Agenda

den 12 november 2020 22:14

- Invers till trigonometriska funktioner

$$\begin{cases} \arcsin x \\ \in [-\frac{\pi}{2}, \frac{\pi}{2}] \\ -1 \le x \le 1 \end{cases}, \begin{cases} \arccos x \\ \in [0, \pi] \\ -1 \le x \le 1 \end{cases}, \begin{cases} \arctan x \\ \in (-\frac{\pi}{2}, \frac{\pi}{2}) \\ x \in \mathbb{R} \end{cases}, \begin{cases} \operatorname{arccot} x \\ \in (0, \pi) \\ x \in \mathbb{R} \end{cases}$$

Den vinkel för vilken ... är x : $f(f^{-1}(x)) = x$

Ex: $\arcsin \frac{\sqrt{3}}{2} = \frac{\pi}{3}$; $\arccos \frac{-1}{2} = \frac{3\pi}{4}$; $\arctan(-1) = -\frac{\pi}{4}$; $\operatorname{arccot} \sqrt{3} = \frac{\pi}{6}$

$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sin \frac{\pi}{6} = \frac{1}{2}$

$\cot \frac{\pi}{6} = \ldots = \sqrt{3}$

- Mer om formler: Halva vinkeln

- Trigonometriska ekvationer

(- Varför radianer?)

Uppgift 1

den 17 november 2020 09:38

Beräkna

$$\sin\frac{\pi}{24} \qquad \left(\frac{\pi}{24}\text{ rad} = 7.5^\circ\right)$$

Lösning

$$\sin\frac{\pi}{24} = \left[\begin{array}{l} v = \frac{\pi}{12} \\ \Rightarrow \frac{\pi}{24} = \frac{v}{2} \end{array}\right] = \left[\begin{array}{l} \sin\frac{\pi}{24} > 0 \\ \frac{\pi}{24} \in \text{1:a kvadranten} \end{array}\right]$$

$$= \sqrt{\frac{1-\cos\frac{\pi}{12}}{2}}$$

Bestäm $\cos\frac{\pi}{12}$: $\left[\text{Tag } v = \frac{\pi}{6}\right]$

$$\cos\frac{\pi}{12} \overset{*}{=} \sqrt{\frac{1+\cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

Till sist

$$\sin\frac{\pi}{24} = \sqrt{\frac{1-\cos\frac{\pi}{12}}{2}} = \sqrt{\frac{1-\frac{\sqrt{2+\sqrt{3}}}{2}}{2}}$$

$$= \sqrt{\frac{2-\sqrt{2+\sqrt{3}}}{4}} = \underline{\underline{\frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{3}}}}}$$

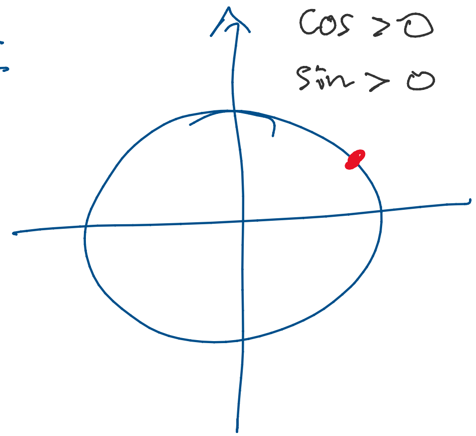
Formler för "halva vinkeln"

$$\cos\frac{v}{2} = \pm\sqrt{\frac{1+\cos v}{2}}$$

$$\sin\frac{v}{2} = \pm\sqrt{\frac{1-\cos v}{2}}$$

$$2\cdot\frac{\pi}{24} = \frac{\pi}{12}, \quad 2\cdot\frac{\pi}{12} = \frac{\pi}{6}$$

$$\boxed{\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}}$$



cos > 0

sin > 0

* Alternativ

$$\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) =$$

$$= \cos\frac{\pi}{4}\cdot\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\cdot\sin\frac{\pi}{6}$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2} =$$

$$= \frac{1}{2}\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right) \overset{\text{Se 1) \& 2)}}{=} \frac{1}{2}\sqrt{2+\sqrt{3}}\ !$$

1) $\left(\sqrt{2+\sqrt{3}}\right)^2 = 2+\sqrt{3}$

2) $\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right)^2 = \frac{3+2\sqrt{3}+1}{2} = \frac{4+2\sqrt{3}}{2} = \underline{\underline{2+\sqrt{3}}}$

Dvs $\sqrt{2+\sqrt{3}} = \frac{\sqrt{3}+1}{\sqrt{2}}$

Breakout 1

den 17 november 2020 00:53

Låt $f(x) = \arcsin\left(\frac{\sqrt{x-1}}{2}\right)$

och $g(x) = \arccos\left(-\frac{\sqrt{x+1}}{2}\right)$

Bestäm

a) $f(1)$, $f(3)$, $f(4)$

b) $g(-1)$, $g(0)$, $g(3)$

c) Definitionsmängderna

d) Värdemängderna

Lösning

a) $f(1) = \arcsin\frac{\sqrt{1-1}}{2} = \arcsin 0 = 0 \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$f(3) = \arcsin\frac{\sqrt{3-1}}{2} = \arcsin\frac{\sqrt{2}}{2} = \frac{\pi}{4}$

