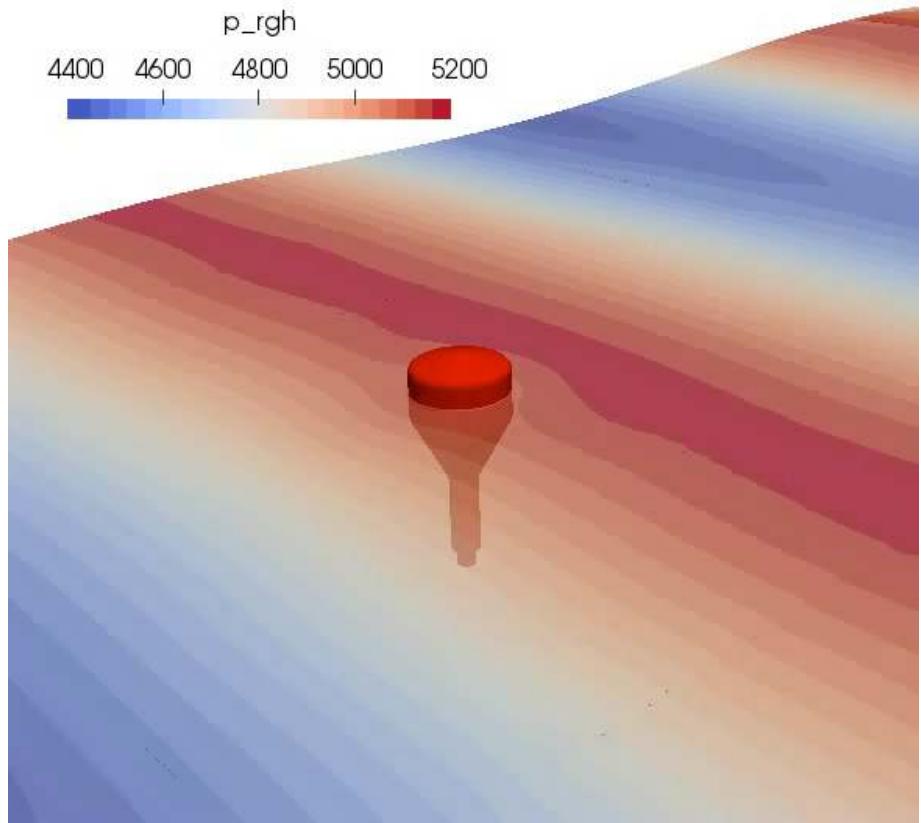
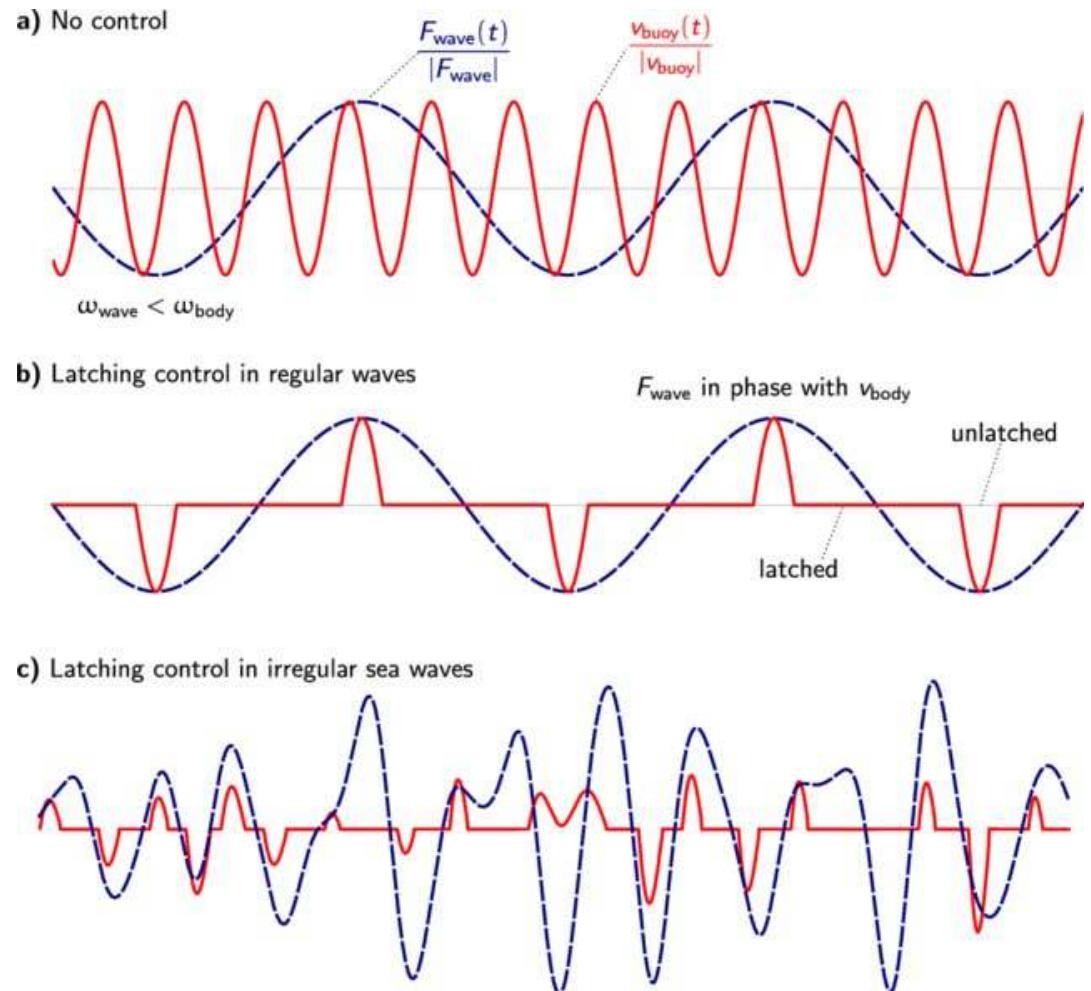


