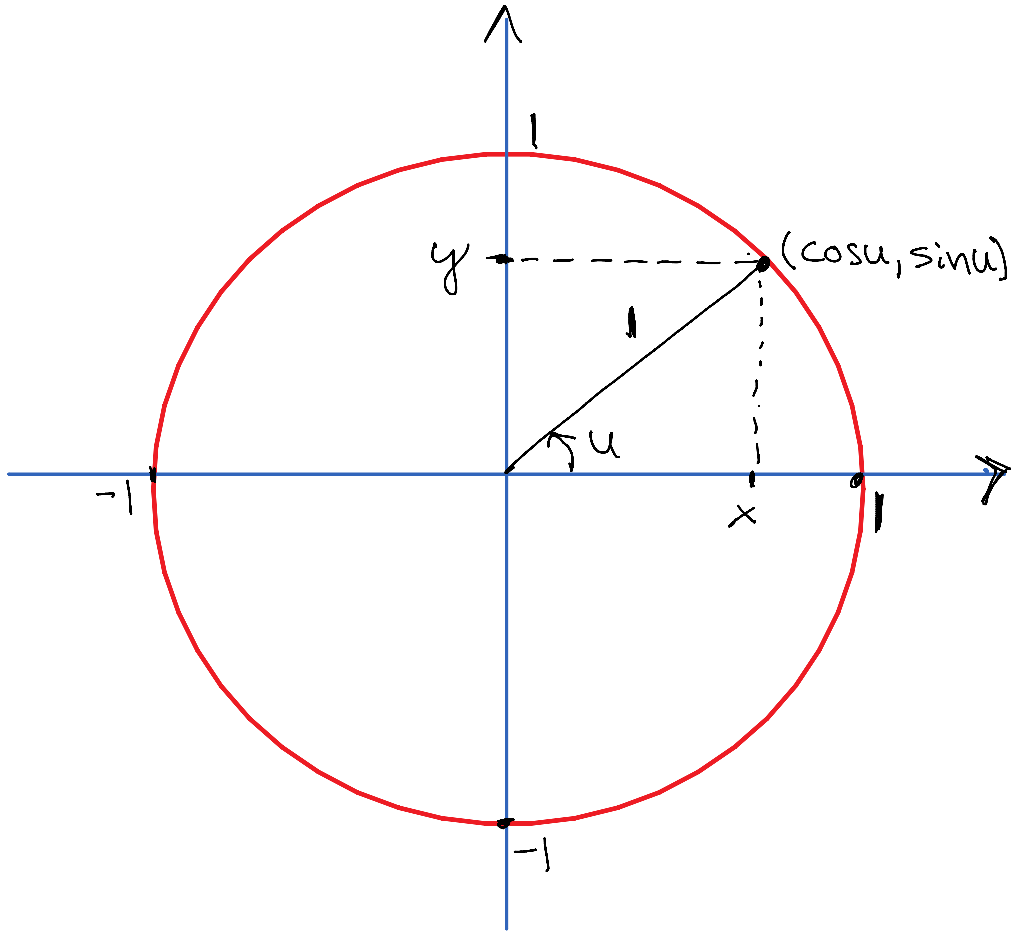
Agenda

den 5 november 2020 17:55

Trigonometri

- $\cos u$, $\sin u$, $\tan u$, $\cot u$ — Utökad definition
- Areasatsen, Sinussatsen, cosinussatsen — Trianglar
- Trigonometriska ettan

  $\cos^2 u + \sin^2 u = 1$



$\cos u$ och $\sin u$ är definierade för alla vinklar $u \in \mathbb{R}$

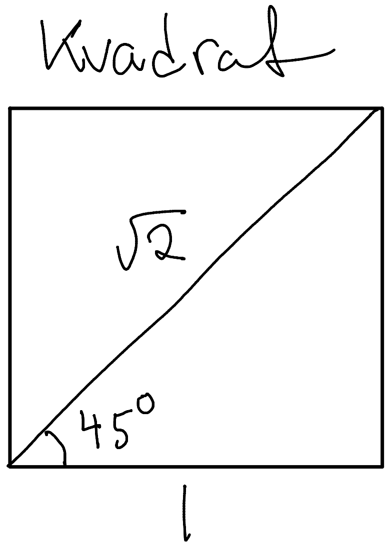
$\tan u = \dfrac{\sin u}{\cos u}$ är def. för alla u där $\cos u \neq 0$, $u \neq 90^\circ + n \cdot 180^\circ$, n heltal