$f(4) = \ldots$

Kommentarer

$\frac{\sqrt{x-1}}{2} \geq 0 \Rightarrow f(x) \geq 0$

$V_{\arcsin} = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Rightarrow -\frac{\pi}{2} \leq f(x) \leq \frac{\pi}{2}$

$\Rightarrow 0 \leq f(x) \leq \frac{\pi}{2}$

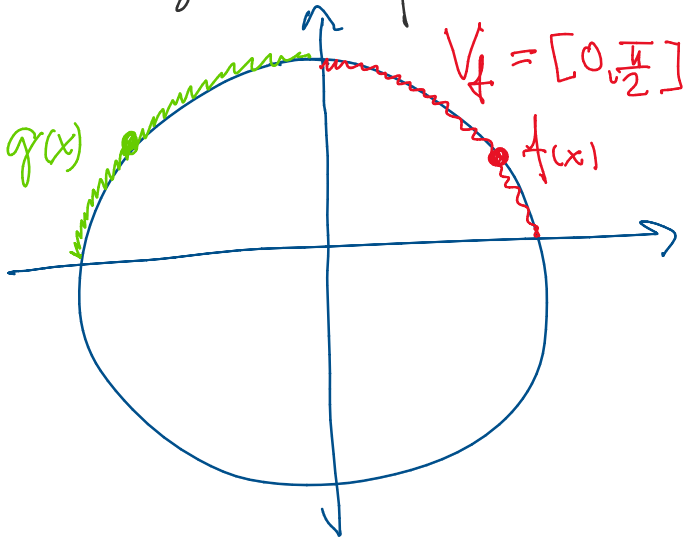
$-\frac{\sqrt{x+1}}{2} \leq 0 \Rightarrow g(x) \geq \frac{\pi}{2}$

$V_{\arccos} = [0, \pi] \Rightarrow 0 \leq g(x) \leq \pi$

$\Rightarrow \frac{\pi}{2} \leq g(x) \leq \pi$

$V_f = \left[0, \frac{\pi}{2}\right]$

$V_g = \left[\frac{\pi}{2}, \pi\right]$



b) $g(-1) = \arccos\left(-\frac{\sqrt{x+1}}{2}\right)\Big|_{x=-1} = \arccos\left(-\frac{\sqrt{0}}{2}\right)$

$= \arccos(0) = \pi/2$

$g(0) = \ldots \quad , \quad g(3) = \ldots$

c) Bestäm D_f och D_g

D_f: $\sqrt{x-1}$ är definierad för $\underline{x \geq 1}$

Eftersom $D_{\arcsin} = [-1, 1]$

måste dessutom $\frac{\sqrt{x-1}}{2} \leq 1$

Dvs

$\sqrt{x-1} \leq 2 \Leftrightarrow x-1 \leq 4 \Leftrightarrow \underline{x \leq 5}$

Alltså är $D_f = [1, 5]$

D_g: $\sqrt{x+1}$ är definierad för $\underline{x \geq -1}$

Eftersom $D_{\arccos} = [-1, 1]$ är

$-1 \leq -\frac{\sqrt{x+1}}{2}$

Dvs

$\sqrt{x+1} \leq 2 \Leftrightarrow x+1 \leq 4 \Leftrightarrow \underline{x \leq 3}$.

Alltså är

$D_g = [-1, 3]$

d) Se figur.