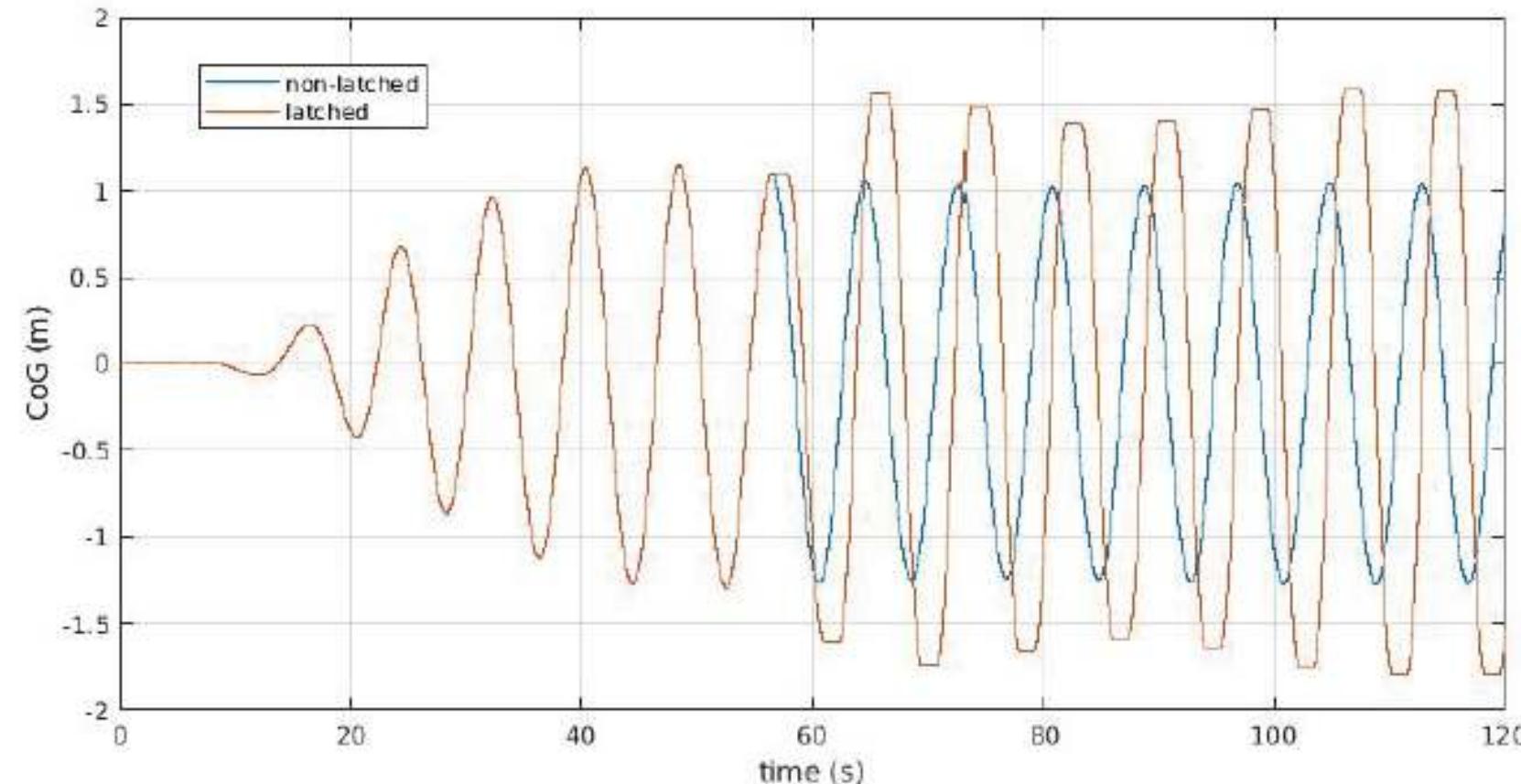
Figure 7. Motion response and force with linear damper RTO:



# LATCHING: REACTIVE CONTROL



# CFD LATCHING



# QUESTIONS

- I CAN RECOMMEND “HANDBOOK OF OCEAN WAVE ENERGY” OPEN ACCESS BOOK FROM SPRINGER

Arthur Pecher  
Jens Peter Kofoed *Editors*

# Handbook of Ocean Wave Energy



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