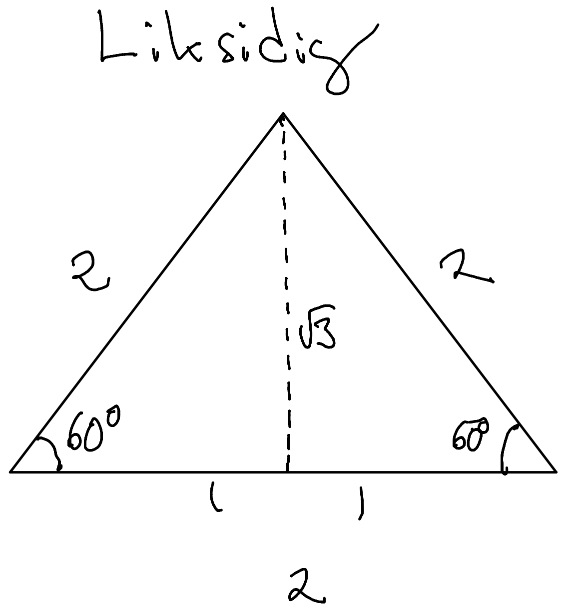
Tabell

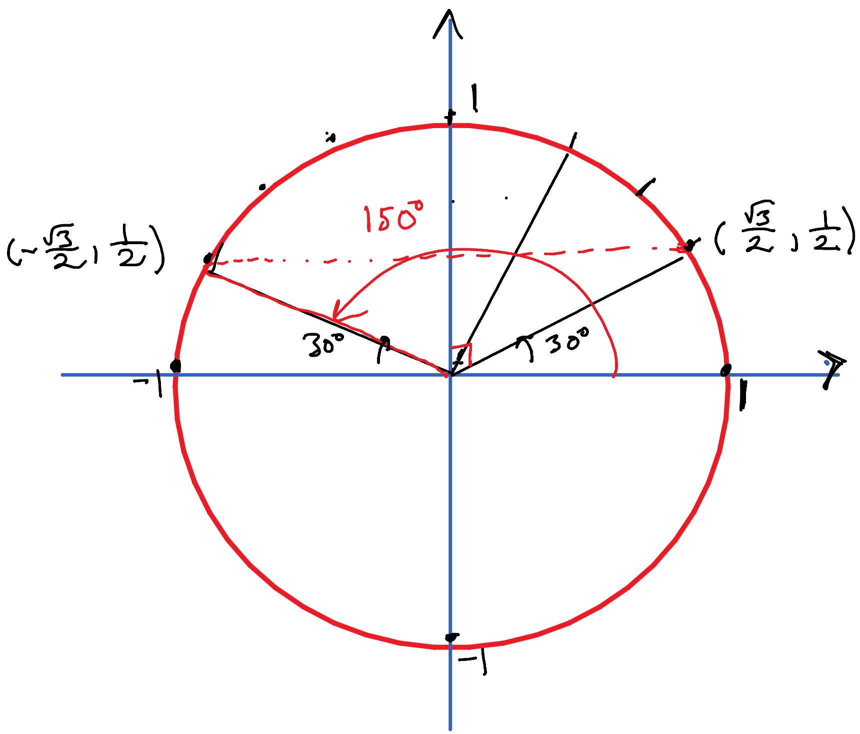
den 10 november 2020 00:17

vinkel	x	y
u	$\cos u$	$\sin u$
0°	1	0
30°	$\sqrt{3}/2$	$1/2$
45°	$1/\sqrt{2}$	$1/\sqrt{2}$
60°	$1/2$	$\sqrt{3}/2$
90°	0	1
$\vdots$	$\vdots$	$\vdots$
$\underbrace{150^\circ}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$