Uppgift 2

den 17 november 2020 00:54

Lös ekvationen

$$\sin 2x = \cos x$$

på intervallet $[0, 2\pi]$

Lösning

$$\sin 2x = 2 \sin x \cdot \cos x$$

Dvs

$$2 \sin x \cdot \cos x = \cos x$$

$$2 \sin x \cdot \cos x - \cos x = 0$$

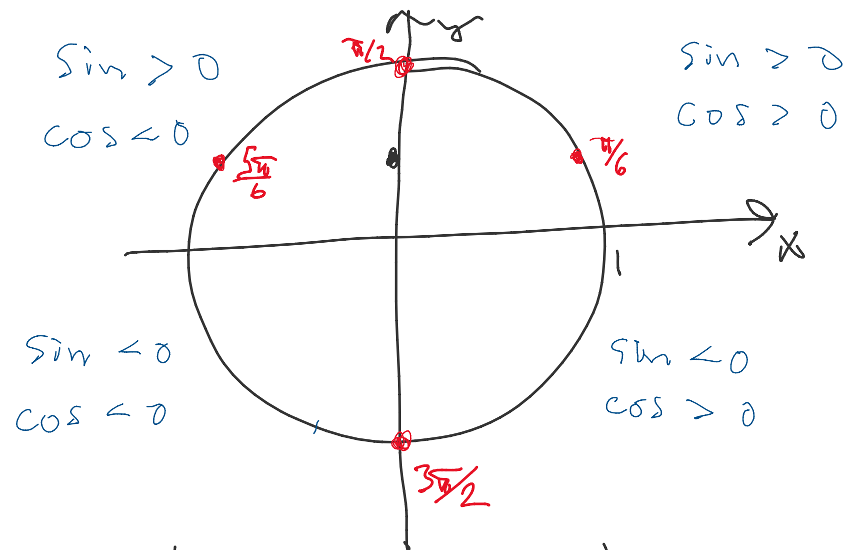
$$(2 \sin x - 1) \cdot \cos x = 0$$

Dvs

1) $2 \sin x - 1 = 0$

$\sin x = \frac{1}{2}$

2) $\cos x = 0$



1) $\sin x = \frac{1}{2} = \sin \frac{\pi}{6}$

$$\begin{cases} x = \frac{\pi}{6} + n \cdot 2\pi \\ (n \text{ heltal}) \\ x = \pi - \frac{\pi}{6} + n \cdot 2\pi \\ \quad = \frac{5\pi}{6} + n \cdot 2\pi \end{cases}$$

2) $\cos x = 0 = \cos \frac{\pi}{2}$

$$\begin{cases} x = \frac{\pi}{2} + n 2\pi \\ \text{eller} \\ x = -\frac{\pi}{2} + n \cdot 2\pi \end{cases}$$

med $n = 1$ får vi

$$x = -\frac{\pi}{2} + 2\pi = \frac{3\pi}{2} \in [0, 2\pi]$$

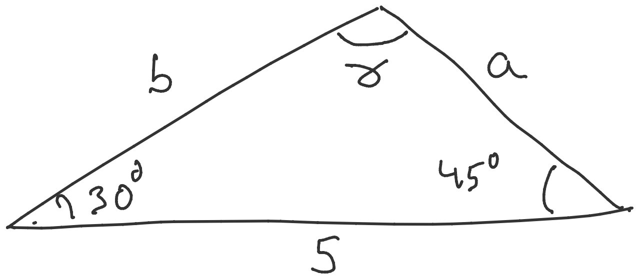
Svar $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ eller $\frac{3\pi}{2}$

Hemuppgift

den 17 november 2020 00:53

Solvera triangeln

$\gamma + 30^\circ + 45^\circ = 180^\circ$



Lösning: Enligt sinussatsen är

$$\frac{a}{\sin 30^\circ} = \frac{b}{\sin 45^\circ} = \frac{5}{\sin \gamma}$$

Men $\gamma = 180^\circ - 30^\circ - 45^\circ = 105^\circ$

$= 60^\circ + 45^\circ$

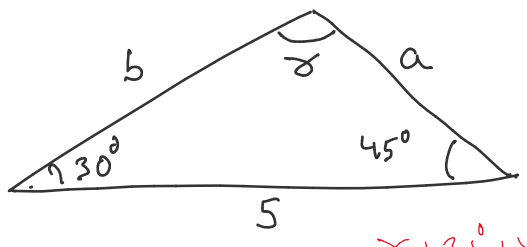
Bestäm $\sin \gamma$ (exakt!) och sedan a och b.

⋮

Lösning hemuppgift

den 17 november 2020 01:04

Solvera triangeln



$\gamma + 30° + 45° = 180°$

Lösning: Enligt sinussatsen är

$$\frac{a}{\sin 30°} = \frac{b}{\sin 45°} = \frac{5}{\sin \gamma}$$

Men $\gamma = 180° - 30° - 45° = \underline{\underline{105°}}$

$= 60° + 45°$

Bestäm $\sin\gamma$ (exakt!)

och sedan a och b:

$$\sin(105°) = \sin(60° + 45°) = \sin 60° \cdot \cos 45° + \sin 45° \cos 60°$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3} + 1}{2\sqrt{2}} = \ldots = \frac{\sqrt{6} + \sqrt{2}}{4}$$

Då är

$$a = \sin 30° \cdot \frac{5}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{1}{2} \cdot \frac{5 \cdot 4}{\sqrt{6}+\sqrt{2}} = \frac{5 \cdot 2}{\sqrt{6}+\sqrt{2}} \cdot \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}} = \frac{10(\sqrt{6}-\sqrt{2})}{6-2}$$

$$= \underline{\underline{\frac{5(\sqrt{6}-\sqrt{2})}{2}}}$$

$$b = \sin 45° \cdot \frac{5}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{1}{\sqrt{2}} \cdot \frac{5 \cdot 4}{\sqrt{6}+\sqrt{2}} = \frac{\sqrt{2}}{2} \cdot 5 \cdot \underbrace{(\sqrt{6}-\sqrt{2})}_{= \sqrt{2}(\sqrt{3}-1)}$$

$$= \underline{5 \cdot (\sqrt{3} - 1)}$$

Kommentar/ Liten kontroll

$a \approx 2.6$ står mot minsta vinkeln

$b \approx 3.7$

$c = 5$ står mot största vinkeln

och $a < b < c$! Bra!