$= 180^\circ - 30^\circ$







Breakout 1

den 9 november 2020 23:43

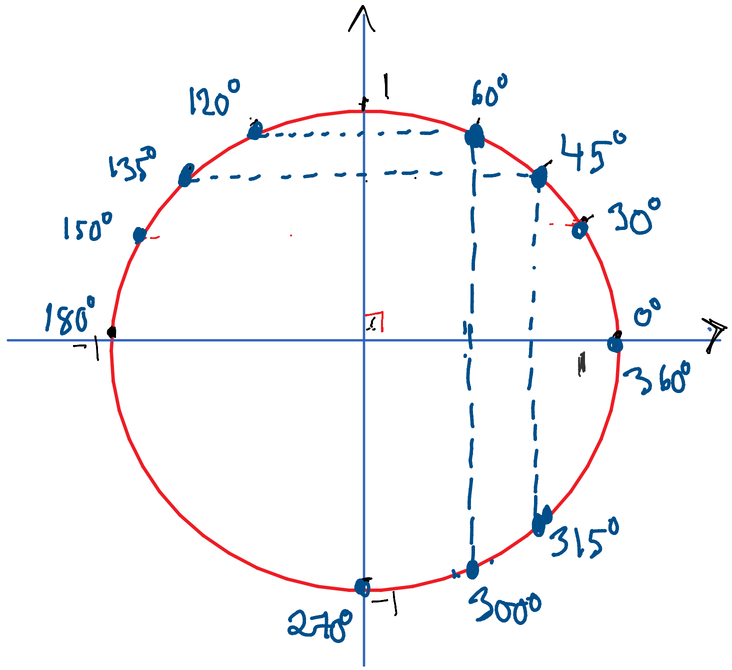
Beräkna

$\sin 120^\circ = \sin(180^\circ - 60^\circ)$

$\cos 300^\circ$

$\cot 135^\circ$

$\tan 315^\circ = \ldots = \frac{-1/\sqrt{2}}{1/\sqrt{2}} = \underline{\underline{-1}}$



$$\sin(180^\circ - u) = \sin u$$

$$\sin 120^\circ = \sin(180^\circ - 60^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

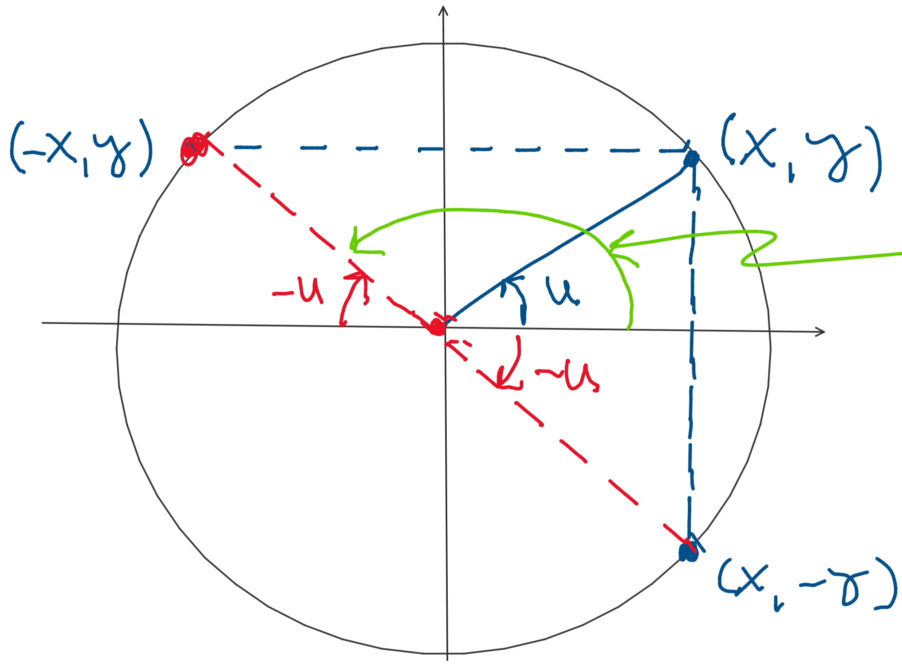
$$\cos 300^\circ = \cos(360^\circ - 60^\circ) = [\text{helt varv kan tas bort}] = \cos(-60^\circ)$$
$$= \cos 60^\circ = \frac{1}{2}.$$

$$\cot 135^\circ = \frac{\cos 135^\circ}{\sin 135^\circ} = [135^\circ = 180^\circ - 45^\circ] = \frac{\cos(180^\circ - 45^\circ)}{\sin(180^\circ - 45^\circ)} =$$

$$= \frac{-\cos 45^\circ}{\sin 45^\circ} = \frac{-1/\sqrt{2}}{1/\sqrt{2}} = \underline{\underline{-1}}$$

$$\left.\begin{aligned}\cos(180^\circ - u) &= -\cos u \\ \sin(180^\circ - u) &= \sin u\end{aligned}\right\}$$

$$\left.\begin{aligned}\cos(-u) &= \cos u \\ \sin(-u) &= -\sin u\end{aligned}\right\}$$



$$\begin{cases} x = \cos u \\ y = \sin u \end{cases}$$

$$\cos(u + n \cdot 360^\circ) = \cos u$$

$$\sin(u + n \cdot 360^\circ) = \sin u$$

Hela varv kan tas bort eller läggas till.

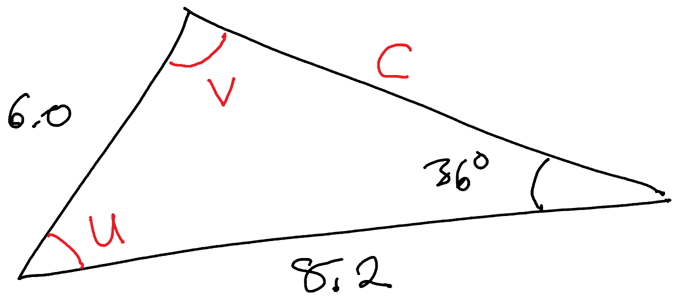
n är ett heltal: $\ldots -3, -2, -1, 0, 1, 2, 3, \ldots$

Uppgift 1

den 10 november 2020 01:00

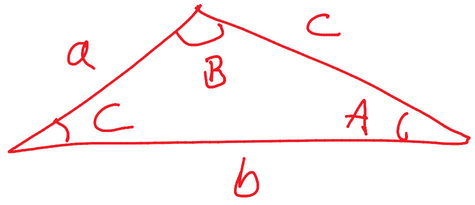
Solvera triangeln

(cm)



Sinussatsen

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



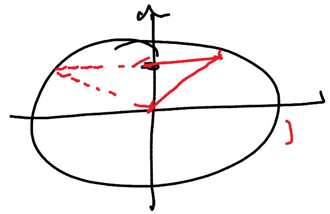
eller

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Lösning:

$$\frac{\sin V}{8.2} = \frac{\sin 36^\circ}{6.0} \iff \sin V = \frac{8.2}{6.0} \sin 36^\circ \approx 0.80,$$

*) $V = 53.45^\circ$ eller 126.55°



$180^\circ - 53.45^\circ$

Vet att $u + V + 36^\circ = 180^\circ$

$u = 180^\circ - 36^\circ - V \overset{*}{=} 90.55^\circ$ eller 17.45°

Sinussatsen ger nu

$$\frac{\sin u}{c} = \frac{\sin 36^\circ}{6.0}, \quad \text{Dvs} \quad c = 6.0 \cdot \frac{\sin u}{\sin 36^\circ}$$

Fall 1: $u = 90.55^\circ$; $c = 6.0 \cdot \frac{\sin 90.55^\circ}{\sin 36^\circ} \approx 10.2$

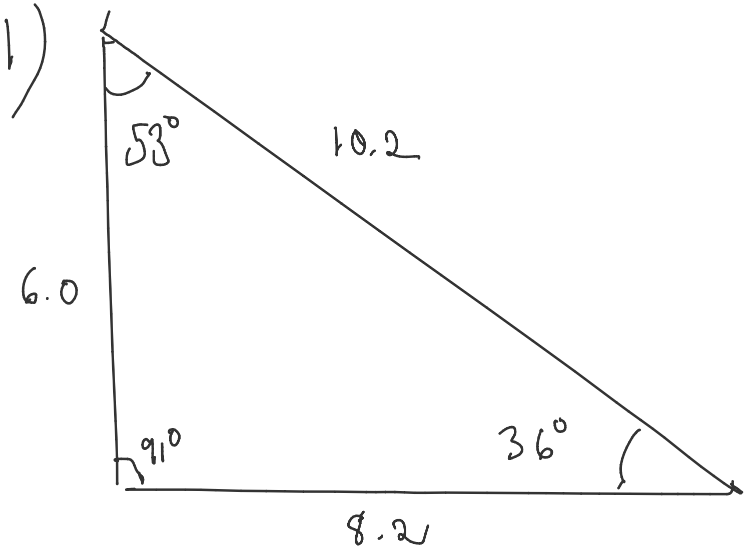
Fall 2: $u = 17.45°$; $c = 6.0 \cdot \frac{\sin 17.45°}{\sin 36°} \approx 3.1$

Svar: Vi får två lösningar

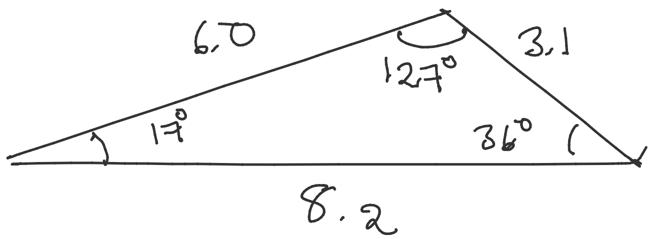
1) $c = 10.2$ cm , $u = 91°$, $v = 53°$

2) $c = 3.1$ cm , $u = 17°$, $v = 127°$

1)



2)



Kommentar:

Vår ursprungliga figur var något missvisande, vilket är ganska vanligt!

Det är viktigt att inte dra förhastade slutsatser baserade på figurer om de inte är noggrant ritade.

Men våra räkningar krävde inte en noggrann figur, bara tillräckligt för att använda sinussatsen rätt.

Breakout 2

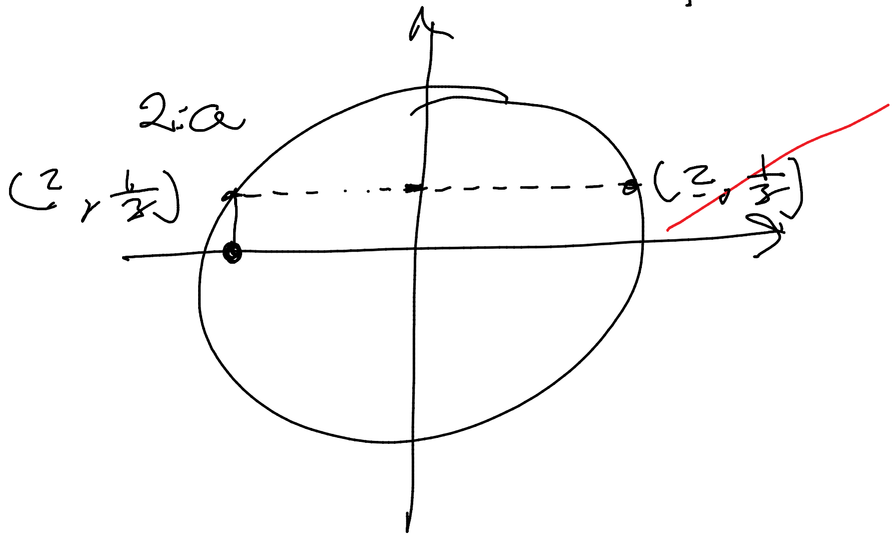
den 10 november 2020 01:05

Trigonometriska ettan!

Beräkna $\cos v$ (exakt!)

om $\sin v = \frac{1}{3}$ och $90^\circ < v < 180^\circ$

$\cos^2 v + \sin^2 v = 1$



Lösning

$$\cos^2 v + \sin^2 v = 1$$

$$\cos^2 v + \left(\frac{1}{3}\right)^2 = 1$$

$$\cos^2 v = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\cos v = \pm\sqrt{\frac{8}{9}} = (\pm)\ \frac{2\sqrt{2}}{3}$$

Svar: $\cos v = -\frac{2\sqrt{2}}{3}$